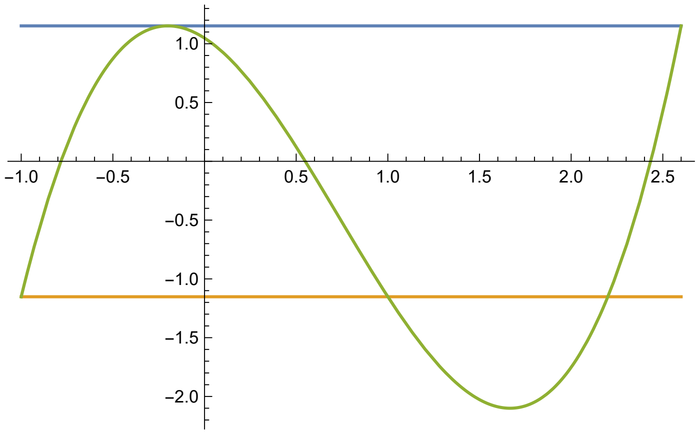
(* Consider the cubic proper Zolotarev polynomial $Z_{n=3,s=11/15}$ on [-1,1] U [α_0=11/5,β_0=13/5] *)

$$Z_{3,11/15}[x\_] = \frac{131}{125} + (-x) + \left(-\frac{11}{5}\right) x^2 + x^3$$

(* there holds γ_0=5/3 and the (least) deviation from zero is $L_{s=11/15}$=144/125 *)

In[∘]:= Plot$\left[\left\{144/125, -144/125, \frac{131}{125} + (-x) + \left(-\frac{11}{5}\right) x^2 + x^3\right\}, \{x, -1, 13/5\}\right]$
stelle Funktion graphisch dar

Out[∘]=



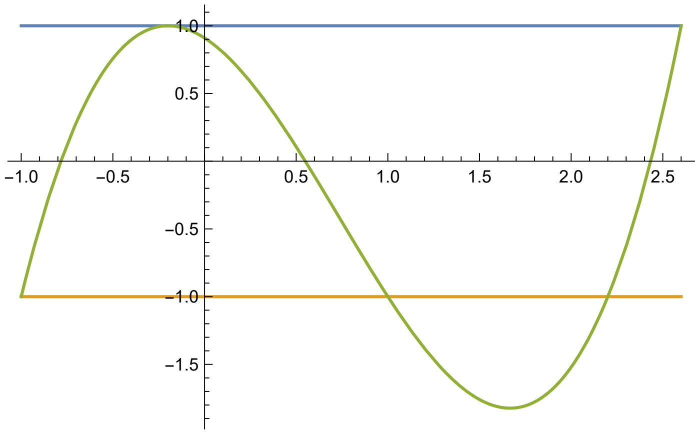
(* Consider the scaled Zolotarev polynomial y(x) = $Z_{3,11/15}$(x)/$L_{11/15}$ with uniform norm equal to 1 on [-1,1] U [11/5,13/5] *)

In[∘]:= $\left(\frac{131}{125} + (-x) + \left(-\frac{11}{5}\right) x^2 + x^3\right) \Big/ (144/125)$ // Expand
multipliziere aus

$$\frac{131}{144} - \frac{125\,x}{144} - \frac{275\,x^2}{144} + \frac{125\,x^3}{144}$$

In[∘]:= Plot$\left[\left\{1, -1, \left(\frac{131}{125} + (-x) + \left(-\frac{11}{5}\right) x^2 + x^3\right) \Big/ (144/125)\right\}, \{x, -1, 13/5\}\right]$
stelle Funktion graphisch dar

Out[∘]=



In[∘]:= $y[x\_] := \left(\frac{131}{125} + (-x) + \left(-\frac{11}{5}\right) x^2 + x^3\right) \Big/ (144/125)$

In[ ]:= y'[x]

Out[ ]=

$$\frac{125}{144}\left(-1-\frac{22\,x}{5}+3\,x^2\right)$$

In[ ]:= y''[x]

Out[ ]=

$$\frac{125}{144}\left(-\frac{22}{5}+6\,x\right)$$

In[ ]:= G0[x_] := 9 (x - 5 / 3) ^3

In[ ]:= $G1[x\_] := \left(-\frac{13}{5}+x\right)\left(-\frac{11}{5}+x\right)-\left(-\frac{12}{5}+x\right)\left(-\frac{5}{3}+x\right)$

In[ ]:= $G2[x\_] := \left(-\frac{13}{5}+x\right)\left(-\frac{11}{5}+x\right)\left(-\frac{5}{3}+x\right)$

In[ ]:= Expand[G2[x] * ((1 - x^2) * D[y[x], x, x] - x * D[y[x], x]) -
multipliziere aus | leite ab | leite ab

(1 - x^2) * G1[x] * D[y[x], x] + G0[x] * y[x]]
leite ab

Out[ ]=

0

(* above is the second order linear differential equation,
Formula (63), by Vlcek & Unbehauen, Reference [56] in [6] *)

(* substituting here
$\left(y[x]=f[w]=\sum_{m=0}^{n} b(m)\,w^m\right)$ and analogously f'[w] and f''[w] yields after comparison
of coefficients the recursive Formulae as given in Table IV of Vlcek &
Tabelle

Unbehauen (Reference [56] in [6]) *)

(* We identify the parameters in that Table IV as follows: *)
Tabelle

In[ ]:= n = 3 (* n = 3 = 1+2 = p+q *)
xp = 13 / 5 (* = w_p = β_0 = 13/5 *)
xs = 11 / 5 (* = w_s = α_0 = 11/5 *)
xq = (xp + xs) / 2 (* $w_q=\left(w_p+w_s\right)/2$ = 12/5 *)
xm = 5 / 3 (* = w_m = γ_0 = 5/3 *)
β[3] = 1; β[4] = 0; β[5] = 0; β[6] = 0; β[7] = 0
Table[d1 = (m + 2) (m + 1) xp xs xm;
Tabelle
d2 = - (m + 1) (m - 1) xp xs - (m + 1) (2 m + 1) xm xq;
d3 = xm (9 xm^2 - m^2 xp xs) + m^2 (xm - xq) + 3 m (m - 1) xq;
d4 = (m - 1) (m - 2) (xp xs - xm xq - 1) - 3 xm (9 xm - (m - 1) ^2 xq);
d5 = (2 m - 5) (m - 2) (xm - xq) + 3 (xm) (9 - (m - 2) ^2);
d6 = 9 - (m - 3) ^2;
β[m - 3] = 1 / d6 (d1 β[m + 3 - 1] + d2 β[m + 3 - 2] +
d3 β[m + 3 - 3] + d4 β[m + 3 - 4] + d5 β[m + 3 - 5]), {m, 5, 3, -1}]

Out[ ]=

$$\left\{-\frac{11}{5},\,-1,\,\frac{131}{125}\right\}$$

```
In[∘]:= β[0]
Out[∘]=
```

$$\frac{131}{125}$$

```
In[∘]:= s[n_] := Sum[β[m], {m, 0, n}]
                 summiere
In[∘]:= Table[-β[m] / s[3], {m, 0, 3}]
        Tabelle
```

$$\left\{\frac{131}{144}, -\frac{125}{144}, -\frac{275}{144}, \frac{125}{144}\right\}$$

```
(* We have retrieved along Table IV by Vlcek & Unbehauen (1999) the list
                           Tabelle
  of coefficients of the above scaled proper Zolotarev polynomial y(x) =
 Z3,11/15(x)/L11/15 with uniform norm equal to 1 on [-1,1] U [11/5,13/5] *)
```