We are now going to examine, for the degree $n = n_0 = 4$, the equations in [P&S, pp. 80-81] for the computation of the deviation points z_j of T-polynomials on two intervals, and will then specialize to $Z_{4,s}$. According to [P&S, Formula (7.1)] one has to consider the intervals

(1) $[-1, \min\{A, B(A)\}] \cup [\max\{A, B(A)\}, 1]$ and $[-1, CC - \sqrt{DT}] \cup [CC + \sqrt{DT}, 1]$

where

(2) $CC = CC(A) = \frac{B(A)+A}{2}$ and $DT = DT(A) = \frac{(B(A)-A)^2}{4}$.

Here we have set $A, B(A), CC, DT$ in place of the variables $\alpha \in (-1,1), b^{(m)}(\alpha) \in (-1,1), c, \tilde{d}$ as used in [P&S], in order to avoid notational confusion.

Key is the system of equations as given in [P&S, Formula (7.3)] (we correct here the second upper index of summation to $n-2$): $For\ k = 1, 2, \ldots, n-1\ there\ holds$

(3) $2\sum_{j=1}^{m}(-1)^j z_j^k - 2\sum_{j=m+1}^{n-2}(-1)^j z_j^k + (-1)^k + (-1)^n + (-1)^{m+1}\left(\left(CC - \sqrt{DT}\right)^k + \left(CC + \sqrt{DT}\right)^k\right) = 0$

Due to exploitation of Gröbner basis, it is shown that it suffices to consider the special case $m = 0$, so that (3)7.4) reduces, if $n = 4$, to

(4) $-2\left(-z_1^k + z_2^k\right) + (-1)^k + 1 - \left(\left(CC - \sqrt{DT}\right)^k + \left(CC + \sqrt{DT}\right)^k\right) = 0,\ for\ k = 1, 2, 3.$

The assumption $n = 4$ and $m = 0$ implies, see [P&S, pp. 78-79], that for a given $A \in (-1,1)$ there is a $B(A) \in (-1,1)$ such that there exists a T-polynomial on $[-1, \min\{A, B(A)\}] \cup [\max\{A, B(A)\}, 1]$ with $m = 0$ deviation points in $(-1, \min\{A, B(A)\})$. Consequently, both of the two inner deviation points of the T-polynomial must be situated in $(\max\{A, B(A)\}, 1)$. Therefore, the goal is to determine $B(A)$. To this end, we deploy the two identities in (2) and, furthermore, the first equation given in [P&S, p. 80], for $n = 4$, i.e., $2(1 + DT)CC^2 - (1 - DT)^2 = 0$. With these three equations we execute, with $Mathematica$,

GroebnerBasis[{$(B[A] + A)/2 - CC, (B[A] - A)^2 - 4DT, 2(1 + DT)CC^2 - (1 - DT)^2$}, {$A, B[A]$}, {$DT, CC$}].

We so get the equation $-16 + 16A^2 + A^4 + 4A^3B[A] + 16B[A]^2 - 10A^2B[A]^2 + 4AB[A]^3 + B[A]^4 = 0$. Choosing e.g. $A = -19/35$, it turns to $-\frac{16804079}{1500625} - \frac{27436B[A]}{42875} + \frac{3198B[A]^2}{245} - \frac{76B[A]^3}{35} + B[A]^4$.
Among the two real zeros of this equation in the interval $(-1,1)$ we choose that one which is least, and this is $B[A] = B(A) = -29/35$. Hence there exists a T-polynomial on $[-1, -29/35] \cup [-19/35, 1]$. From (2) we deduce $CC = -24/35$ and $DT = 1/49$, and we know that for the chosen $A = -19/35$ the two inner deviation points of the T-polynomial must be situated in $(-19/35, 1)$.
In order to determine them we deploy the second respectively third equation (in the variable z) given in [P&S, p. 80] for $n = 4$, i.e.,
$2CCz - 2CC^2 + 1 - DT = 2(-24/35)z - 2(-24/35)^2 + 1 - \frac{1}{49} = 0$, yielding $z = z_1 = 1/35$, respectively

$2CCz + 1 - DT = 2(-24/35)z + 1 - \frac{1}{49} = 0$, yielding $z = z_2 = 5/7$.
It is readily verified that with $CC = -24/35$, $DT = 1/49$, $z_1 = 1/35$ and $z_2 = 5/7$ the three equations in (4) are indeed satisfied.
Finally, we reconstruct the Zolotarev polynomial on $\boldsymbol{I} \cup [\alpha, \beta]$ which corresponds to the normed quartic T-polynomial (with fixed $A = -19/35$) on $[-1, -29/35] \cup [-19/35, 1]$, call it $t4A$:

EXAMPLE. Let $t4A$ be of form $t4A(x) = \sum_{i=0}^{4} t_i x^i$. Exploiting its interpolatory conditions $t4A(-1) = 1$, $t4A(-29/35) = -1$, $t4A(-19/35) = -1$, $t4A(1/35) = 1$, $t4A(5/7) = -1$, $t4A(1) = 1$ we get

(5) $t4A(x) = \frac{6863}{6912} + \frac{1715x}{3456} - \frac{833x^2}{96} - \frac{1715x^3}{3456} + \frac{60025x^4}{6912}$, see the left Figure below.

Knowing that its two deviation points are situated in $(-19/35, 1)$, we have to consider $t4A(-x)$ in order to be compliant with Theorem 1.1 in [11]. Its graph has the shape of a compressed normed quartic Zolotarev polynomial on $\boldsymbol{I}$. Decompressing it by means of the linear transformation $x \to (-8 + 27x)/35$ yields the normed Zolotarev polynomial $t4A((8 - 27x)/35)$ on $\boldsymbol{I} \cup [\alpha, \beta]$ with $\alpha = \alpha_0 = 37/27$ and $\beta = \beta_0 = 43/27$. After division by the leading coefficient, $19683/6400$, we obtain the monic quartic Zolotarev polynomial which corresponds to the normed T-polynomial $t4A$ above, see the right Figure below:

(6) $Z_{4,s_0} = \frac{53}{243} + \frac{15470x}{19683} - \frac{296x^2}{243} - \frac{10x^3}{9} + x^4$ with $s_0 = \frac{5}{18}$ and equioscillation points $\frac{-17}{27}, \frac{7}{27}$. □