(* Goal: Algebraic solution of ZFP if n=12 and s=7 and exemplarily one coefficient, $a^{*}_{10,12}(7)$, is explicitly expressed as a root object (see Appendix 3 in[6]) *)

(* The proper monic Zolotarev polynomial of degree n= 12 and with parameter s=7 can be generated by *)

(* $Z_{12,7}(x) = \sum_{k=0}^{12} a^{*}_{k,12}(7)\, x^{k} = (R_{12,\alpha,\beta}(x))$ /.s→7/.α→Root[$F_{m(12),7}(\alpha)$,4]/.β→Root[$G_{m(12),7}(\beta)$,4] *)

Nullstelle Nullstelle

(* The first insertion, $(R_{12,\alpha,\beta}(x))$/.s→7, will yield the following polynomial of degree n=12, still depending on α and β: *)

Out[•]=

$$
\begin{aligned}
&-84\,x^{11}+x^{12}-\frac{3}{4}\,x^{10}\left(-4700+\alpha^2-2\,\alpha\,(-14+\beta)+28\,\beta+\beta^2\right)-\\
&\frac{1}{2}\,x^9\left(197\,092+\alpha^3-3528\,\beta-105\,\beta^2+\beta^3-\alpha^2\,(105+\beta)-\alpha\left(3528-266\,\beta+\beta^2\right)\right)-\\
&\frac{3}{128}\,x^8\left(-88\,109\,200+3\,\alpha^4+3\,158\,624\,\beta+65\,768\,\beta^2-2184\,\beta^3+3\,\beta^4+12\,\alpha^3\,(-182+3\,\beta)+\right.\\
&\qquad\left.\alpha^2\left(65\,768+2632\,\beta-78\,\beta^2\right)+4\,\alpha\left(789\,656-67\,380\,\beta+658\,\beta^2+9\,\beta^3\right)\right)+\\
&\frac{3}{80}\,x^7\left(-922\,780\,880+3\,\alpha^5-25\,\alpha^4\,(-21+\beta)+55\,197\,520\,\beta+490\,336\,\beta^2-61\,704\,\beta^3+\right.\\
&\qquad 525\,\beta^4+3\,\beta^5+2\,\alpha^3\left(-30\,852+770\,\beta+11\,\beta^2\right)+2\,\alpha^2\left(245\,168+50\,452\,\beta-2289\,\beta^2+11\,\beta^3\right)+\\
&\qquad\left.\alpha\left(55\,197\,520-5\,588\,576\,\beta+100\,904\,\beta^2+1540\,\beta^3-25\,\beta^4\right)\right)+\\
&\frac{1}{1280}\,x^6\left(618\,345\,627\,200+145\,\alpha^6-55\,532\,487\,360\,\beta+336\,530\,928\,\beta^2+\right.\\
&\qquad 78\,863\,008\,\beta^3-1\,763\,700\,\beta^4-1596\,\beta^5+145\,\beta^6-2\,\alpha^5\,(798+275\,\beta)-\\
&\qquad 5\,\alpha^4\left(352\,740-22\,372\,\beta+141\,\beta^2\right)+4\,\alpha^3\left(19\,715\,752-505\,020\,\beta-29\,806\,\beta^2+555\,\beta^3\right)+\\
&\qquad \alpha^2\left(336\,530\,928-197\,367\,968\,\beta+11\,004\,616\,\beta^2-119\,224\,\beta^3-705\,\beta^4\right)-\\
&\qquad\left.2\,\alpha\left(27\,766\,243\,680-3\,419\,939\,216\,\beta+98\,683\,984\,\beta^2+1\,010\,040\,\beta^3-55\,930\,\beta^4+275\,\beta^5\right)\right)+\\
&\frac{1}{4480}\times 3\,x^5\left(-8\,630\,977\,252\,800+99\,\alpha^7+1\,085\,963\,244\,800\,\beta-26\,075\,370\,160\,\beta^2-\right.\\
&\qquad 1\,540\,955\,536\,\beta^3+75\,986\,652\,\beta^4-601\,272\,\beta^5-10\,633\,\beta^6+99\,\beta^7-7\,\alpha^6\,(1519+9\,\beta)-\\
&\qquad 7\,\alpha^5\left(85\,896-11\,746\,\beta+195\,\beta^2\right)+\alpha^4\left(75\,986\,652-5\,745\,304\,\beta+49\,049\,\beta^2+1329\,\beta^3\right)+\\
&\qquad \alpha^3\left(-1\,540\,955\,536+9\,027\,536\,\beta+8\,190\,544\,\beta^2-259\,196\,\beta^3+1329\,\beta^4\right)+\\
&\qquad \alpha^2\left(-26\,075\,370\,160+6\,956\,777\,360\,\beta-455\,162\,456\,\beta^2+8\,190\,544\,\beta^3+49\,049\,\beta^4-1365\,\beta^5\right)+\\
&\qquad \alpha\left(1\,085\,963\,244\,800-164\,092\,726\,560\,\beta+6\,956\,777\,360\,\beta^2+\right.\\
&\qquad\qquad\left.\left.9\,027\,536\,\beta^3-5\,745\,304\,\beta^4+82\,222\,\beta^5-63\,\beta^6\right)\right)+\\
&\frac{1}{327\,680}\times 3\,x^4\left(6\,608\,775\,245\,049\,600+3271\,\alpha^8-1\,109\,260\,562\,457\,600\,\beta+49\,882\,770\,680\,064\,\beta^2+\right.\\
&\qquad 997\,036\,311\,808\,\beta^3-131\,528\,938\,976\,\beta^4+2\,800\,431\,424\,\beta^5+7\,631\,824\,\beta^6-653\,840\,\beta^7+\\
&\qquad 3271\,\beta^8+8\,\alpha^7\,(-81\,730+1293\,\beta)+\alpha^6\left(7\,631\,824+2\,092\,496\,\beta-66\,812\,\beta^2\right)-\\
&\qquad 8\,\alpha^5\left(-350\,053\,928+42\,755\,644\,\beta-1\,136\,478\,\beta^2+2693\,\beta^3\right)+\\
&\qquad 2\,\alpha^4\left(-65\,764\,469\,488+5\,579\,244\,064\,\beta-22\,833\,512\,\beta^2-5\,551\,960\,\beta^3+74\,741\,\beta^4\right)-\\
&\qquad 8\,\alpha^3\left(-124\,629\,538\,976-15\,530\,473\,264\,\beta+3\,062\,436\,528\,\beta^2-126\,243\,896\,\beta^3+\right.\\
&\qquad\qquad\left.1\,387\,990\,\beta^4+2693\,\beta^5\right)-4\,\alpha^2\left(-12\,470\,692\,670\,016+3\,019\,128\,751\,936\,\beta-\right.\\
&\qquad\qquad\left.231\,702\,244\,912\,\beta^2+6\,124\,873\,056\,\beta^3+11\,416\,756\,\beta^4-2\,272\,956\,\beta^5+16\,703\,\beta^6\right)+\\
&\qquad 8\,\alpha\left(-138\,657\,570\,307\,200+25\,658\,770\,049\,472\,\beta-1\,509\,564\,375\,968\,\beta^2+\right.
\end{aligned}
$$

