(* Goal: Algebraic solution of ZFP if n=11 and s=6 and exemplarily one coefficient, $a^*_{9,11}(6)$, is explicitly expressed as a root object (see Appendix 3 in[6]) *)

(* The proper monic Zolotarev polynomial of degree n= 11 and with parameter s=6 can be generated by *)

(* $Z_{11,6}(x) = \sum_{k=0}^{11} a^*_{k,11}(6)\, x^k = (R_{11,\alpha,\beta}(x))$ /.s→6/.α→Root$[F_{m(11),6}(\alpha),10]$/.β→Root$[G_{m(11),6}(\beta),10]$ *)

Nullstelle Nullstelle

(* The first insertion, $(R_{11,\alpha,\beta}(x))$/.s→6, will yield the following polynomial of degree n=11, still depending on α and β: *)

Out[•]=

$$
\begin{aligned}
&\frac{1}{371\,589\,120}\\
&\Big(12\,049\,561\,154\,138\,872\,320 - 24\,524\,881\,920\,x^{10} + 371\,589\,120\,x^{11} - 1\,653\,727\,002\,870\,009\,600\,\beta +\\
&\quad 11\,801\,008\,826\,507\,526\,912\,\beta^2 - 4\,870\,566\,482\,290\,172\,928\,\beta^3 + 843\,897\,002\,707\,725\,120\,\beta^4 -\\
&\quad 78\,983\,858\,154\,079\,296\,\beta^5 + 4\,294\,555\,757\,057\,808\,\beta^6 - 135\,060\,663\,592\,800\,\beta^7 +\\
&\quad 2\,307\,184\,553\,322\,\beta^8 - 21\,092\,817\,185\,\beta^9 + 238\,676\,922\,\beta^{10} - 3\,394\,331\,\beta^{11} -\\
&\quad 2\,200\,022\,\alpha^9\,(1+\beta^2) - 255\,467\,520\,x^9\,(-3164 + \alpha^2 - 2\,\alpha\,(-12+\beta) + 24\,\beta + \beta^2) -\\
&\quad 363\,\alpha^8\,(-1\,085\,886 + 15\,139\,\beta - 1\,085\,886\,\beta^2 + 42\,145\,\beta^3) +\\
&\quad 8712\,\alpha^7\,(-1\,069\,839 - 141\,082\,\beta - 1\,063\,144\,\beta^2 - 1678\,\beta^3 + 2703\,\beta^4) +\\
&\quad 44\,\alpha^6\,(-21\,421\,486\,548 + 3\,815\,770\,932\,\beta - 21\,545\,695\,710\,\beta^2 + 4\,064\,153\,923\,\beta^3 -\\
&\qquad 225\,680\,994\,\beta^4 + 3\,676\,753\,\beta^5) + 132\,\alpha^5\,(410\,667\,252\,696 - 42\,184\,772\,808\,\beta +\\
&\qquad 410\,178\,436\,374\,\beta^2 - 76\,917\,320\,940\,\beta^3 + 4\,882\,085\,549\,\beta^4 - 116\,722\,860\,\beta^5 + 731\,975\,\beta^6) -\\
&\quad 66\,\alpha^4\,(12\,992\,040\,756\,576 - 19\,618\,556\,832\,\beta + 12\,758\,773\,005\,960\,\beta^2 - 2\,229\,979\,293\,984\,\beta^3 +\\
&\qquad 103\,834\,042\,626\,\beta^4 + 1\,206\,207\,299\,\beta^5 - 177\,811\,158\,\beta^6 + 2\,859\,881\,\beta^7) -\\
&\quad 88\,\alpha^3\,(55\,895\,102\,526\,384 - 37\,916\,753\,698\,080\,\beta + 60\,625\,922\,298\,408\,\beta^2 - 31\,724\,402\,382\,936\,\beta^3 +\\
&\qquad 5\,619\,455\,135\,739\,\beta^4 - 457\,382\,021\,310\,\beta^5 + 18\,505\,784\,902\,\beta^6 - 351\,408\,690\,\beta^7 + 2\,357\,923\,\beta^8) -\\
