

(* Goal: Algebraic solution of ZFP if n=13 and s=8 and exemplarily one coefficient, $a_{11,13}^*(8)$, is explicitly expressed as a root object (see Appendix 3 in[6]) *)

(* The proper monic Zolotarev polynomial of degree n=13 and with parameter s=8 can be generated by *)

(* $Z_{13,8}(x) = \sum_{k=0}^{13} a_{k,13}^*(8) x^k = (R_{13,\alpha,\beta}(x)) / .s \rightarrow 8 / .\alpha \rightarrow \text{Root}[F_{m(13),8}(\alpha), 12] / .\beta \rightarrow \text{Root}[G_{m(13),8}(\beta), 12]$ *)

(* The first insertion, $(R_{13,\alpha,\beta}(x)) / .s \rightarrow 8$, will yield the following polynomial of degree n=13, still depending on α and β : *)

Out[*]=

$$\frac{1}{326998425600} \left(62576334451615886443110400 - 34007836262400x^{12} + 326998425600x^{13} - 996249330624025374489600\beta + 62141734629329271418644480\beta^2 - 22986997810298577806915840\beta^3 + 3688957136404219067621376\beta^4 - 331950413540972413444608\beta^5 + 18200173556098050676224\beta^6 - 614599187687306011008\beta^7 + 12095089771026072096\beta^8 - 115416141536766356\beta^9 + 336273189825144\beta^{10} - 9836312711823\beta^{11} + 314773225624\beta^{12} - 2435333421\beta^{13} + 136247800\alpha^{11}(1 + \beta^2) - 265686220800x^{11}(-6652 + \alpha^2 - 2\alpha(-16 + \beta) + 32\beta + \beta^2) - 1859\alpha^{10}(-92607016 + 3945361\beta - 92607016\beta^2 + 4127359\beta^3) - 18590\alpha^9(1729166512 - 69004872\beta + 1729037319\beta^2 - 113245736\beta^3 + 1601265\beta^4) + 1859\alpha^8(802453575264 + 9624336036\beta + 799382185992\beta^2 - 48431124689\beta^3 + 127051368\beta^4 + 18403453\beta^5) + 1144\alpha^7(34566367645896 - 11172629763360\beta + 35209800080232\beta^2 - 9243629149352\beta^3 + 695216527025\beta^4 - 19685642600\beta^5 + 180738803\beta^6) + 26\alpha^6(-218594611823579904 + 30648052157608320\beta - 21950850485353632\beta^2 + 41979957948401704\beta^3 - 2968691552221032\beta^4 + 94226651202129\beta^5 - 1266983946536\beta^6 + 4813669719\beta^7) - 52\alpha^5(-3480994658859608064 + 296135196154210560\beta - 3466797876308809008\beta^2 + 650182044814799552\beta^3 - 44336698551646560\beta^4 + 1222298893988792\beta^5 - 4850847730001\beta^6 - 33225222184\beta^7 + 3940230937\beta^8) - 26\alpha^4(46872261757755239424 + 10197640332845043072\beta + 4431716324404621440\beta^2 - 1938972178799665216\beta^3 - 950328729508421984\beta^4 + 127068624085091372\beta^5 - 6728020140638872\beta^6 + 174682092011531\beta^7 - 2140711949816\beta^8 + 9328839641\beta^9) - 208\alpha^3(288823813863953208000 - 85886734081469093376\beta + 296717729340879990240\beta^2 - 104252380620586848640\beta^3 + 15271572364180768860\beta^4 - 1194901437509343600\beta^5 + 54122378862931884\beta^6 - 1429194574875684\beta^7 + 20769937307502\beta^8 - 140103652964\beta^9 + 248843331\beta^{10}) + 13\alpha^2(125962242486438941378560 - 26159956078675821400320\beta + 127185075810452471597056\beta^2 - 42540334241510846004480\beta^3 + 6117687042232832011776\beta^4 - 485866838404328020992\beta^5 + 23099478010313732736\beta^6 - 664369029625947312\beta^7 + 11000893273729656\beta^8 - 89354841596867\beta^9 + 171547605240\beta^{10} + 906183619\beta^{11}) - 26\alpha(637735916165267674726400 - 113654517487276157265920\beta + 638374748751816764468480\beta^2 - 222752977624431793031168\beta^3 + 33632479960704584454144\beta^4 - 2833231215767235036672\beta^5 +$$