$$15\,530\,473\,264\,\beta^3 + 1\,394\,811\,016\,\beta^4 - 42\,755\,644\,\beta^5 + 261\,562\,\beta^6 + 1293\,\beta^7)) +$$

$$\frac{1}{1\,720\,320}\,x^3\,(-968\,722\,250\,017\,150\,720 + 19\,632\,\alpha^9 + 209\,113\,581\,765\,473\,280\,\beta -$$

$$14\,414\,114\,854\,406\,400\,\beta^2 + 81\,951\,252\,404\,992\,\beta^3 + 33\,306\,547\,950\,816\,\beta^4 -$$

$$1\,422\,787\,914\,048\,\beta^5 + 13\,603\,518\,768\,\beta^6 + 310\,983\,696\,\beta^7 - 6\,180\,111\,\beta^8 + 19\,632\,\beta^9 +$$

$$3\,\alpha^8\,(-2\,060\,037 + 55\,568\,\beta) + \alpha^7\,(310\,983\,696 + 569\,688\,\beta - 427\,584\,\beta^2) -$$

$$4\,\alpha^6\,(-3\,400\,879\,692 + 706\,797\,684\,\beta - 29\,662\,983\,\beta^2 + 265\,168\,\beta^3) +$$

$$8\,\alpha^5\,(-177\,848\,489\,256 + 23\,147\,055\,036\,\beta - 772\,303\,710\,\beta^2 + 637\,581\,\beta^3 + 162\,740\,\beta^4) +$$

$$2\,\alpha^4\,(16\,653\,273\,975\,408 - 1\,408\,260\,630\,048\,\beta - 31\,889\,242\,392\,\beta^2 + 5\,263\,732\,824\,\beta^3 -$$

$$126\,170\,317\,\beta^4 + 650\,960\,\beta^5) - 8\,\alpha^3\,(-10\,243\,906\,550\,624 + 11\,044\,893\,936\,048\,\beta -$$

$$1\,521\,827\,988\,144\,\beta^2 + 75\,232\,559\,416\,\beta^3 - 1\,315\,933\,206\,\beta^4 - 637\,581\,\beta^5 + 132\,584\,\beta^6) -$$

$$4\,\alpha^2\,(3\,603\,528\,713\,601\,600 - 920\,482\,981\,762\,880\,\beta + 83\,558\,555\,055\,024\,\beta^2 -$$

$$3\,043\,655\,976\,288\,\beta^3 + 15\,944\,621\,196\,\beta^4 + 1\,544\,607\,420\,\beta^5 - 29\,662\,983\,\beta^6 + 106\,896\,\beta^7) + 8\,\alpha$$

$$(26\,139\,197\,720\,684\,160 - 5\,881\,484\,099\,986\,880\,\beta + 460\,241\,490\,881\,440\,\beta^2 - 11\,044\,893\,936\,048$$

$$\beta^3 - 352\,065\,157\,512\,\beta^4 + 23\,147\,055\,036\,\beta^5 - 353\,398\,842\,\beta^6 + 71\,211\,\beta^7 + 20\,838\,\beta^8)) +$$

$$\frac{1}{45\,875\,200}\,x^2\,(216\,436\,980\,027\,142\,784\,000 + 176\,769\,\alpha^{10} - 58\,405\,994\,519\,315\,788\,800\,\beta +$$

$$5\,600\,155\,872\,127\,837\,440\,\beta^2 - 160\,039\,740\,433\,771\,520\,\beta^3 - 9\,077\,583\,480\,653\,184\,\beta^4 +$$

$$799\,582\,654\,641\,792\,\beta^5 - 19\,539\,788\,153\,184\,\beta^6 + 767\,731\,776\,\beta^7 + 6\,915\,966\,036\,\beta^8 - 85\,161\,300\,\beta^9 +$$