&\quad 396\,\alpha^2\,(-759\,290\,873\,660\,352 + 175\,612\,690\,179\,200\,\beta - 766\,764\,104\,792\,352\,\beta^2 +\\
&\qquad 282\,062\,914\,689\,536\,\beta^3 - 42\,784\,221\,728\,180\,\beta^4 + 3\,384\,151\,531\,324\,\beta^5 -\\
&\qquad 147\,948\,481\,426\,\beta^6 + 3\,464\,910\,709\,\beta^7 - 37\,722\,230\,\beta^8 + 122\,015\,\beta^9) -\\
&\quad 66\,\alpha\,(49\,952\,146\,094\,991\,360 - 8\,599\,616\,239\,327\,488\,\beta + 49\,638\,841\,582\,156\,608\,\beta^2 -\\
&\qquad 19\,033\,695\,061\,934\,976\,\beta^3 + 3\,050\,829\,153\,154\,128\,\beta^4 - 261\,205\,117\,951\,344\,\beta^5 +\\
&\qquad 12\,779\,825\,562\,444\,\beta^6 - 352\,353\,543\,192\,\beta^7 + 4\,963\,652\,839\,\beta^8 - 27\,449\,976\,\beta^9 + 34\,555\,\beta^{10}) -\\
&\quad 170\,311\,680\,x^8\,(104\,172 + \alpha^3 - 2376\,\beta - 81\,\beta^2 + \beta^3 - \alpha^2\,(81+\beta) - \alpha\,(2376 - 210\,\beta + \beta^2)) -\\
&\quad 31\,933\,440\,x^7\,(-9\,138\,656 + \alpha^4 + 8\,\alpha^3\,(-53+\beta) + 417\,696\,\beta + 9484\,\beta^2 -\\
&\qquad 424\,\beta^3 + \beta^4 + \alpha^2\,(9484 + 520\,\beta - 18\,\beta^2) + 8\,\alpha\,(52\,212 - 5275\,\beta + 65\,\beta^2 + \beta^3)) +\\
&\quad 2\,128\,896\,x^6\,(-1\,802\,784\,000 + 13\,\alpha^5 + 137\,554\,560\,\beta + 1\,030\,488\,\beta^2 - 221\,564\,\beta^3 +\\
&\qquad 2670\,\beta^4 + 13\,\beta^5 - 15\,\alpha^4\,(-178 + 9\,\beta) + 2\,\alpha^3\,(-110\,782 + 3120\,\beta + 61\,\beta^2) +\\
&\qquad 2\,\alpha^2\,(515\,244 + 189\,982\,\beta - 10\,062\,\beta^2 + 61\,\beta^3) +\\
&\qquad \alpha\,(137\,554\,560 - 16\,687\,728\,\beta + 379\,964\,\beta^2 + 6240\,\beta^3 - 135\,\beta^4)) +\\
&\quad 22\,176\,x^5\,(1\,896\,056\,596\,800 + 1415\,\alpha^6 + \alpha^5\,(18\,792 - 6370\,\beta) - 217\,284\,854\,400\,\beta +\\
&\qquad 2\,512\,338\,192\,\beta^2 + 429\,345\,024\,\beta^3 - 13\,348\,980\,\beta^4 + 18\,792\,\beta^5 + 1415\,\beta^6 -\\
&\qquad 5\,\alpha^4\,(2\,669\,796 - 187\,944\,\beta + 1291\,\beta^2) + 4\,\alpha^3\,(107\,336\,256 - 2\,844\,900\,\beta - 262\,668\,\beta^2 + 5705\,\beta^3) +\\
&\qquad \alpha^2\,(2\,512\,338\,192 - 1\,181\,727\,744\,\beta + 77\,234\,184\,\beta^2 - 1\,050\,672\,\beta^3 - 6455\,\beta^4) - 2\,\alpha\\
&\qquad\; (108\,642\,427\,200 - 16\,217\,359\,344\,\beta + 590\,863\,872\,\beta^2 + 5\,689\,800\,\beta^3 - 469\,860\,\beta^4 + 3185\,\beta^5)) +
\end{aligned}
$$