$$\begin{aligned}
& 144\,632\,137\,120\,445\,368\,800\,\beta^6 - 4\,527\,635\,647\,210\,062\,720\,\beta^7 + 82\,615\,571\,694\,982\,320\,\beta^8 - \\
& 735\,032\,983\,515\,160\,\beta^9 + 1\,055\,313\,862\,141\,\beta^{10} + 12\,459\,863\,240\,\beta^{11} + 95\,061\,835\,\beta^{12} - \\
& 177\,124\,147\,200\,x^{10} \left(345\,520 + \alpha^3 - 4992\,\beta - 132\,\beta^2 + \beta^3 - \alpha^2 (132 + \beta) - \alpha (4992 - 328\,\beta + \beta^2) \right) - \\
& 5\,535\,129\,600\,x^9 \\
& \left(-287\,033\,584 + 3\,\alpha^4 + 8\,302\,080\,\beta + 159\,600\,\beta^2 - 4096\,\beta^3 + 3\,\beta^4 + \alpha^3 (-4096 + 60\,\beta) + \right. \\
& \left. \alpha^2 (159\,600 + 4864\,\beta - 126\,\beta^2) + \alpha (8\,302\,080 - 612\,064\,\beta + 4864\,\beta^2 + 60\,\beta^3) \right) + \\
& 2\,214\,051\,840\,x^8 \left(-14\,898\,100\,160 + 23\,\alpha^5 + 718\,781\,440\,\beta + 6\,892\,544\,\beta^2 - 582\,164\,\beta^3 + \right. \\
& \left. 3580\,\beta^4 + 23\,\beta^5 - 5\,\alpha^4 (-716 + 33\,\beta) + 2\,\alpha^3 (-291\,082 + 6520\,\beta + 71\,\beta^2) + \right. \\
& \left. 2\,\alpha^2 (3\,446\,272 + 457\,482\,\beta - 18\,156\,\beta^2 + 71\,\beta^3) + \right. \\
& \left. \alpha (718\,781\,440 - 62\,225\,664\,\beta + 914\,964\,\beta^2 + 13\,040\,\beta^3 - 165\,\beta^4) \right) + \\
& 46\,126\,080\,x^7 \left(12\,369\,377\,528\,960 + 1025\,\alpha^6 - 895\,793\,856\,000\,\beta + 2\,209\,519\,632\,\beta^2 + \right. \\
& \left. 946\,991\,488\,\beta^3 - 15\,773\,280\,\beta^4 - 38\,976\,\beta^5 + 1025\,\beta^6 - 2\,\alpha^5 (19\,488 + 1685\,\beta) - \right. \\
& \left. 5\,\alpha^4 (3\,154\,656 - 184\,256\,\beta + 1069\,\beta^2) + 4\,\alpha^3 (236\,747\,872 - 5\,735\,940\,\beta - 235\,936\,\beta^2 + 3845\,\beta^3) + \right. \\
& \left. \alpha^2 (2\,209\,519\,632 - 2\,192\,725\,888\,\beta + 106\,613\,984\,\beta^2 - 943\,744\,\beta^3 - 5345\,\beta^4) - \right. \\
& \left. 2\,\alpha (447\,896\,928\,000 - 46\,695\,813\,104\,\beta + \right. \\
& \left. 1\,096\,362\,944\,\beta^2 + 11\,471\,880\,\beta^3 - 460\,640\,\beta^4 + 1685\,\beta^5) \right) + \\
& 6\,589\,440\,x^6 \left(-1\,283\,553\,168\,732\,160 + 3921\,\alpha^7 + 130\,210\,189\,025\,280\,\beta - 2\,206\,457\,878\,656\,\beta^2 - \right. \\
& \left. 145\,640\,995\,088\,\beta^3 + 5\,229\,723\,072\,\beta^4 - 23\,570\,400\,\beta^5 - 632\,968\,\beta^6 + \right. \\
& \left. 3921\,\beta^7 + 7\,\alpha^6 (-90\,424 + 69\,\beta) - 7\,\alpha^5 (3\,367\,200 - 552\,176\,\beta + 8505\,\beta^2) + \right. \\
& \left. \alpha^4 (5\,229\,723\,072 - 357\,096\,416\,\beta + 2\,976\,904\,\beta^2 + 55\,131\,\beta^3) + \right. \\
& \left. \alpha^3 (-145\,640\,995\,088 + 1\,878\,846\,336\,\beta + 474\,596\,288\,\beta^2 - 13\,155\,616\,\beta^3 + 55\,131\,\beta^4) + \right. \\
& \left. \alpha^2 (-2\,206\,457\,878\,656 + 568\,644\,544\,784\,\beta - 32\,200\,901\,248\,\beta^2 + 474\,596\,288\,\beta^3 + \right. \\
& \left. 2\,976\,904\,\beta^4 - 59\,535\,\beta^5) + \alpha (130\,210\,189\,025\,280 - 16\,508\,585\,579\,264\,\beta + \right. \\
& \left. 568\,644\,544\,784\,\beta^2 + 1\,878\,846\,336\,\beta^3 - 357\,096\,416\,\beta^4 + 3\,865\,232\,\beta^5 + 483\,\beta^6) \right) + \\
& 823\,680\,x^5 \left(133\,186\,208\,367\,529\,600 + 12\,467\,\alpha^8 - 18\,022\,358\,283\,345\,920\,\beta + 610\,009\,635\,687\,680\,\beta^2 + \right. \\
& \left. 15\,388\,282\,147\,328\,\beta^3 - 1\,272\,668\,519\,080\,\beta^4 + 18\,793\,643\,008\,\beta^5 + 127\,662\,220\,\beta^6 - 3\,876\,704\,\beta^7 + \right. \\
& \left. 12\,467\,\beta^8 + 4\,\alpha^7 (-969\,176 + 16\,387\,\beta) - 28\,\alpha^6 (-4\,559\,365 - 304\,392\,\beta + 10\,873\,\beta^2) - \right. \\
& \left. 28\,\alpha^5 (-671\,201\,536 + 79\,540\,434\,\beta - 1\,936\,344\,\beta^2 + 4861\,\beta^3) + \right. \\
& \left. 2\,\alpha^4 (-636\,334\,259\,540 + 48\,658\,223\,104\,\beta - 304\,709\,062\,\beta^2 - 30\,722\,192\,\beta^3 + 362\,537\,\beta^4) - \right. \\
& \left. 4\,\alpha^3 (-3\,847\,070\,536\,832 - 202\,286\,690\,920\,\beta + 47\,205\,620\,480\,\beta^2 - 1\,702\,366\,460\,\beta^3 + \right. \\
& \left. 15\,361\,096\,\beta^4 + 34\,027\,\beta^5) - 4\,\alpha^2 (-152\,502\,408\,921\,920 + 33\,151\,984\,479\,360\,\beta - \right. \\
& \left. 2\,179\,896\,039\,556\,\beta^2 + 47\,205\,620\,480\,\beta^3 + 152\,354\,531\,\beta^4 - 13\,554\,408\,\beta^5 + 76\,111\,\beta^6) + \right. \\
& \left. 4\,\alpha (-4\,505\,589\,570\,836\,480 + 694\,573\,823\,973\,760\,\beta - 33\,151\,984\,479\,360\,\beta^2 + \right. \\
& \left. 202\,286\,690\,920\,\beta^3 + 24\,329\,111\,552\,\beta^4 - 556\,783\,038\,\beta^5 + 2\,130\,744\,\beta^6 + 16\,387\,\beta^7) \right) + \\
& 91\,520\,x^4 \left(-13\,820\,520\,301\,623\,034\,880 + 31\,837\,\alpha^9 + 2\,405\,156\,402\,368\,512\,000\,\beta - \right. \\
& \left. 127\,884\,033\,631\,577\,088\,\beta^2 - 239\,431\,917\,910\,272\,\beta^3 + 241\,557\,115\,060\,800\,\beta^4 - \right. \\
& \left. 7\,305\,490\,337\,328\,\beta^5 + 30\,518\,167\,008\,\beta^6 + 1\,594\,174\,104\,\beta^7 - 19\,336\,824\,\beta^8 + 31\,837\,\beta^9 + \right. \\
& \left. 39\,\alpha^8 (-495\,816 + 13\,415\,\beta) - 24\,\alpha^7 (-66\,423\,921 + 966\,316\,\beta + 39\,279\,\beta^2) - \right. \\
& \left. 8\,\alpha^6 (-3\,814\,770\,876 + 1\,180\,148\,625\,\beta - 48\,435\,324\,\beta^2 + 401\,234\,\beta^3) + \right. \\
& \left. 6\,\alpha^5 (-1\,217\,581\,722\,888 + 145\,850\,661\,344\,\beta - 4\,448\,864\,596\,\beta^2 + 9\,803\,824\,\beta^3 + 599\,591\,\beta^4) + \right. \\
& \left. 6\,\alpha^4 (40\,259\,519\,176\,800 - 3\,190\,229\,038\,952\,\beta - 26\,310\,079\,568\,\beta^2 + 6\,729\,221\,884\,\beta^3 - \right. \\
& \left. 141\,473\,656\,\beta^4 + 599\,591\,\beta^5) - 8\,\alpha^3 (29\,928\,989\,738\,784 + 61\,340\,646\,256\,416\,\beta - \right. \\
& \left. 8\,147\,465\,165\,652\,\beta^2 + 350\,457\,965\,008\,\beta^3 - 5\,046\,916\,413\,\beta^4 - 7\,352\,868\,\beta^5 + 401\,234\,\beta^6) - \right.
\end{aligned}$$