$$176\,769\,\beta^{10} + 150\,\alpha^9\,(-567\,742 + 19\,617\,\beta) - 3\,\alpha^8\,(-2\,305\,322\,012 + 79\,561\,020\,\beta + 675\,441\,\beta^2) -$$

$$24\,\alpha^7\,(-31\,988\,824 + 1\,116\,519\,564\,\beta - 72\,812\,698\,\beta^2 + 1\,006\,677\,\beta^3) + 2\,\alpha^6$$

$$(-9\,769\,894\,076\,592 + 1\,678\,776\,040\,032\,\beta - 84\,712\,312\,296\,\beta^2 + 1\,261\,183\,000\,\beta^3 + 723\,177\,\beta^4) +$$

$$4\,\alpha^5\,(199\,895\,663\,660\,448 - 28\,406\,944\,334\,448\,\beta + 1\,033\,776\,542\,736\,\beta^2 +$$

$$10\,752\,802\,824\,\beta^3 - 1\,034\,675\,558\,\beta^4 + 10\,810\,449\,\beta^5) +$$

$$2\,\alpha^4\,(-4\,538\,791\,740\,326\,592 + 192\,293\,069\,172\,288\,\beta + 60\,131\,883\,805\,104\,\beta^2 -$$

$$5\,943\,145\,313\,952\,\beta^3 + 192\,080\,118\,908\,\beta^4 - 2\,069\,351\,116\,\beta^5 + 723\,177\,\beta^6) - 8\,\alpha^3$$

$$(20\,004\,967\,554\,221\,440 - 7\,897\,057\,097\,743\,296\,\beta + 1\,036\,745\,242\,083\,936\,\beta^2 - 60\,130\,766\,224\,880$$

$$\beta^3 + 1\,485\,786\,328\,488\,\beta^4 - 5\,376\,401\,412\,\beta^5 - 315\,295\,750\,\beta^6 + 3\,020\,031\,\beta^7) +$$

$$\alpha^2\,(5\,600\,155\,872\,127\,837\,440 - 1\,593\,531\,048\,851\,338\,240\,\beta + 172\,007\,239\,262\,654\,208\,\beta^2 -$$

$$8\,293\,961\,936\,671\,488\,\beta^3 + 120\,263\,767\,610\,208\,\beta^4 + 4\,135\,106\,170\,944\,\beta^5 -$$

$$169\,424\,624\,592\,\beta^6 + 1\,747\,504\,752\,\beta^7 - 2\,026\,323\,\beta^8) +$$

$$2\,\alpha\,(-29\,202\,997\,259\,657\,894\,400 + 7\,912\,668\,481\,688\,615\,680\,\beta - 796\,765\,524\,425\,669\,120\,\beta^2 +$$

$$31\,588\,228\,390\,973\,184\,\beta^3 + 192\,293\,069\,172\,288\,\beta^4 - 56\,813\,888\,668\,896\,\beta^5 +$$

$$1\,678\,776\,040\,032\,\beta^6 - 13\,398\,234\,768\,\beta^7 - 119\,341\,530\,\beta^8 + 1\,471\,275\,\beta^9)) +$$

$$\frac{1}{45\,875\,200}\,(-219\,234\,922\,034\,110\,259\,200 - 176\,769\,\alpha^{10} + 58\,873\,877\,637\,010\,867\,200\,\beta -$$

$$5\,621\,118\,626\,333\,763\,840\,\beta^2 + 159\,618\,156\,384\,368\,640\,\beta^3 +$$

$$9\,132\,888\,849\,256\,704\,\beta^4 - 800\,758\,778\,639\,232\,\beta^5 + 19\,536\,577\,590\,304\,\beta^6 -$$

$$493\,118\,976\,\beta^7 - 6\,917\,339\,856\,\beta^8 + 85\,161\,300\,\beta^9 - 176\,769\,\beta^{10} -$$

$$150\,\alpha^9\,(-567\,742 + 19\,617\,\beta) + 3\,\alpha^8\,(-2\,305\,779\,952 + 79\,561\,020\,\beta + 675\,441\,\beta^2) +$$

$$24\,\alpha^7\,(-20\,546\,624 + 1\,116\,338\,544\,\beta - 72\,812\,698\,\beta^2 + 1\,006\,677\,\beta^3) -$$

$$2\,\alpha^6\,(-9\,768\,288\,795\,152 + 1\,679\,215\,464\,192\,\beta - 84\,726\,342\,816\,\beta^2 + 1\,261\,183\,000\,\beta^3 + 723\,177\,\beta^4) -$$

$$4\,\alpha^5\,(200\,189\,694\,659\,808 - 28\,442\,864\,003\,408\,\beta + 1\,034\,731\,184\,256\,\beta^2 +$$

$$10\,750\,540\,704\,\beta^3 - 1\,034\,675\,558\,\beta^4 + 10\,810\,449\,\beta^5) -$$