$$\begin{aligned}
& 24 \alpha^2 \left(5\,328\,501\,401\,315\,712 - 1\,184\,290\,331\,648\,608 \beta + 91\,152\,472\,108\,912 \beta^2 - \right. \\
& \quad \left. 2\,715\,821\,721\,884 \beta^3 + 6\,577\,519\,892 \beta^4 + 1\,112\,216\,149 \beta^5 - 16\,145\,108 \beta^6 + 39\,279 \beta^7 \right) + \\
& 3 \alpha \left(801\,718\,800\,789\,504\,000 - 149\,421\,136\,916\,916\,224 \beta + 9\,474\,322\,653\,188\,864 \beta^2 - \right. \\
& \quad \left. 163\,575\,056\,683\,776 \beta^3 - 6\,380\,458\,077\,904 \beta^4 + 291\,701\,322\,688 \beta^5 - \right. \\
& \quad \left. 3\,147\,063\,000 \beta^6 - 7\,730\,528 \beta^7 + 174\,395 \beta^8 \right) + \\
& 286 x^3 \left(45\,898\,622\,624\,379\,342\,003\,200 + 1\,182\,987 \alpha^{10} - 9\,985\,968\,415\,148\,372\,582\,400 \beta + \right. \\
& \quad \left. 748\,065\,471\,646\,220\,847\,360 \beta^2 - 13\,293\,380\,743\,098\,470\,400 \beta^3 - \right. \\
& \quad \left. 1\,114\,567\,221\,270\,396\,672 \beta^4 + 64\,713\,551\,525\,028\,864 \beta^5 - 1\,032\,977\,168\,870\,592 \beta^6 - \right. \\
& \quad \left. 7\,739\,928\,338\,688 \beta^7 + 377\,653\,480\,308 \beta^8 - 2\,875\,656\,160 \beta^9 + 1\,182\,987 \beta^{10} + \right. \\
& \quad \left. 130 \alpha^9 \left(-22\,120\,432 + 786\,861 \beta \right) + 39 \alpha^8 \left(9\,683\,422\,572 - 374\,412\,320 \beta + 152\,409 \beta^2 \right) - \right. \\
& \quad \left. 24 \alpha^7 \left(322\,497\,014\,112 + 29\,903\,776\,352 \beta - 2\,593\,249\,776 \beta^2 + 34\,993\,371 \beta^3 \right) - \right. \\
& \quad \left. 2 \alpha^6 \left(516\,488\,584\,435\,296 - 88\,579\,236\,991\,104 \beta + 4\,174\,625\,325\,768 \beta^2 - 57\,803\,846\,720 \beta^3 + \right. \right. \\
& \quad \left. \left. 16\,264\,269 \beta^4 \right) + 12 \alpha^5 \left(5\,392\,795\,960\,419\,072 - 692\,606\,154\,581\,248 \beta + \right. \right. \\
& \quad \left. \left. 23\,625\,562\,439\,744 \beta^2 + 90\,149\,131\,264 \beta^3 - 13\,859\,448\,112 \beta^4 + 127\,158\,429 \beta^5 \right) - \right. \\
& \quad \left. 6 \alpha^4 \left(185\,761\,203\,545\,066\,112 - 11\,415\,059\,624\,095\,232 \beta - 1\,075\,222\,173\,103\,904 \beta^2 + \right. \right. \\
& \quad \left. \left. 110\,119\,585\,341\,568 \beta^3 - 3\,118\,906\,010\,548 \beta^4 + 27\,718\,896\,224 \beta^5 + 5\,421\,423 \beta^6 \right) - \right. \\
& \quad \left. 8 \alpha^3 \left(1\,661\,672\,592\,887\,308\,800 - 682\,017\,712\,642\,939\,968 \beta + 81\,112\,499\,909\,428\,992 \beta^2 - \right. \right. \\
& \quad \left. \left. 4\,056\,218\,649\,028\,480 \beta^3 + 82\,589\,689\,006\,176 \beta^4 - 135\,223\,696\,896 \beta^5 - 14\,450\,961\,680 \beta^6 + \right. \right. \\
& \quad \left. \left. 104\,980\,113 \beta^7 \right) + 3 \alpha^2 \left(249\,355\,157\,215\,406\,949\,120 - 60\,623\,289\,254\,863\,278\,080 \beta + \right. \right. \\
& \quad \left. \left. 5\,496\,599\,233\,945\,353\,728 \beta^2 - 216\,299\,999\,758\,477\,312 \beta^3 + 2\,150\,444\,346\,207\,808 \beta^4 + \right. \right. \\
& \quad \left. \left. 94\,502\,249\,758\,976 \beta^5 - 2\,783\,083\,550\,512 \beta^6 + 20\,745\,998\,208 \beta^7 + 1\,981\,317 \beta^8 \right) + \right. \\
& \quad \left. 6 \alpha \left(-1\,664\,328\,069\,191\,395\,430\,400 + 371\,862\,812\,518\,489\,488\,640 \beta - 30\,311\,644\,627\,431\,639\,040 \right. \right. \\
& \quad \left. \left. \beta^2 + 909\,356\,950\,190\,586\,624 \beta^3 + 11\,415\,059\,624\,095\,232 \beta^4 - 1\,385\,212\,309\,162\,496 \beta^5 + \right. \right. \\
& \quad \left. \left. 29\,526\,412\,330\,368 \beta^6 - 119\,615\,105\,408 \beta^7 - 2\,433\,680\,080 \beta^8 + 17\,048\,655 \beta^9 \right) \right) - \\
& 22 x \left(601\,694\,592\,186\,994\,015\,308\,800 + 15\,378\,831 \alpha^{10} - 130\,494\,226\,737\,450\,772\,684\,800 \beta + \right. \\
& \quad \left. 9\,747\,694\,484\,561\,333\,064\,960 \beta^2 - 172\,235\,825\,845\,910\,261\,760 \beta^3 - \right. \\
& \quad \left. 14\,537\,055\,657\,514\,081\,536 \beta^4 + 841\,979\,722\,100\,954\,112 \beta^5 - 13\,423\,921\,372\,744\,896 \beta^6 - \right. \\
& \quad \left. 100\,764\,212\,200\,704 \beta^7 + 4\,909\,962\,008\,484 \beta^8 - 37\,383\,530\,080 \beta^9 + 15\,378\,831 \beta^{10} + \right. \\
& \quad \left. 1690 \alpha^9 \left(-22\,120\,432 + 786\,861 \beta \right) + 507 \alpha^8 \left(9\,684\,343\,212 - 374\,412\,320 \beta + 152\,409 \beta^2 \right) - \right. \\
& \quad \left. 312 \alpha^7 \left(322\,962\,218\,592 + 29\,895\,910\,592 \beta - 2\,593\,249\,776 \beta^2 + 34\,993\,371 \beta^3 \right) - \right. \\
& \quad \left. 26 \alpha^6 \left(516\,304\,668\,182\,496 - 88\,591\,510\,076\,544 \beta + 4\,175\,063\,725\,128 \beta^2 - 57\,803\,846\,720 \beta^3 + \right. \right. \\
& \quad \left. \left. 16\,264\,269 \beta^4 \right) + 156 \alpha^5 \left(5\,397\,305\,910\,903\,552 - 693\,140\,711\,590\,528 \beta + \right. \right. \\
& \quad \left. \left. 23\,638\,574\,671\,424 \beta^2 + 90\,116\,465\,344 \beta^3 - 13\,859\,448\,112 \beta^4 + 127\,158\,429 \beta^5 \right) - \right. \\
& \quad \left. 78 \alpha^4 \left(186\,372\,508\,429\,667\,712 - 11\,461\,796\,282\,281\,472 \beta - 1\,074\,929\,508\,730\,784 \beta^2 + \right. \right. \\
& \quad \left. \left. 110\,149\,078\,645\,888 \beta^3 - 3\,119\,254\,046\,068 \beta^4 + 27\,718\,896\,224 \beta^5 + 5\,421\,423 \beta^6 \right) - \right. \\
& \quad \left. 104 \alpha^3 \left(1\,656\,113\,710\,056\,829\,440 - 682\,308\,542\,786\,511\,168 \beta + 81\,180\,495\,028\,799\,232 \beta^2 - \right. \right. \\
& \quad \left. \left. 4\,058\,670\,366\,791\,680 \beta^3 + 82\,611\,808\,984\,416 \beta^4 - 135\,174\,698\,016 \beta^5 - 14\,450\,961\,680 \beta^6 + \right. \right. \\
& \quad \left. \left. 104\,980\,113 \beta^7 \right) + 39 \alpha^2 \left(249\,940\,884\,219\,521\,360\,640 - 60\,750\,710\,787\,586\,396\,160 \beta + \right. \right. \\
& \quad \left. \left. 5\,504\,975\,767\,117\,879\,808 \beta^2 - 216\,481\,320\,076\,797\,952 \beta^3 + 2\,149\,859\,017\,461\,568 \beta^4 + \right. \right. \\
& \quad \left. \left. 94\,554\,298\,685\,696 \beta^5 - 2\,783\,375\,816\,752 \beta^6 + 20\,745\,998\,208 \beta^7 + 1\,981\,317 \beta^8 \right) + 78 \alpha \right. \\
& \quad \left. \left(-1\,673\,002\,906\,890\,394\,521\,600 + 373\,198\,906\,602\,014\,874\,880 \beta - 30\,375\,355\,393\,793\,198\,080 \right. \right. \\
& \quad \left. \left. \beta^2 + 909\,744\,723\,715\,348\,224 \beta^3 + 11\,461\,796\,282\,281\,472 \beta^4 - 1\,386\,281\,423\,181\,056 \beta^5 + \right. \right. \\
& \quad \left. \left. 29\,530\,503\,358\,848 \beta^6 - 119\,583\,642\,368 \beta^7 - 2\,433\,680\,080 \beta^8 + 17\,048\,655 \beta^9 \right) \right) - \\
& 208 x^2 \left(595\,573\,672\,799\,206\,958\,464\,000 + 1\,310\,075 \alpha^{11} - 158\,371\,577\,510\,180\,810\,547\,200 \beta + \right.
\end{aligned}$$

$$\begin{aligned}
& 15\,688\,941\,508\,901\,330\,397\,440\,\beta^2 - 577\,757\,597\,876\,195\,936\,640\,\beta^3 - \\
& 11\,611\,614\,593\,077\,898\,496\,\beta^4 + 1\,737\,282\,167\,215\,930\,752\,\beta^5 - \\
& 54\,635\,245\,014\,837\,696\,\beta^6 + 380\,931\,604\,927\,896\,\beta^7 + 14\,335\,349\,455\,284\,\beta^8 - \\
& 309\,074\,505\,740\,\beta^9 + 1\,655\,350\,411\,\beta^{10} + 1\,310\,075\,\beta^{11} - \\
& 143\,\alpha^{10} \left(-11\,575\,877 + 504\,545\,\beta \right) - 715\,\alpha^9 \left(432\,272\,036 - 22\,781\,326\,\beta + 184\,009\,\beta^2 \right) + \\
& 143\,\alpha^8 \left(100\,247\,198\,988 - 2\,426\,976\,900\,\beta - 184\,021\,119\,\beta^2 + 4\,312\,625\,\beta^3 \right) + \\
& 22\,\alpha^7 \left(17\,315\,072\,951\,268 - 5\,102\,335\,385\,472\,\beta + 334\,652\,100\,816\,\beta^2 - 7\,114\,585\,036\,\beta^3 + \right. \\
& \quad \left. 36\,470\,305\,\beta^4 \right) + \alpha^6 \left(-54\,635\,245\,014\,837\,696 + 9\,077\,696\,769\,045\,384\,\beta - \right. \\
& \quad \left. 485\,337\,358\,915\,728\,\beta^2 + 8\,429\,123\,758\,400\,\beta^3 + 26\,405\,491\,046\,\beta^4 - 1\,216\,645\,790\,\beta^5 \right) - \\
& 2\,\alpha^5 \left(-868\,641\,083\,607\,965\,376 + 118\,368\,988\,705\,257\,984\,\beta - 3\,770\,922\,895\,557\,612\,\beta^2 - \right. \\
& \quad \left. 119\,038\,875\,023\,872\,\beta^3 + 8\,699\,576\,532\,396\,\beta^4 - 145\,918\,183\,662\,\beta^5 + 608\,322\,895\,\beta^6 \right) + \\
& 2\,\alpha^4 \left(-5\,805\,807\,296\,538\,949\,248 - 510\,135\,203\,950\,066\,752\,\beta + 218\,847\,299\,217\,840\,096\,\beta^2 - \right. \\
& \quad \left. 18\,183\,439\,100\,725\,372\,\beta^3 + 627\,656\,437\,669\,852\,\beta^4 - \right. \\
& \quad \left. 8\,699\,576\,532\,396\,\beta^5 + 13\,202\,745\,523\,\beta^6 + 401\,173\,355\,\beta^7 \right) + \\
& \alpha^3 \left(-577\,757\,597\,876\,195\,936\,640 + 189\,774\,859\,559\,396\,669\,952\,\beta - \right. \\
& \quad \left. 23\,154\,560\,662\,110\,651\,648\,\beta^2 + 1\,343\,254\,576\,526\,679\,040\,\beta^3 - 36\,366\,878\,201\,450\,744\,\beta^4 + \right. \\
& \quad \left. 238\,077\,750\,047\,744\,\beta^5 + 8\,429\,123\,758\,400\,\beta^6 - 156\,520\,870\,792\,\beta^7 + 616\,705\,375\,\beta^8 \right) + \alpha^2 \\
& \left(15\,688\,941\,508\,901\,330\,397\,440 - 4\,288\,476\,526\,938\,447\,672\,960\,\beta + 458\,826\,058\,787\,478\,431\,232\, \right. \\
& \quad \left. \beta^2 - 23\,154\,560\,662\,110\,651\,648\,\beta^3 + 437\,694\,598\,435\,680\,192\,\beta^4 + 7\,541\,845\,791\,115\,224\,\beta^5 - \right. \\
& \quad \left. 485\,337\,358\,915\,728\,\beta^6 + 7\,362\,346\,217\,952\,\beta^7 - 26\,315\,020\,017\,\beta^8 - 131\,566\,435\,\beta^9 \right) - \\
& \alpha \left(158\,371\,577\,510\,180\,810\,547\,200 - 42\,018\,143\,798\,304\,166\,530\,560\,\beta + \right. \\
& \quad \left. 4\,288\,476\,526\,938\,447\,672\,960\,\beta^2 - 189\,774\,859\,559\,396\,669\,952\,\beta^3 + \right. \\
& \quad \left. 1\,020\,270\,407\,900\,133\,504\,\beta^4 + 236\,737\,977\,410\,515\,968\,\beta^5 - 9\,077\,696\,769\,045\,384\,\beta^6 + \right. \\
& \quad \left. 112\,251\,378\,480\,384\,\beta^7 + 347\,057\,696\,700\,\beta^8 - 16\,288\,648\,090\,\beta^9 + 72\,149\,935\,\beta^{10} \right) \Big)
\end{aligned}$$

(* In particular, the coefficient at the monomial

[\[einggegebenes Objekt\]](#)

x^{11} still depends on α and β and reads as follows: *)

Out[*]=

$$-\frac{13}{16} \left(-6652 + \alpha^2 - 2\,\alpha \left(-16 + \beta \right) + 32\,\beta + \beta^2 \right)$$

(* The parameter $\alpha = \text{Root}[F_{m(13),8}(\alpha), 12]$ reads as follows: *)

[\[Nullstelle\]](#)

Root[

[\[Nullstelle\]](#)

$$\begin{aligned}
& 21\,301\,229\,214\,181\,630\,908\,497\,244\,475\,668\,490\,898\,502\,202\,548\,470\,325\,984\,297\,398\,587\,035\,625\,541 \searrow \\
& 599\,232 + \\
& 14\,315\,000\,429\,330\,324\,202\,339\,757\,152\,291\,235\,775\,784\,717\,759\,767\,154\,361\,740\,306\,614\,782\,181\,574 \searrow \\
& 180\,864\,\alpha - \\
& 34\,726\,563\,371\,460\,111\,111\,099\,299\,999\,251\,360\,351\,580\,156\,202\,230\,695\,468\,777\,742\,816\,069\,560\,951 \searrow \\
& 635\,968\,\alpha^2 + \\
& 16\,616\,689\,137\,429\,852\,285\,357\,593\,360\,093\,967\,338\,384\,353\,671\,269\,408\,170\,866\,271\,871\,787\,523\,769 \searrow \\
& 368\,576\,\alpha^3 + \\
& 2\,344\,849\,746\,546\,022\,694\,254\,645\,602\,769\,652\,573\,440\,800\,666\,916\,915\,292\,271\,839\,922\,373\,724\,783 \searrow \\
& 771\,648\,\alpha^4 - \\
& 7\,202\,337\,372\,223\,763\,240\,793\,946\,549\,960\,481\,433\,897\,623\,078\,638\,841\,146\,474\,254\,819\,186\,039\,553 \searrow \\
& 261\,568\,\alpha^5 +
\end{aligned}$$

$$\begin{aligned}
& 4\,912\,346\,300\,998\,193\,397\,975\,442\,133\,484\,948\,362\,031\,870\,160\,093\,854\,757\,080\,420\,400\,476\,112\,841 \, \backslash \\
& \quad 342\,976 \, \alpha^6 - \\
& 2\,176\,545\,720\,565\,886\,442\,519\,688\,246\,255\,001\,717\,148\,170\,686\,284\,646\,037\,792\,475\,365\,580\,028\,998 \, \backslash \\
& \quad 844\,416 \, \alpha^7 + \\
& 728\,588\,421\,668\,960\,088\,930\,514\,863\,589\,676\,472\,019\,876\,067\,843\,397\,948\,376\,218\,377\,384\,339\,150\,209 \, \backslash \\
& \quad 024 \, \alpha^8 - \\
& 193\,334\,737\,214\,796\,931\,640\,959\,652\,681\,674\,493\,592\,629\,346\,219\,281\,393\,552\,748\,950\,022\,224\,596\,697 \, \backslash \\
& \quad 088 \, \alpha^9 + \\
& 41\,064\,201\,764\,990\,252\,882\,896\,903\,337\,688\,170\,005\,653\,112\,198\,842\,061\,100\,969\,675\,119\,222\,337\,830\,912 \\
& \quad \alpha^{10} - \\
& 6\,830\,930\,028\,371\,481\,305\,098\,141\,005\,273\,908\,053\,827\,654\,677\,022\,163\,602\,246\,807\,456\,227\,111\,469\,056 \\
& \quad \alpha^{11} + \\
& 813\,010\,120\,658\,281\,295\,867\,573\,766\,517\,900\,312\,981\,079\,136\,131\,787\,605\,508\,571\,181\,680\,285\,450\,240 \\
& \quad \alpha^{12} - \\
& 41\,021\,944\,629\,268\,534\,439\,537\,468\,636\,321\,241\,871\,439\,657\,183\,861\,793\,892\,400\,585\,342\,013\,407\,232 \\
& \quad \alpha^{13} - \\
& 9\,918\,250\,851\,725\,657\,776\,804\,000\,138\,186\,897\,130\,248\,886\,761\,840\,347\,882\,120\,707\,106\,826\,878\,976 \\
& \quad \alpha^{14} + \\
& 3\,649\,026\,394\,902\,730\,171\,926\,067\,274\,286\,543\,497\,418\,545\,152\,837\,591\,594\,967\,186\,054\,724\,976\,640 \\
& \quad \alpha^{15} - \\
& 730\,192\,355\,970\,653\,063\,045\,183\,460\,205\,838\,581\,805\,357\,748\,452\,060\,344\,882\,653\,386\,226\,270\,208 \, \alpha^{16} + \\
& 111\,072\,630\,458\,131\,594\,982\,671\,839\,062\,904\,146\,416\,264\,142\,304\,558\,813\,541\,918\,355\,219\,283\,968 \, \alpha^{17} - \\
& 13\,942\,403\,578\,263\,155\,601\,609\,203\,191\,813\,775\,324\,710\,851\,501\,868\,761\,178\,895\,813\,361\,598\,464 \, \alpha^{18} + \\
& 1\,493\,843\,075\,506\,299\,325\,208\,134\,541\,638\,923\,355\,738\,997\,809\,588\,759\,823\,182\,070\,861\,004\,800 \, \alpha^{19} - \\
& 138\,993\,878\,115\,847\,449\,733\,327\,526\,403\,806\,152\,780\,937\,892\,064\,366\,827\,490\,836\,247\,347\,200 \, \alpha^{20} + \\
& 11\,340\,173\,229\,165\,917\,180\,117\,850\,638\,717\,835\,007\,851\,727\,932\,032\,327\,684\,079\,671\,050\,240 \, \alpha^{21} - \\
& 815\,783\,778\,692\,959\,465\,339\,666\,635\,879\,738\,628\,983\,880\,025\,345\,111\,137\,080\,817\,188\,864 \, \alpha^{22} + \\
& 51\,890\,425\,130\,337\,317\,677\,174\,692\,250\,091\,319\,391\,985\,663\,327\,256\,791\,590\,235\,275\,264 \, \alpha^{23} - \\
& 2\,920\,811\,343\,504\,887\,296\,967\,802\,116\,229\,338\,451\,815\,090\,346\,251\,487\,227\,503\,058\,944 \, \alpha^{24} + \\
& 145\,348\,258\,393\,403\,751\,794\,133\,371\,504\,239\,359\,121\,459\,942\,220\,876\,841\,858\,891\,776 \, \alpha^{25} - \\
& 6\,377\,129\,231\,793\,349\,945\,658\,302\,983\,514\,169\,845\,841\,549\,043\,724\,050\,116\,636\,672 \, \alpha^{26} + \\
& 245\,511\,712\,421\,450\,800\,672\,640\,723\,058\,676\,429\,965\,458\,011\,799\,340\,477\,562\,880 \, \alpha^{27} - \\
& 8\,231\,206\,181\,288\,195\,442\,626\,458\,579\,312\,174\,025\,152\,818\,727\,413\,744\,478\,208 \, \alpha^{28} + \\
& 237\,481\,644\,499\,231\,937\,848\,615\,445\,531\,058\,282\,208\,357\,852\,523\,313\,983\,488 \, \alpha^{29} - \\
& 5\,780\,867\,778\,324\,609\,177\,789\,813\,562\,399\,461\,653\,725\,194\,680\,863\,629\,056 \, \alpha^{30} + \\
& 114\,418\,437\,698\,002\,802\,166\,736\,017\,196\,618\,299\,634\,185\,963\,766\,194\,176 \, \alpha^{31} - \\
& 1\,687\,471\,914\,819\,374\,208\,852\,645\,639\,659\,988\,136\,332\,565\,720\,364\,544 \, \alpha^{32} + \\
& 12\,999\,945\,833\,409\,413\,876\,714\,223\,573\,696\,636\,254\,252\,565\,749\,760 \, \alpha^{33} + \\
& 168\,869\,802\,830\,620\,475\,707\,978\,524\,205\,389\,694\,171\,688\,967\,552 \, \alpha^{34} - \\
& 8\,808\,803\,403\,082\,408\,133\,057\,575\,942\,268\,693\,864\,513\,366\,016 \, \alpha^{35} + \\
& 193\,848\,198\,901\,558\,336\,773\,051\,149\,533\,990\,201\,035\,253\,920 \, \alpha^{36} - \\
& 2\,906\,160\,018\,831\,012\,322\,019\,079\,871\,690\,422\,444\,973\,056 \, \alpha^{37} + \\
& 31\,822\,247\,002\,700\,473\,787\,416\,009\,929\,430\,018\,881\,600 \, \alpha^{38} - \\
& 253\,343\,909\,200\,997\,917\,797\,526\,648\,928\,014\,704\,000 \, \alpha^{39} + \\
& 1\,398\,048\,199\,740\,466\,270\,079\,046\,637\,080\,330\,000 \, \alpha^{40} - \\
& 4\,792\,772\,087\,525\,409\,762\,808\,425\,442\,050\,000 \, \alpha^{41} + 7\,680\,724\,499\,239\,438\,722\,449\,399\,746\,875 \, \alpha^{42}, 12]
\end{aligned}$$

```
(* Its numerical approximation is  $\alpha =$ 
104.00480758118828422142150735958071113763240063717077590934409847478213985827484704 \
57359749697997779105575647161718801`99. *)
```

```
(* The parameter  $\beta = \text{Root}[G_{m(13)}, 8(\alpha), 12]$  reads as follows: *)
```