$$
\begin{aligned}
&2\,\alpha^4\,\big(-4\,566\,444\,424\,628\,352+194\,638\,356\,210\,368\,\beta+60\,122\,281\,096\,464\,\beta^2-\\
&\quad 5\,945\,477\,137\,152\,\beta^3+192\,111\,510\,128\,\beta^4-2\,069\,351\,116\,\beta^5+723\,177\,\beta^6\big)+8\,\alpha^3\\
&\quad\big(19\,952\,269\,548\,046\,080-7\,903\,570\,841\,717\,376\,\beta+1\,038\,031\,999\,549\,216\,\beta^2-60\,183\,798\,606\,800\\
&\quad\quad \beta^3+1\,486\,369\,284\,288\,\beta^4-5\,375\,270\,352\,\beta^5-315\,295\,750\,\beta^6+3\,020\,031\,\beta^7\big)+\\
&\alpha^2\,\big(-5\,621\,118\,626\,333\,763\,840+1\,598\,610\,261\,652\,490\,240\,\beta-172\,396\,893\,523\,409\,408\,\beta^2+\\
&\quad 8\,304\,255\,996\,393\,728\,\beta^3-120\,244\,562\,192\,928\,\beta^4-4\,138\,924\,737\,024\,\beta^5+\\
&\quad 169\,452\,685\,632\,\beta^6-1\,747\,504\,752\,\beta^7+2\,026\,323\,\beta^8\big)-\\
&2\,\alpha\,\big(-29\,436\,938\,818\,505\,433\,600+7\,955\,897\,930\,921\,588\,480\,\beta-799\,305\,130\,826\,245\,120\,\beta^2+\\
&\quad 31\,614\,283\,366\,869\,504\,\beta^3+194\,638\,356\,210\,368\,\beta^4-56\,885\,728\,006\,816\,\beta^5+\\
&\quad 1\,679\,215\,464\,192\,\beta^6-13\,396\,062\,528\,\beta^7-119\,341\,530\,\beta^8+1\,471\,275\,\beta^9\big)\big)-\\
&\frac{1}{137\,625\,600}\,x\,\big(579\,406\,779\,228\,117\,299\,200+2\,336\,247\,\alpha^{11}-159\,792\,178\,882\,189\,593\,600\,\beta+\\
&\quad 15\,707\,826\,122\,364\,660\,480\,\beta^2-472\,440\,701\,943\,592\,960\,\beta^3-\\
&\quad 24\,727\,137\,072\,352\,512\,\beta^4+2\,288\,397\,905\,049\,216\,\beta^5-57\,522\,431\,206\,752\,\beta^6+\\
&\quad 26\,367\,176\,448\,\beta^7+20\,257\,610\,688\,\beta^8-253\,913\,340\,\beta^9+530\,307\,\beta^{10}+\\
&\quad 3\,\alpha^{10}\,(-64\,687\,911+120\,310\,\beta)+3\,\alpha^9\,\big(2\,436\,836\,076-55\,014\,910\,\beta+1\,554\,237\,\beta^2\big)+\\
&\quad \alpha^8\,\big(-1\,111\,332\,288\,384+148\,495\,128\,612\,\beta-6\,994\,888\,953\,\beta^2+105\,301\,816\,\beta^3\big)+\\
&\quad 2\,\alpha^7\,\big(61\,973\,060\,463\,888-10\,162\,118\,311\,488\,\beta+589\,799\,793\,000\,\beta^2-13\,812\,183\,292\,\beta^3+\\
&\quad\quad 102\,409\,111\,\beta^4\big)+2\,\alpha^6\,\big(-3\,346\,468\,529\,358\,768+615\,765\,377\,591\,328\,\beta-\\
&\quad\quad 41\,515\,050\,287\,712\,\beta^2+1\,217\,703\,564\,584\,\beta^3-13\,566\,215\,677\,\beta^4+18\,641\,014\,\beta^5\big)-\\
&\quad 2\,\alpha^5\,\big(-101\,053\,560\,328\,560\,832+20\,501\,669\,845\,524\,384\,\beta-1\,548\,664\,850\,901\,168\,\beta^2+\\
&\quad\quad 51\,468\,908\,444\,672\,\beta^3-595\,440\,251\,996\,\beta^4-3\,445\,451\,506\,\beta^5+77\,456\,749\,\beta^6\big)-\\
&\quad 2\,\alpha^4\,\big(1\,813\,264\,429\,608\,189\,056-401\,064\,926\,116\,798\,016\,\beta+33\,077\,365\,561\,780\,560\,\beta^2-\\
&\quad\quad 1\,164\,286\,889\,341\,504\,\beta^3+9\,247\,666\,659\,872\,\beta^4+422\,434\,840\,964\,\beta^5-9\,280\,473\,371\,\beta^6+\\
&\quad\quad 42\,265\,092\,\beta^7\big)+\alpha^3\,\big(38\,254\,809\,244\,239\,845\,120-9\,141\,223\,405\,828\,811\,776\,\beta+\\
&\quad\quad 806\,918\,195\,957\,251\,840\,\beta^2-28\,330\,947\,854\,433\,664\,\beta^3-49\,884\,676\,216\,608\,\beta^4+\\
&\quad\quad 29\,915\,419\,601\,280\,\beta^5-706\,276\,426\,736\,\beta^6+4\,379\,720\,712\,\beta^7+11\,378\,811\,\beta^8\big)+\\
&\quad \alpha^2\,\big(-213\,988\,513\,613\,135\,985\,920+54\,881\,256\,217\,533\,002\,240\,\beta-5\,102\,866\,095\,826\,582\,016\,\beta^2+\\
&\quad\quad 166\,358\,284\,242\,771\,200\,\beta^3+3\,803\,228\,117\,972\,832\,\beta^4-424\,920\,360\,076\,224\,\beta^5+\\
&\quad\quad 10\,447\,179\,070\,656\,\beta^6-51\,058\,511\,472\,\beta^7-1\,199\,079\,249\,\beta^8+10\,398\,210\,\beta^9\big)+\\
&\quad \alpha\,\big(419\,614\,600\,345\,927\,705\,600-115\,836\,092\,767\,052\,833\,280\,\beta+11\,207\,191\,549\,003\,129\,600\,\beta^2-\\
&\quad\quad 289\,822\,893\,935\,850\,496\,\beta^3-23\,785\,158\,252\,138\,624\,\beta^4+1\,961\,905\,229\,810\,880\,\beta^5-\\
&\quad\quad 47\,673\,319\,486\,560\,\beta^6-53\,963\,623\,680\,\beta^7+19\,554\,897\,828\,\beta^8-245\,085\,690\,\beta^9+530\,307\,\beta^{10}\big)\big)
\end{aligned}
$$

(* In particular, the coefficient at the monomial

eingegebenes Objekt

x^10 still depends on α and β and reads as follows: *)

$$-\frac{3}{4}\left(-4700+\alpha^2-2\,\alpha\,(-14+\beta)+28\,\beta+\beta^2\right)$$

(* The parameter α=Root[$F_{m(12),7}$(α),4] reads as follows: *)

Nullstelle

```
In[•]:= Root[-107 848 865 858 158 208 685 982 531 273 759 493 499 860 591 730 275 673 813 647 +
        Nullstelle
   5 896 390 467 832 664 197 397 063 579 930 767 052 162 624 915 964 005 522 656 α +
   60 468 584 846 611 294 096 619 151 936 522 735 830 307 775 093 619 753 313 264 α^2 -
   37 687 080 190 910 356 549 385 088 653 134 434 998 316 561 920 003 471 283 808 α^3 +
   11 213 506 557 431 859 110 268 685 515 321 866 965 676 966 249 606 637 084 152 α^4 -
   2 890 483 939 571 150 356 005 505 951 720 207 430 498 271 439 566 596 611 488 α^5 +
   1 407 343 031 407 743 573 729 790 472 766 575 133 321 871 381 624 453 236 688 α^6 -
   768 522 437 733 631 393 516 123 089 228 747 757 931 566 373 608 599 529 184 α^7 +
   309 089 648 482 344 375 465 416 490 183 670 375 902 954 624 023 157 419 100 α^8 -
   91 665 965 263 919 406 170 654 880 827 944 077 842 962 655 653 819 153 568 α^9 +
   21 090 584 050 592 309 741 254 996 886 955 373 769 587 989 100 209 813 232 α^10 -
   3 921 902 502 327 597 976 379 827 313 664 749 858 484 716 493 517 988 064 α^11 +
   607 687 627 992 668 886 219 605 788 806 685 921 953 063 673 659 671 496 α^12 -
   80 328 111 971 043 477 959 138 752 923 731 931 858 721 328 910 620 448 α^13 +
   9 228 330 831 773 787 640 502 677 381 635 654 561 811 853 766 113 616 α^14 -
   934 469 294 882 594 744 951 636 706 892 194 457 549 463 812 805 984 α^15 +
   84 183 418 001 687 840 841 016 769 565 964 538 271 323 818 850 278 α^16 -
   6 775 149 364 939 541 659 670 273 118 716 783 325 813 571 127 136 α^17 +
   486 824 030 992 857 804 601 196 578 024 690 350 273 984 821 072 α^18 -
   31 095 507 033 209 700 702 451 777 309 791 965 699 731 794 208 α^19 +
   1 753 818 594 517 767 513 944 854 455 153 976 181 197 535 688 α^20 -
   86 653 634 332 316 513 980 840 145 024 029 079 821 296 352 α^21 +
   3 718 523 653 595 524 900 441 065 631 330 182 658 688 240 α^22 -
   137 315 359 285 901 700 622 944 618 402 731 836 972 704 α^23 +
   4 318 590 666 564 168 545 637 310 473 402 802 574 940 α^24 -
   114 269 021 034 715 595 201 925 084 740 972 769 504 α^25 +
   2 505 131 797 973 747 722 149 189 942 013 423 056 α^26 -
   44 594 826 811 354 733 918 577 364 928 251 808 α^27 + 626 811 420 142 725 808 750 299 240 311 800 α^28 -
   6 675 916 114 653 511 081 182 462 972 000 α^29 + 50 448 848 241 079 938 293 552 790 000 α^30 -
   239 819 039 658 942 704 522 100 000 α^31 + 535 282 896 407 331 842 240 625 α^32, 4]

(* Its numerical approximation is α=
 84.00595217006947300041528590967861014567349221184310468438115211985486362649496641 2\
  66904719385932732062086260314284 37`99. *)

(* The parameter β=Root[G_m(12),7(β),4]reads as follows: *)
                    Nullstelle
```

In[ ]:= Root[36 661 005 940 422 965 034 511 511 934 416 181 790 904 605 809 579 497 795 569 +
Nullstelle