[Nullstelle](#)

Root[
[Nullstelle](#)

```
- 49 485 324 297 702 701 746 881 134 366 017 970 361 059 487 566 263 715 090 387 704 985 475 246 119 \
999 592 464 384 +
21 920 058 451 521 432 307 603 438 061 253 998 544 394 860 646 354 736 127 350 049 146 694 131 959 \
565 169 721 344  $\beta$  +
9 426 882 325 090 023 014 546 813 342 966 091 566 181 574 249 038 424 548 968 588 292 241 875 999 \
137 401 405 440  $\beta^2$  -
20 307 594 399 745 427 104 750 183 896 394 927 492 515 583 359 906 585 431 736 651 206 518 473 012 \
159 205 867 520  $\beta^3$  +
20 508 211 439 215 485 172 890 486 447 997 994 063 615 916 765 441 431 420 567 634 044 958 707 463 \
861 066 268 672  $\beta^4$  -
13 519 557 256 306 081 817 884 338 264 133 955 876 481 672 131 067 361 799 295 448 498 368 043 699 \
545 611 173 888  $\beta^5$  +
5 700 844 060 427 289 127 932 429 816 406 522 691 361 364 937 198 859 328 922 792 960 999 963 581 \
334 159 360 000  $\beta^6$  -
1 367 859 014 776 887 491 605 807 007 786 844 842 858 991 189 380 458 701 838 267 174 715 969 558 \
558 829 182 976  $\beta^7$  +
15 123 836 106 408 362 166 451 058 362 242 845 667 576 936 135 312 589 653 201 039 242 873 084 013 \
656 932 352  $\beta^8$  +
143 806 674 072 257 114 319 807 606 967 241 067 193 449 867 512 583 915 880 845 097 879 881 001 743 \
167 782 912  $\beta^9$  -
73 417 684 301 905 579 529 448 063 543 491 672 232 637 369 966 354 322 646 744 270 342 819 645 646 \
421 622 784  $\beta^{10}$  +
23 642 454 840 801 547 359 684 452 583 460 620 824 324 370 696 944 636 942 444 743 691 178 063 105 \
194 196 992  $\beta^{11}$  -
5 816 686 223 535 625 614 495 920 134 090 023 497 043 004 661 561 200 491 431 257 446 014 412 083 \
851 952 128  $\beta^{12}$  +
1 165 956 662 972 050 523 790 199 429 469 171 766 348 539 613 359 167 077 624 158 136 713 255 985 \
340 219 392  $\beta^{13}$  -
196 253 807 083 920 474 853 329 443 650 958 129 594 991 024 748 034 565 846 242 395 957 340 642 468 \
691 968  $\beta^{14}$  +
28 175 621 060 209 883 910 711 454 275 442 666 361 341 060 934 073 228 283 750 924 608 616 519 870 \
447 616  $\beta^{15}$  -
3 475 289 148 645 986 450 197 967 008 611 884 229 567 160 814 718 489 717 267 964 253 741 380 957 \
110 272  $\beta^{16}$  +
368 511 266 435 785 666 556 050 601 255 282 864 842 539 671 376 596 833 154 710 725 767 874 722 398 \
208  $\beta^{17}$  -
33 365 499 081 433 579 592 966 384 139 190 046 708 723 153 438 371 081 429 028 135 418 088 646 836 224 \
 $\beta^{18}$  +
2 530 504 197 057 335 853 634 131 518 990 407 334 143 601 180 527 413 473 275 359 786 299 543 257 088 \
 $\beta^{19}$  -
153 132 132 460 278 110 496 983 964 392 338 941 025 691 301 452 500 087 048 421 748 728 371 871 744
```

$\beta^{20} +$
 6 308 614 692 754 723 771 909 439 000 382 075 807 948 796 702 422 622 077 627 509 542 769 655 808
 $\beta^{21} -$
 17 052 261 185 718 103 206 855 425 289 746 843 989 008 109 292 555 784 620 787 838 031 265 792 $\beta^{22} -$
 27 613 569 282 779 835 348 485 863 966 035 482 427 259 094 630 684 392 564 933 042 525 962 240 $\beta^{23} +$
 3 296 785 358 363 913 395 605 509 075 543 778 098 059 387 926 066 890 062 703 662 107 516 928 $\beta^{24} -$
 261 397 821 868 677 676 974 413 310 527 782 470 317 353 205 204 249 643 898 711 136 862 208 $\beta^{25} +$
 16 459 254 589 079 257 079 388 256 543 490 708 035 422 078 022 365 247 923 008 233 287 680 $\beta^{26} -$
 868 220 488 127 781 820 900 858 719 052 637 705 314 582 336 610 338 475 828 424 753 152 $\beta^{27} +$
 39 218 456 705 035 846 084 309 984 160 839 370 554 404 032 849 751 705 431 779 423 232 $\beta^{28} -$
 1 531 902 239 964 547 346 069 674 969 894 600 631 131 541 576 227 616 917 662 314 496 $\beta^{29} +$
 51 933 438 180 593 371 254 244 656 093 148 847 727 861 573 222 631 675 847 348 480 $\beta^{30} -$
 1 527 946 784 816 613 176 171 235 660 853 132 299 965 990 215 778 085 487 747 072 $\beta^{31} +$
 38 902 145 292 606 085 713 923 850 808 709 617 205 335 536 677 097 089 227 776 $\beta^{32} -$
 852 561 541 082 714 415 984 024 139 384 448 107 840 425 511 723 148 193 792 $\beta^{33} +$
 15 956 362 885 440 142 355 846 551 007 885 195 073 665 826 273 063 219 072 $\beta^{34} -$
 252 271 475 988 593 809 388 499 848 028 572 223 461 351 044 680 333 312 $\beta^{35} +$
 3 319 968 508 232 820 139 289 386 946 874 228 495 195 666 343 695 264 $\beta^{36} -$
 35 647 359 297 401 610 595 813 050 644 611 422 151 867 716 232 704 $\beta^{37} +$
 303 655 440 183 630 170 335 617 469 324 929 580 244 458 310 400 $\beta^{38} -$
 1 969 463 017 800 487 416 799 219 073 065 773 868 771 056 000 $\beta^{39} +$
 9 111 338 700 186 653 832 066 964 314 295 506 684 457 500 $\beta^{40} -$
 26 692 840 900 403 579 505 936 430 614 826 059 750 000 $\beta^{41} +$
 37 073 390 139 449 415 980 467 264 742 813 971 875 $\beta^{42}, 12]$