495 930 905 777 285 619 757 015 828 001 648 623 927 393 797 314 303 210 144 z −
22 461 685 700 725 690 285 747 084 636 139 707 907 998 078 567 088 220 308 880 z^2 +
7 064 460 938 205 024 342 650 833 343 297 517 980 690 839 174 183 605 342 688 z^3 +
6 677 564 912 854 871 800 117 765 678 773 867 316 523 842 506 039 458 978 936 z^4 −
7 286 735 297 559 400 208 020 663 638 945 555 606 537 622 563 868 260 654 816 z^5 +
3 618 448 011 507 442 669 328 434 769 659 421 088 607 200 433 331 658 492 496 z^6 −
1 205 456 273 698 954 496 034 313 124 668 645 770 048 274 871 490 702 817 696 z^7 +
307 701 892 904 036 687 277 331 170 820 291 655 032 178 006 992 872 427 740 z^8 −
65 881 644 244 646 650 658 539 284 425 532 711 776 216 275 623 163 006 432 z^9 +
12 714 941 847 331 759 604 314 827 744 709 741 159 346 252 497 241 227 120 z^{10} −
2 297 099 581 468 061 331 521 240 588 954 718 037 597 549 449 520 541 088 z^{11} +
386 042 027 329 072 605 523 055 200 968 280 817 690 519 961 404 206 152 z^{12} −
58 587 037 612 902 351 617 172 268 626 476 460 286 095 743 617 189 728 z^{13} +
7 817 558 797 031 564 644 627 997 793 863 855 334 782 095 675 823 568 z^{14} −
903 790 764 982 825 707 122 764 110 357 285 245 023 964 751 600 928 z^{15} +
90 067 668 597 693 047 168 435 778 736 505 495 249 665 745 850 214 z^{16} −
7 736 908 981 033 457 461 397 375 440 854 372 916 610 233 285 152 z^{17} +
574 000 126 780 516 483 443 998 200 442 802 083 607 071 222 480 z^{18} −
36 857 807 601 021 884 315 769 018 269 553 683 819 542 959 200 z^{19} +
2 051 114 955 760 924 631 883 735 510 210 715 788 911 105 352 z^{20} −
98 908 353 792 471 368 521 262 573 883 774 503 621 931 680 z^{21} +
4 124 397 458 952 224 589 377 931 683 256 444 374 428 784 z^{22} −
148 063 788 988 980 393 862 713 891 670 527 430 763 232 z^{23} +
4 543 041 658 663 972 115 276 667 095 588 196 390 876 z^{24} −
117 874 876 905 229 559 705 130 477 343 396 147 872 z^{25} +
2 547 907 073 150 637 884 378 151 091 304 889 168 z^{26} −
44 943 403 579 896 042 223 632 021 086 423 008 z^{27} + 628 484 870 915 406 340 339 654 443 239 800 z^{28} −
6 678 536 935 248 185 216 935 191 732 000 z^{29} + 50 438 713 730 261 337 896 985 990 000 z^{30} −
239 794 435 522 026 626 125 500 000 z^{31} + 535 282 896 407 331 842 240 625 z^{32}, 4]

(* Its numerical approximation is β=
84.005952170069473000415397532382698852748841436237702915699403058686910338856326247\
1542126828162916294720896651923 61`99. *)

(* After insertion,
Fläche nach dem Objekt
a numerical approximation of $Z_{12,7}(x)=\sum_{k=0}^{12} a_{k,12}^{*}(7)\, x^{k}$ reads as follows: *)

Out[∘]=

0.00097652790324861920483670972088375342758172141338761622540242335512002089901902106
8172865691655925010 +
0.9023175946500449614799742528348622212161610039946145999822976494869293571528521563
00759792734899376 x −
0.0595688940135826272950669482509784101589441840937292739149945521755884550256738457
637276238365146155 x^{2} −
18.046526258156822420054142846940572345221446498908305328866071784756582258881999266
8440243457855634 x^{3} +
0.6054590626938925845349234013353083099653181273440950985545759146984603678391574076
16580484058768694 x^{4} +
101.06124452013929148532916475978217390978730020331310248047010392205639481289584558
5197777401062936 x^{5} −
2.2968511967682720109004226403924246341095099803671201835609892690711789030471338257
2371602605133569 x^{6} −
230.99832601806630122694407201540025900204576352960684226466495118119085314320246011
5514385637957786 x^{7} +
3.9999756431025794519000817608746999477445609347450473954439616600904059961978678267
2327112783748711 x^{8} +
230.99925600510074546532559179614907321699655586865653900284361460141322696800325744
4368680488191498 x^{9} −
3.2499911429178660174443522832874889668690066190416806527479561768972190268632365839
205808277006142 x^{10} −
84.000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000 x^{11} + x^{12}

(* To obtain a non-numerical explicit representation of $Z_{12,7}(x)=\sum_{k=0}^{12} a_{k,12}^{*}(7)\, x^{k}$ would require to insert into $(R_{12,\alpha,\beta}(x))$ /. s→7 the parameters α and β *)

(* In particular, inserting the parameters α and β into the coefficient at the
eingegebenes Objekt
monomial x^10 (i.e., into $-\frac{3}{4}\left(-4700+\alpha^{2}-2\,\alpha\,(-14+\beta)+28\,\beta+\beta^{2}\right)$) would yield,
after simplification with RootReduce or FullSimplify,
reduziere auf Null·· vereinfache vollständig
the following algebraic representation of $a_{10,12}^{*}(7)$ as a root object: *)

Root[
Nullstelle

−1 026 327 805 681 015 105 257 383 782 617 772 629 023 751 409 307 131 841 668 004 454 065 605 091
493 343 023 167 833 970 191 679 563 968 273 085 342 968 580 528 242 955 224 946 311 168 −
305 228 603 968 759 014 623 415 661 059 570 593 739 124 512 206 048 404 590 173 573 490 489 958 790
023 062 727 210 495 310 533 100 274 766 583 710 186 007 030 538 795 070 802 362 368 z +
3 198 294 811 369 299 767 353 012 476 751 966 033 690 872 658 704 035 205 021 317 575 952 334 096
577 963 047 850 187 414 389 775 166 949 274 351 799 258 263 037 994 288 206 053 376 z^{2} −
16 020 417 545 136 845 820 831 565 447 888 019 822 951 350 656 506 405 709 721 769 469 829 995 305
947 098 174 759 708 374 594 406 751 369 528 971 416 814 069 613 274 748 420 096 z^{3} +
51 550 616 804 128 254 207 352 732 395 587 220 572 594 905 269 783 754 968 959 753 466 039 350 213

615 893 099 492 992 372 070 652 944 876 987 248 343 890 687 908 149 985 280 z^{4} –
119 950 385 546 737 204 741 170 765 513 302 213 077 009 054 996 808 489 052 621 327 242 483 460 917 ⋱
848 877 141 625 667 717 206 643 983 245 179 449 912 786 967 119 003 648 z^{5} +
215 132 738 942 344 358 550 275 286 203 622 335 382 033 790 616 529 062 258 211 773 735 345 120 786 ⋱
990 075 742 955 947 099 618 053 118 589 457 183 440 649 425 256 448 z^{6} –
309 449 994 157 006 793 268 674 675 099 002 609 757 809 248 289 886 216 544 357 886 189 067 280 152 ⋱
038 822 864 409 496 613 170 531 893 151 650 360 583 121 797 120 z^{7} +
366 690 912 648 408 652 824 175 739 440 536 928 833 759 646 183 312 781 395 040 870 739 226 276 886 ⋱
720 214 332 098 410 821 146 977 588 668 432 760 396 840 960 z^{8} –
364 859 669 953 547 629 433 318 967 333 074 365 399 907 754 816 282 461 228 926 152 845 273 122 564 ⋱
187 927 481 599 745 377 436 444 759 561 457 662 689 280 z^{9} +
309 151 457 531 259 178 400 917 319 022 475 860 447 250 618 464 663 873 821 149 836 235 473 203 157 ⋱
425 506 729 494 020 742 551 558 285 902 875 721 728 z^{10} –
225 435 379 838 193 774 084 232 923 050 347 154 748 688 366 778 076 870 372 851 379 154 307 781 080 ⋱
494 346 708 675 361 371 975 332 760 433 721 344 z^{11} +
142 611 083 309 666 798 577 838 806 958 737 534 819 027 017 005 623 146 237 181 276 836 998 324 199 ⋱
546 585 885 549 735 175 934 976 137 887 744 z^{12} –
78 738 890 271 310 355 127 104 078 916 330 696 865 992 573 448 580 347 242 389 057 520 214 741 294 ⋱
305 196 925 960 167 012 655 463 137 280 z^{13} +
38 113 088 029 350 657 088 339 676 152 602 056 329 663 617 668 241 781 303 242 913 205 827 429 991 ⋱
478 440 858 906 849 677 903 134 720 z^{14} –
16 224 982 244 981 231 846 899 186 882 492 943 051 865 712 309 727 247 726 074 501 764 962 956 809 ⋱
746 062 753 845 938 241 929 216 z^{15} +
6 086 975 571 430 911 627 496 927 529 624 199 367 838 940 408 016 439 231 249 174 192 333 491 281 ⋱
140 020 713 161 391 734 784 z^{16} –
2 014 456 234 881 041 533 353 828 715 991 448 945 545 932 281 408 289 806 859 374 956 762 317 772 ⋱
734 751 798 014 771 200 z^{17} +
588 105 612 874 435 357 774 663 486 167 248 200 084 471 276 361 916 139 386 898 392 300 237 050 588 ⋱
309 366 243 328 z^{18} –
151 309 757 198 248 690 938 208 738 761 856 589 681 423 897 938 046 863 807 058 868 492 309 712 890 ⋱
305 445 888 z^{19} +
34 238 570 494 025 665 720 279 630 957 409 727 249 823 466 632 692 004 357 138 523 225 459 160 564 ⋱
793 344 z^{20} –
6 792 513 842 505 720 716 194 672 612 043 673 580 083 281 778 103 503 327 365 093 845 035 295 899 648
z^{21} +
1 176 180 944 184 237 621 574 217 389 213 175 416 928 385 344 562 257 788 230 984 040 574 533 632
z^{22} –
176 697 372 017 412 693 210 814 634 610 268 425 791 979 024 855 077 709 062 254 453 997 568 z^{23} +
22 847 238 860 986 958 892 270 647 154 055 909 878 640 226 185 037 899 115 127 327 744 z^{24} –
2 516 018 014 698 023 808 053 490 372 810 410 681 292 294 386 647 076 866 240 512 z^{25} +
232 695 232 700 747 325 098 125 792 349 091 046 618 315 643 608 297 423 872 z^{26} –
17 733 635 485 948 730 330 129 883 253 668 696 324 618 074 007 585 792 z^{27} +
1 084 315 112 236 477 102 205 512 127 708 371 576 527 104 470 400 z^{28} –
51 136 463 718 769 726 940 974 542 376 374 252 685 152 000 z^{29} +
1 746 030 200 710 086 122 433 779 153 198 852 280 000 z^{30} –
38 413 686 899 622 615 133 357 948 235 400 000 z^{31} + 408 865 911 763 997 509 841 936 503 125 z^{32}, 1]

```
(* Its numerical value is indeed *)
```