(* Its numerical approximation is $\beta =$

104.00480758118828422142150742299729132045950005548794425579518068336174744900276564
 20751953803414177551995680646447783`99. *)

(* After insertion,
 Fläche nach dem Objekt

a numerical approximation of $Z_{13,8}(x) = \sum_{k=0}^{13} a_{k,13}^*(8) x^k$ reads as follows: *)

Out[*]=

```
-0.0507800763283789694945170720121218932116529040060345721187423819256003162778712307 \
88705498571979449 +
0.0058592621464378575712965115589612062089280704443142485827219966251504665214285236 \
376068887588315361 x +
3.6561913159306733991521003582968299192732873862153890691721670119351462901657567804 \
9130959887609198 x^2 -
0.1425762064769235763730989736647563695542677787897881304848367181136508448074994346 \
93488826520799916 x^3 -
42.655780524841206643731296013861941298073243535598187719517242015023452266264042674 \
4251735609726469 x^4 +
1.0117093604809651905921211071868290693043158940762347594776119875182042446972173113 \
3337389060935075 x^5 +
181.99868546434696132614305395721043225389597862939127911037440868144874331476636268 \
7817883699245559 x^6 -
3.1249808597654262014227977100838749995979164847355500405721834097919697913880707300 \
4245087287004630 x^7 -
350.99849766907499460553380880575479707869952757165333178500600458127072958772692560 \
1496758393844630 x^8 +
4.7499826654057262435508634120309050040283583081315167242188409495160997708196123077 \
7666537273305935 x^9 +
311.99939906624106944751197209091864668049947294917961408065602355707317760505078001 \
9780404388747697 x^10 -
3.4999942217907795139183843470280639103894180091267275612221548057538338458426879780 \
1170645271039541 x^11 -
104.00000000000000000000000000000000000000000000000000000000000000000000000000000000 \
0000000000000000 x^12 +
1.00000000000000000000000000000000000000000000000000000000000000000000000000000000 \
0000000000000000 x^13
```

(* To obtain a non-numerical explicit representation of $Z_{13,8}(x) =$

$\sum_{k=0}^{13} a_{k,13}^*(8) x^k$ would require to insert into $(R_{13,\alpha,\beta}(x)) / .$

s→8 the parameters α and β *)

(* In particular, inserting the parameters α and β into the coefficient at the

Leingegebenes Objekt

monomial x^{11} (i.e., into $-\frac{13}{16} (-6652+\alpha^2-2\alpha(-16+\beta)+32\beta+\beta^2)$) would yield,

after simplification with RootReduce or FullSimplify,

reduziere auf Null· vereinfache vollständig

the following algebraic representation of $a_{11,13}^*(8)$ as a root object: *)

In[*]:= Root[
 Nullstelle

```
3 121 777 889 148 235 296 041 179 991 201 416 588 386 095 496 931 920 648 729 059 313 393 442 705 946 \
841 910 797 976 577 140 232 957 981 138 794 244 423 422 822 411 359 588 178 733 442 636 286 068 613 \
234 898 979 992 658 482 035 852 423 007 003 889 602 279 398 233 +
864 853 673 311 940 283 348 867 108 425 593 000 259 060 102 108 844 153 476 128 409 290 459 977 558 \
207 780 092 818 451 658 618 890 566 322 650 372 233 651 298 528 791 177 450 763 860 766 608 655 \
```

978 793 796 282 127 806 222 462 983 828 361 601 052 796 403 007 448 $z -$
 7 623 966 371 083 387 280 202 526 309 871 038 804 254 074 406 142 111 691 338 804 197 707 274 952 $\setminus$
 121 546 471 569 572 774 737 388 900 450 581 920 493 778 372 278 181 362 572 135 616 912 173 564 $\setminus$
 392 087 873 937 524 771 883 122 644 508 030 952 653 532 223 133 984 $z^2 +$
 32 380 372 951 943 072 672 163 481 192 644 290 472 319 459 880 712 023 776 240 268 035 604 736 118 $\setminus$
 279 921 381 901 616 822 319 784 033 449 050 663 284 527 625 752 358 187 756 603 705 359 759 870 $\setminus$
 771 796 316 849 016 054 012 857 935 669 631 309 143 402 800 112 $z^3 -$
 88 993 282 053 435 762 047 474 546 846 745 358 359 939 844 009 589 364 803 886 778 374 093 222 167 $\setminus$
 896 679 054 726 079 481 045 257 908 674 120 776 032 641 967 501 793 025 348 458 300 258 035 034 $\setminus$
 087 161 616 298 238 738 891 304 258 523 632 227 446 213 440 $z^4 +$
 178 200 662 887 901 538 359 801 976 607 834 111 995 808 707 965 888 795 179 556 908 167 205 114 032 $\setminus$
 266 149 409 624 818 206 681 312 787 185 445 213 566 623 345 539 888 554 561 166 726 627 938 535 $\setminus$
 198 583 405 457 382 614 124 352 063 070 088 293 345 216 $z^5 -$
 277 225 352 674 367 656 805 346 256 490 156 017 683 568 779 542 733 571 009 187 712 673 068 777 944 $\setminus$
 716 933 985 765 660 409 758 068 073 227 091 121 623 940 838 612 074 417 522 364 337 448 049 672 $\setminus$
 886 012 648 828 604 073 540 886 837 902 559 479 200 $z^6 +$
 348 792 285 365 773 658 182 860 793 624 669 307 728 662 174 745 236 216 095 527 566 472 518 844 300 $\setminus$
 281 145 002 781 853 333 711 331 293 522 176 966 968 479 384 648 333 124 704 421 888 314 508 965 $\setminus$
 422 964 569 371 069 584 664 382 613 887 125 504 $z^7 -$
 364 735 827 096 134 414 809 096 004 972 824 246 819 173 856 282 614 832 710 998 837 969 861 537 783 $\setminus$
 069 368 503 985 780 248 422 043 695 578 823 200 680 971 675 156 631 822 361 087 159 081 143 787 $\setminus$
 393 079 302 439 776 375 964 919 923 072 640 $z^8 +$
 323 297 129 871 290 475 610 157 787 119 467 037 134 661 551 906 088 515 491 172 321 679 453 591 051 $\setminus$
 289 377 020 446 779 516 879 854 112 529 150 790 714 693 935 345 358 883 967 622 092 432 181 920 $\setminus$
 579 805 497 652 866 550 295 336 544 384 $z^9 -$
 246 495 517 431 163 495 499 019 055 468 430 282 879 611 949 527 272 090 432 680 431 302 579 337 897 $\setminus$
 542 513 865 509 035 846 644 035 489 619 277 698 939 677 569 634 514 994 285 061 750 514 111 807 $\setminus$
 658 270 392 976 837 788 593 071 616 $z^{10} +$
 163 485 849 626 549 708 379 004 841 589 917 274 637 165 633 927 570 415 560 303 984 526 723 731 096 $\setminus$
 626 436 930 756 121 168 902 490 307 939 218 546 682 346 740 426 682 951 161 129 983 792 672 859 $\setminus$
 980 254 618 551 091 996 081 152 $z^{11} -$
 95 151 130 313 194 606 513 414 445 702 542 070 815 411 511 326 643 874 112 935 361 898 357 006 982 $\setminus$
 860 912 016 386 345 395 814 269 585 418 054 189 401 434 232 203 737 788 942 042 284 973 161 978 $\setminus$
 391 574 941 544 077 837 056 $z^{12} +$
 48 931 693 796 259 883 251 582 489 937 957 076 881 070 813 846 690 874 935 756 421 681 614 060 992 $\setminus$
 752 141 339 950 599 522 951 546 687 304 199 507 315 873 203 307 582 130 131 585 369 720 525 484 $\setminus$
 820 192 003 700 062 208 $z^{13} -$
 22 353 397 878 536 225 183 826 259 947 150 944 708 093 328 623 492 572 361 543 849 425 249 615 632 $\setminus$
 017 599 684 755 123 526 607 153 724 964 932 928 224 983 594 979 616 966 097 304 243 631 862 580 $\setminus$
 409 504 341 921 792 $z^{14} +$
 9 109 165 481 739 880 458 792 018 434 565 513 379 730 575 354 742 552 626 253 962 532 346 372 967 $\setminus$
 623 024 941 992 341 112 967 624 243 867 397 103 099 067 373 695 895 716 317 782 044 542 476 734 $\setminus$
 959 925 501 952 $z^{15} -$
 3 321 588 815 706 321 344 186 324 101 586 279 111 437 250 905 467 152 741 975 051 792 288 834 692 $\setminus$
 021 038 593 919 074 211 042 217 475 983 503 257 876 596 203 642 453 257 904 426 892 335 995 643 $\setminus$
 962 044 416 $z^{16} +$
 1 086 141 634 588 401 180 057 438 607 634 555 742 205 932 905 853 790 014 787 668 086 956 274 633 $\setminus$
 470 097 288 444 535 648 186 647 596 365 834 669 687 375 741 998 848 860 839 494 000 695 036 533 $\setminus$
 465 088 $z^{17} -$
 318 896 846 656 084 170 274 553 706 967 053 931 537 904 927 548 307 076 949 590 710 835 402 888 705 $\setminus$

```

455 540 343 381 427 334 255 667 242 359 871 152 258 413 161 221 728 153 321 899 850 935 070 720
z18 +
84 098 156 077 028 780 931 684 743 894 420 541 872 870 337 538 552 926 754 744 407 344 520 765 429 \
418 645 198 952 623 954 698 140 829 569 502 436 412 161 038 931 708 608 948 169 475 424 256 z19 -
19 906 750 339 099 909 044 639 142 591 792 907 188 841 656 589 583 039 901 033 401 301 524 207 564 \
092 424 782 014 703 225 092 466 946 279 857 442 027 426 093 062 535 309 961 080 602 624 z20 +
4 221 419 141 622 188 308 349 754 963 342 552 657 716 264 789 227 146 307 776 202 367 466 636 053 \
453 165 080 813 240 792 372 117 737 370 151 739 957 565 756 181 738 863 667 445 760 z21 -
799 111 577 221 148 042 198 873 912 278 949 168 927 149 865 677 499 626 870 040 451 960 069 935 283 \
632 693 752 347 295 975 151 375 109 506 210 945 373 893 100 290 367 160 320 z22 +
134 222 380 095 780 707 957 419 580 556 956 234 288 004 090 848 571 404 215 873 673 335 842 112 513 \
608 771 917 845 100 061 412 066 855 046 248 948 413 003 206 475 382 784 z23 -
19 801 844 501 950 492 926 634 701 114 439 418 047 170 090 615 832 360 712 783 726 409 852 165 358 \
155 206 221 701 547 574 710 601 015 549 152 028 410 115 535 273 984 z24 +
2 520 158 725 133 412 876 899 031 557 428 382 203 828 543 280 182 779 646 168 539 139 594 109 780 \
199 412 628 693 635 812 347 615 147 092 679 804 559 224 209 408 z25 -
266 869 036 967 489 902 916 324 740 285 336 116 302 908 497 615 427 266 740 764 395 051 125 630 354 \
052 674 710 632 849 355 490 570 272 613 526 817 734 656 z26 +
21 445 616 776 132 002 562 045 146 186 593 729 488 854 736 021 051 992 116 577 178 220 513 919 487 \
454 895 950 746 587 285 712 627 868 630 956 638 208 z27 -
855 103 895 069 967 926 737 413 184 520 912 039 882 872 807 508 524 353 880 603 984 012 848 652 988 \
040 533 541 898 781 852 093 148 335 964 160 z28 -
96 950 604 501 376 233 795 203 067 434 287 725 965 127 454 848 080 640 575 814 537 019 473 069 752 \
813 423 841 054 504 514 403 184 934 912 z29 +
28 070 052 163 053 753 249 315 431 747 227 788 871 282 094 001 285 464 468 587 320 597 366 424 373 \
344 700 193 978 450 804 682 522 624 z30 -
4 069 286 719 369 008 100 786 375 137 262 351 733 984 163 784 894 937 256 067 158 485 284 463 598 \
420 791 180 027 790 601 748 480 z31 +
440 137 209 942 182 350 105 528 019 832 708 376 274 664 482 706 303 759 607 528 407 717 504 854 221 \
875 990 165 442 265 088 z32 -
38 513 522 273 484 523 764 008 123 041 271 774 502 073 097 768 130 314 471 071 825 255 614 194 635 \
063 390 397 005 824 z33 +
2 795 935 463 895 791 659 110 101 304 712 609 826 314 013 817 351 850 825 240 680 232 451 254 117 \
919 247 826 944 z34 -
169 310 791 696 975 632 925 544 371 939 570 978 300 377 936 766 371 319 538 148 499 757 448 816 475 \
766 784 z35 +
8 507 439 393 550 563 127 042 703 145 730 200 615 686 070 772 345 276 249 279 868 359 863 856 791 552
z36 -
349 841 989 984 736 510 381 931 954 929 777 117 556 364 590 463 922 000 522 888 865 090 895 872 z37 +
11 501 018 820 175 115 773 401 985 095 682 265 236 455 603 489 322 970 476 295 120 486 400 z38 -
291 200 041 569 987 757 451 405 128 444 656 911 645 338 967 039 884 375 621 632 000 z39 +
5 338 134 077 826 309 196 117 107 988 557 031 200 550 182 676 888 289 280 000 z40 -
63 113 660 147 934 028 661 403 650 355 234 097 887 514 617 446 400 000 z41 +
361 604 121 440 225 215 491 203 350 303 281 221 782 732 800 000 z42, 2]

```

In[*]:= (* Its numerical value is indeed *)

In[*]:= N[Root[
 [Nullstelle
 3 121 777 889 148 235 296 041 179 991 201 416 588 386 095 496 931 920 648 729 059 313 393 442 705 \

946 841 910 797 976 577 140 232 957 981 138 794 244 423 422 822 411 359 588 178 733 442 636 286 \\
 068 613 234 898 979 992 658 482 035 852 423 007 003 889 602 279 398 233 +
 864 853 673 311 940 283 348 867 108 425 593 000 259 060 102 108 844 153 476 128 409 290 459 977 \\
 558 207 780 092 818 451 658 618 890 566 322 650 372 233 651 298 528 791 177 450 763 860 766 608 \\
 655 978 793 796 282 127 806 222 462 983 828 361 601 052 796 403 007 448 z -
 7 623 966 371 083 387 280 202 526 309 871 038 804 254 074 406 142 111 691 338 804 197 707 274 952 \\
 121 546 471 569 572 774 737 388 900 450 581 920 493 778 372 278 181 362 572 135 616 912 173 564 \\
 392 087 873 937 524 771 883 122 644 508 030 952 653 532 223 133 984 z² +
 32 380 372 951 943 072 672 163 481 192 644 290 472 319 459 880 712 023 776 240 268 035 604 736 118 \\
 279 921 381 901 616 822 319 784 033 449 050 663 284 527 625 752 358 187 756 603 705 359 759 870 \\
 771 796 316 849 016 054 012 857 935 669 631 309 143 402 800 112 z³ -
 88 993 282 053 435 762 047 474 546 846 745 358 359 939 844 009 589 364 803 886 778 374 093 222 167 \\
 896 679 054 726 079 481 045 257 908 674 120 776 032 641 967 501 793 025 348 458 300 258 035 034 \\
 087 161 616 298 238 738 891 304 258 523 632 227 446 213 440 z⁴ +
 178 200 662 887 901 538 359 801 976 607 834 111 995 808 707 965 888 795 179 556 908 167 205 114 \\
 032 266 149 409 624 818 206 681 312 787 185 445 213 566 623 345 539 888 554 561 166 726 627 938 \\
 535 198 583 405 457 382 614 124 352 063 070 088 293 345 216 z⁵ -
 277 225 352 674 367 656 805 346 256 490 156 017 683 568 779 542 733 571 009 187 712 673 068 777 \\
 944 716 933 985 765 660 409 758 068 073 227 091 121 623 940 838 612 074 417 522 364 337 448 049 \\
 672 886 012 648 828 604 073 540 886 837 902 559 479 200 z⁶ +
 348 792 285 365 773 658 182 860 793 624 669 307 728 662 174 745 236 216 095 527 566 472 518 844 \\
 300 281 145 002 781 853 333 711 331 293 522 176 966 968 479 384 648 333 124 704 421 888 314 508 \\
 965 422 964 569 371 069 584 664 382 613 887 125 504 z⁷ -
 364 735 827 096 134 414 809 096 004 972 824 246 819 173 856 282 614 832 710 998 837 969 861 537 \\
 783 069 368 503 985 780 248 422 043 695 578 823 200 680 971 675 156 631 822 361 087 159 081 143 \\
 787 393 079 302 439 776 375 964 919 923 072 640 z⁸ +
 323 297 129 871 290 475 610 157 787 119 467 037 134 661 551 906 088 515 491 172 321 679 453 591 \\
 051 289 377 020 446 779 516 879 854 112 529 150 790 714 693 935 345 358 883 967 622 092 432 181 \\
 920 579 805 497 652 866 550 295 336 544 384 z⁹ -
 246 495 517 431 163 495 499 019 055 468 430 282 879 611 949 527 272 090 432 680 431 302 579 337 \\
 897 542 513 865 509 035 846 644 035 489 619 277 698 939 677 569 634 514 994 285 061 750 514 111 \\
 807 658 270 392 976 837 788 593 071 616 z¹⁰ +
 163 485 849 626 549 708 379 004 841 589 917 274 637 165 633 927 570 415 560 303 984 526 723 731 \\
 096 626 436 930 756 121 168 902 490 307 939 218 546 682 346 740 426 682 951 161 129 983 792 672 \\
 859 980 254 618 551 091 996 081 152 z¹¹ -
 95 151 130 313 194 606 513 414 445 702 542 070 815 411 511 326 643 874 112 935 361 898 357 006 982 \\
 860 912 016 386 345 395 814 269 585 418 054 189 401 434 232 203 737 788 942 042 284 973 161 978 \\
 391 574 941 544 077 837 056 z¹² +
 48 931 693 796 259 883 251 582 489 937 957 076 881 070 813 846 690 874 935 756 421 681 614 060 992 \\
 752 141 339 950 599 522 951 546 687 304 199 507 315 873 203 307 582 130 131 585 369 720 525 484 \\
 820 192 003 700 062 208 z¹³ -
 22 353 397 878 536 225 183 826 259 947 150 944 708 093 328 623 492 572 361 543 849 425 249 615 632 \\
 017 599 684 755 123 526 607 153 724 964 932 928 224 983 594 979 616 966 097 304 243 631 862 580 \\
 409 504 341 921 792 z¹⁴ +
 9 109 165 481 739 880 458 792 018 434 565 513 379 730 575 354 742 552 626 253 962 532 346 372 967 \\
 623 024 941 992 341 112 967 624 243 867 397 103 099 067 373 695 895 716 317 782 044 542 476 734 \\
 959 925 501 952 z¹⁵ -
 3 321 588 815 706 321 344 186 324 101 586 279 111 437 250 905 467 152 741 975 051 792 288 834 692 \\
 021 038 593 919 074 211 042 217 475 983 503 257 876 596 203 642 453 257 904 426 892 335 995 643 \\
 962 044 416 z¹⁶ +