In[ ]:= **N[Root[**

Nullstelle

- 1 026 327 805 681 015 105 257 383 782 617 772 629 023 751 409 307 131 841 668 004 454 065 605 091 \
493 343 023 167 833 970 191 679 563 968 273 085 342 968 580 528 242 955 224 946 311 168 -
305 228 603 968 759 014 623 415 661 059 570 593 739 124 512 206 048 404 590 173 573 490 489 958 \
790 023 062 727 210 495 310 533 100 274 766 583 710 186 007 030 538 795 070 802 362 368 z +
3 198 294 811 369 299 767 353 012 476 751 966 033 690 872 658 704 035 205 021 317 575 952 334 096 \
577 963 047 850 187 414 389 775 166 949 274 351 799 258 263 037 994 288 206 053 376 z^{2} -
16 020 417 545 136 845 820 831 565 447 888 019 822 951 350 656 506 405 709 721 769 469 829 995 305 \
947 098 174 759 708 374 594 406 751 369 528 971 416 814 069 613 274 748 420 096 z^{3} +
51 550 616 804 128 254 207 352 732 395 587 220 572 594 905 269 783 754 968 959 753 466 039 350 213 \
615 893 099 492 992 372 070 652 944 876 987 248 343 890 687 908 149 985 280 z^{4} -
119 950 385 546 737 204 741 170 765 513 302 213 077 009 054 996 808 489 052 621 327 242 483 460 \
917 848 877 141 625 667 717 206 643 983 245 179 449 912 786 967 119 003 648 z^{5} +
215 132 738 942 344 358 550 275 286 203 622 335 382 033 790 616 529 062 258 211 773 735 345 120 \
786 990 075 742 955 947 099 618 053 118 589 457 183 440 649 425 256 448 z^{6} -
309 449 994 157 006 793 268 674 675 099 002 609 757 809 248 289 886 216 544 357 886 189 067 280 \
152 038 822 864 409 496 613 170 531 893 151 650 360 583 121 797 120 z^{7} +
366 690 912 648 408 652 824 175 739 440 536 928 833 759 646 183 312 781 395 040 870 739 226 276 \
886 720 214 332 098 410 821 146 977 588 668 432 760 396 840 960 z^{8} -
364 859 669 953 547 629 433 318 967 333 074 365 399 907 754 816 282 461 228 926 152 845 273 122 \
564 187 927 481 599 745 377 436 444 759 561 457 662 689 280 z^{9} +
309 151 457 531 259 178 400 917 319 022 475 860 447 250 618 464 663 873 821 149 836 235 473 203 \
157 425 506 729 494 020 742 551 558 285 902 875 721 728 z^{10} -
225 435 379 838 193 774 084 232 923 050 347 154 748 688 366 778 076 870 372 851 379 154 307 781 \
080 494 346 708 675 361 371 975 332 760 433 721 344 z^{11} +
142 611 083 309 666 798 577 838 806 958 737 534 819 027 017 005 623 146 237 181 276 836 998 324 \
199 546 585 885 549 735 175 934 976 137 887 744 z^{12} -
78 738 890 271 310 355 127 104 078 916 330 696 865 992 573 448 580 347 242 389 057 520 214 741 294 \
305 196 925 960 167 012 655 463 137 280 z^{13} +
38 113 088 029 350 657 088 339 676 152 602 056 329 663 617 668 241 781 303 242 913 205 827 429 991 \
478 440 858 906 849 677 903 134 720 z^{14} -
16 224 982 244 981 231 846 899 186 882 492 943 051 865 712 309 727 247 726 074 501 764 962 956 809 \
746 062 753 845 938 241 929 216 z^{15} +
6 086 975 571 430 911 627 496 927 529 624 199 367 838 940 408 016 439 231 249 174 192 333 491 281 \
140 020 713 161 391 734 784 z^{16} -
2 014 456 234 881 041 533 353 828 715 991 448 945 545 932 281 408 289 806 859 374 956 762 317 772 \
734 751 798 014 771 200 z^{17} +
588 105 612 874 435 357 774 663 486 167 248 200 084 471 276 361 916 139 386 898 392 300 237 050 \
588 309 366 243 328 z^{18} -
151 309 757 198 248 690 938 208 738 761 856 589 681 423 897 938 046 863 807 058 868 492 309 712 \
890 305 445 888 z^{19} +
34 238 570 494 025 665 720 279 630 957 409 727 249 823 466 632 692 004 357 138 523 225 459 160 564 \
793 344 z^{20} -
6 792 513 842 505 720 716 194 672 612 043 673 580 083 281 778 103 503 327 365 093 845 035 295 899 \
648 z^{21} +
1 176 180 944 184 237 621 574 217 389 213 175 416 928 385 344 562 257 788 230 984 040 574 533 632
z^{22} - 176 697 372 017 412 693 210 814 634 610 268 425 791 979 024 855 077 709 062 254 453 997 568
z^{23} + 22 847 238 860 986 958 892 270 647 154 055 909 878 640 226 185 037 899 115 127 327 744 z^{24} -
2 516 018 014 698 023 808 053 490 372 810 410 681 292 294 386 647 076 866 240 512 z^{25} +
232 695 232 700 747 325 098 125 792 349 091 046 618 315 643 608 297 423 872 z^{26} -

17 733 635 485 948 730 330 129 883 253 668 696 324 618 074 007 585 792 z^{27} +
1 084 315 112 236 477 102 205 512 127 708 371 576 527 104 470 400 z^{28} −
51 136 463 718 769 726 940 974 542 376 374 252 685 152 000 z^{29} +
1 746 030 200 710 086 122 433 779 153 198 852 280 000 z^{30} −
38 413 686 899 622 615 133 357 948 235 400 000 z^{31} +
408 865 911 763 997 509 841 936 503 125 z^{32}, 1], 99]

Out[ ]=

−3.2499911429178660174443522832874889668690066190416806527479561768972190268632365839\
2058082770006142