$1\,086\,141\,634\,588\,401\,180\,057\,438\,607\,634\,555\,742\,205\,932\,905\,853\,790\,014\,787\,668\,086\,956\,274\,633 \setminus$
 $470\,097\,288\,444\,535\,648\,186\,647\,596\,365\,834\,669\,687\,375\,741\,998\,848\,860\,839\,494\,000\,695\,036\,533 \setminus$
 $465\,088\,z^{17} -$
 $318\,896\,846\,656\,084\,170\,274\,553\,706\,967\,053\,931\,537\,904\,927\,548\,307\,076\,949\,590\,710\,835\,402\,888 \setminus$
 $705\,455\,540\,343\,381\,427\,334\,255\,667\,242\,359\,871\,152\,258\,413\,161\,221\,728\,153\,321\,899\,850\,935\,070 \setminus$
 $720\,z^{18} +$
 $84\,098\,156\,077\,028\,780\,931\,684\,743\,894\,420\,541\,872\,870\,337\,538\,552\,926\,754\,744\,407\,344\,520\,765\,429 \setminus$
 $418\,645\,198\,952\,623\,954\,698\,140\,829\,569\,502\,436\,412\,161\,038\,931\,708\,608\,948\,169\,475\,424\,256$
 $z^{19} -$
 $19\,906\,750\,339\,099\,909\,044\,639\,142\,591\,792\,907\,188\,841\,656\,589\,583\,039\,901\,033\,401\,301\,524\,207\,564 \setminus$
 $092\,424\,782\,014\,703\,225\,092\,466\,946\,279\,857\,442\,027\,426\,093\,062\,535\,309\,961\,080\,602\,624\,z^{20} +$
 $4\,221\,419\,141\,622\,188\,308\,349\,754\,963\,342\,552\,657\,716\,264\,789\,227\,146\,307\,776\,202\,367\,466\,636\,053 \setminus$
 $453\,165\,080\,813\,240\,792\,372\,117\,737\,370\,151\,739\,957\,565\,756\,181\,738\,863\,667\,445\,760\,z^{21} -$
 $799\,111\,577\,221\,148\,042\,198\,873\,912\,278\,949\,168\,927\,149\,865\,677\,499\,626\,870\,040\,451\,960\,069\,935 \setminus$
 $283\,632\,693\,752\,347\,295\,975\,151\,375\,109\,506\,210\,945\,373\,893\,100\,290\,367\,160\,320\,z^{22} +$
 $134\,222\,380\,095\,780\,707\,957\,419\,580\,556\,956\,234\,288\,004\,090\,848\,571\,404\,215\,873\,673\,335\,842\,112 \setminus$
 $513\,608\,771\,917\,845\,100\,061\,412\,066\,855\,046\,248\,948\,413\,003\,206\,475\,382\,784\,z^{23} -$
 $19\,801\,844\,501\,950\,492\,926\,634\,701\,114\,439\,418\,047\,170\,090\,615\,832\,360\,712\,783\,726\,409\,852\,165\,358 \setminus$
 $155\,206\,221\,701\,547\,574\,710\,601\,015\,549\,152\,028\,410\,115\,535\,273\,984\,z^{24} +$
 $2\,520\,158\,725\,133\,412\,876\,899\,031\,557\,428\,382\,203\,828\,543\,280\,182\,779\,646\,168\,539\,139\,594\,109\,780 \setminus$
 $199\,412\,628\,693\,635\,812\,347\,615\,147\,092\,679\,804\,559\,224\,209\,408\,z^{25} -$
 $266\,869\,036\,967\,489\,902\,916\,324\,740\,285\,336\,116\,302\,908\,497\,615\,427\,266\,740\,764\,395\,051\,125\,630 \setminus$
 $354\,052\,674\,710\,632\,849\,355\,490\,570\,272\,613\,526\,817\,734\,656\,z^{26} +$
 $21\,445\,616\,776\,132\,002\,562\,045\,146\,186\,593\,729\,488\,854\,736\,021\,051\,992\,116\,577\,178\,220\,513\,919\,487 \setminus$
 $454\,895\,950\,746\,587\,285\,712\,627\,868\,630\,956\,638\,208\,z^{27} -$
 $855\,103\,895\,069\,967\,926\,737\,413\,184\,520\,912\,039\,882\,872\,807\,508\,524\,353\,880\,603\,984\,012\,848\,652 \setminus$
 $988\,040\,533\,541\,898\,781\,852\,093\,148\,335\,964\,160\,z^{28} -$
 $96\,950\,604\,501\,376\,233\,795\,203\,067\,434\,287\,725\,965\,127\,454\,848\,080\,640\,575\,814\,537\,019\,473\,069\,752 \setminus$
 $813\,423\,841\,054\,504\,514\,403\,184\,934\,912\,z^{29} +$
 $28\,070\,052\,163\,053\,753\,249\,315\,431\,747\,227\,788\,871\,282\,094\,001\,285\,464\,468\,587\,320\,597\,366\,424\,373 \setminus$
 $344\,700\,193\,978\,450\,804\,682\,522\,624\,z^{30} -$
 $4\,069\,286\,719\,369\,008\,100\,786\,375\,137\,262\,351\,733\,984\,163\,784\,894\,937\,256\,067\,158\,485\,284\,463\,598 \setminus$
 $420\,791\,180\,027\,790\,601\,748\,480\,z^{31} +$
 $440\,137\,209\,942\,182\,350\,105\,528\,019\,832\,708\,376\,274\,664\,482\,706\,303\,759\,607\,528\,407\,717\,504\,854 \setminus$
 $221\,875\,990\,165\,442\,265\,088\,z^{32} -$
 $38\,513\,522\,273\,484\,523\,764\,008\,123\,041\,271\,774\,502\,073\,097\,768\,130\,314\,471\,071\,825\,255\,614\,194\,635 \setminus$
 $063\,390\,397\,005\,824\,z^{33} +$
 $2\,795\,935\,463\,895\,791\,659\,110\,101\,304\,712\,609\,826\,314\,013\,817\,351\,850\,825\,240\,680\,232\,451\,254\,117 \setminus$
 $919\,247\,826\,944\,z^{34} -$
 $169\,310\,791\,696\,975\,632\,925\,544\,371\,939\,570\,978\,300\,377\,936\,766\,371\,319\,538\,148\,499\,757\,448\,816 \setminus$
 $475\,766\,784\,z^{35} +$
 $8\,507\,439\,393\,550\,563\,127\,042\,703\,145\,730\,200\,615\,686\,070\,772\,345\,276\,249\,279\,868\,359\,863\,856\,791 \setminus$
 $552\,z^{36} -$
 $349\,841\,989\,984\,736\,510\,381\,931\,954\,929\,777\,117\,556\,364\,590\,463\,922\,000\,522\,888\,865\,090\,895\,872\,z^{37} +$
 $11\,501\,018\,820\,175\,115\,773\,401\,985\,095\,682\,265\,236\,455\,603\,489\,322\,970\,476\,295\,120\,486\,400\,z^{38} -$
 $291\,200\,041\,569\,987\,757\,451\,405\,128\,444\,656\,911\,645\,338\,967\,039\,884\,375\,621\,632\,000\,z^{39} +$
 $5\,338\,134\,077\,826\,309\,196\,117\,107\,988\,557\,031\,200\,550\,182\,676\,888\,289\,280\,000\,z^{40} -$
 $63\,113\,660\,147\,934\,028\,661\,403\,650\,355\,234\,097\,887\,514\,617\,446\,400\,000\,z^{41} +$
 $361\,604\,121\,440\,225\,215\,491\,203\,350\,303\,281\,221\,782\,732\,800\,000\,z^{42}, 2], 99]$

Out[8]=

– 3.4999942217907795139183843470280639103894180091267275612221548057538338458426879780
1170645271039541