$$
\begin{aligned}
&19008\, x^4 \big(-20769955477440 + 1034\, \alpha^7 + 3335397690240\, \beta - 114936463536\, \beta^2 - \\
&\quad 6066958336\, \beta^3 + 434725116\, \beta^4 - 5673808\, \beta^5 - 65989\, \beta^6 + 1034\, \beta^7 - 7\, \alpha^6\, (9427 + 214\, \beta) - \\
&\quad 14\, \alpha^5 (405272 - 51829\, \beta + 945\, \beta^2) + \alpha^4 (434725116 - 37121424\, \beta + 312277\, \beta^2 + 13694\, \beta^3) + \\
&\quad 2\, \alpha^3 (-3033479168 - 29957736\, \beta + 28848752\, \beta^2 - 1064054\, \beta^3 + 6847\, \beta^4) + \\
&\quad \alpha^2 (-114936463536 + 33075020032\, \beta - 2559982680\, \beta^2 + 57697504\, \beta^3 + \\
&\qquad 312277\, \beta^4 - 13230\, \beta^5) - 2\, \alpha (-1667698845120 + 308374907472\, \beta - \\
&\qquad 16537510016\, \beta^2 + 29957736\, \beta^3 + 18560712\, \beta^4 - 362803\, \beta^5 + 749\, \beta^6)\big) - \\
&18\, x \big(182568293266602240 + 544621\, \alpha^8 - 38878444867046400\, \beta + 2365836944904576\, \beta^2 + \\
&\quad 30978876039936\, \beta^3 - 8164273426848\, \beta^4 + 255624600000\, \beta^5 - 607800424\, \beta^6 - 67471536 \\
&\quad \beta^7 + 544621\, \beta^8 + 1936\, \alpha^7\, (-34851 + 434\, \beta) - 308\, \alpha^6 (1973378 - 1031580\, \beta + 31789\, \beta^2) - \\
&\quad 1232\, \alpha^5 (-207487500 + 27121325\, \beta - 800181\, \beta^2 + 1582\, \beta^3) + \\
&\quad 22\, \alpha^4 (-371103337584 + 35354019360\, \beta + 21284956\, \beta^2 - 60056088\, \beta^3 + 941381\, \beta^4) - \\
&\quad 176\, \alpha^3 (-176016341136 - 61385314176\, \beta + 11362265112\, \beta^2 - 546881830\, \beta^3 + \\
&\qquad 7507011\, \beta^4 + 11074\, \beta^5) - 44\, \alpha^2 (-53769021475104 + 14963059714368\, \beta - \\
&\qquad 1373665409136\, \beta^2 + 45449060448\, \beta^3 - 10642478\, \beta^4 - 22405068\, \beta^5 + 222523\, \beta^6) + \\
&\quad 176\, \alpha (-220900254926400 + 50294832075504\, \beta - 3740764928592\, \beta^2 + \\
&\qquad 61385314176\, \beta^3 + 4419252420\, \beta^4 - 189849275\, \beta^5 + 1805265\, \beta^6 + 4774\, \beta^7)\big) + \\
&198\, x^3 \big(16383281264213760 + 49511\, \alpha^8 - 3510000778152960\, \beta + 214796234892672\, \beta^2 + \\
&\quad 2768106432768\, \beta^3 - 740711428128\, \beta^4 + 23236495296\, \beta^5 - 55413064\, \beta^6 - 6133776\, \beta^7 + \\
&\quad 49511\, \beta^8 + 176\, \alpha^7\, (-34851 + 434\, \beta) - 28\, \alpha^6 (1979038 - 1031580\, \beta + 31789\, \beta^2) - \\
&\quad 112\, \alpha^5 (-207468708 + 27114955\, \beta - 800181\, \beta^2 + 1582\, \beta^3) + \\
&\quad 2\, \alpha^4 (-370355714064 + 35301395040\, \beta + 21646436\, \beta^2 - 60056088\, \beta^3 + 941381\, \beta^4) - \\
&\quad 16\, \alpha^3 (-173006652048 - 61465052016\, \beta + 11354910408\, \beta^2 - 546722090\, \beta^3 + \\
&\qquad 7507011\, \beta^4 + 11074\, \beta^5) + \alpha^2 (214796234892672 - 59719801484544\, \beta + \\
&\qquad 5486008504896\, \beta^2 - 181678566528\, \beta^3 + 43292872\, \beta^4 + 89620272\, \beta^5 - 890092\, \beta^6) + \\
&\quad 16\, \alpha (-219375048634560 + 50067363507408\, \beta - 3732487592784\, \beta^2 + \\
&\qquad 61465052016\, \beta^3 + 4412674380\, \beta^4 - 189804685\, \beta^5 + 1805265\, \beta^6 + 4774\, \beta^7)\big) + \\
&44\, x^2 \big(-538647074742792960 + 100001\, \alpha^9 + 148408881846128640\, \beta - \\
&\quad 13617633346298496\, \beta^2 + 226212060129984\, \beta^3 + 38788191834336\, \beta^4 - \\
&\quad 2461553060112\, \beta^5 + 42871480344\, \beta^6 + 423209556\, \beta^7 - 17917119\, \beta^8 + \\
&\quad 100001\, \beta^9 + 33\, \alpha^8\, (-542943 + 14321\, \beta) - 396\, \alpha^7 (-1068711 - 71380\, \beta + 4699\, \beta^2) - \\
&\quad 4\, \alpha^6 (-10717870086 + 1969885089\, \beta - 87472539\, \beta^2 + 853529\, \beta^3) + \\
&\quad 6\, \alpha^5 (-410258843352 + 59561479176\, \beta - 2196831762\, \beta^2 - 4314504\, \beta^3 + 783721\, \beta^4) + \\
&\quad 6\, \alpha^4 (6464698639056 - 563790453624\, \beta - 32303590980\, \beta^2 + 4365319194\, \beta^3 - \\
&\qquad 121772751\, \beta^4 + 783721\, \beta^5) - 4\, \alpha^3 (-56553015032496 + 34742064165696\, \beta - \\
&\qquad 5171610114936\, \beta^2 + 300748890456\, \beta^3 - 6547978791\, \beta^4 + 6471756\, \beta^5 + 853529\, \beta^6) - \\
&\quad 36\, \alpha^2 (378267592952736 - 114293444807632\, \beta + 12565242461808\, \beta^2 - 574623346104\, \beta^3 + \\
&\qquad 5383931830\, \beta^4 + 366138627\, \beta^5 - 9719171\, \beta^6 + 51689\, \beta^7) + 3\, \alpha (49469627282042880 - \\
&\qquad 13812140973581568\, \beta + 1371521337691584\, \beta^2 - 46322752220928\, \beta^3 - \\
&\qquad 1127580907248\, \beta^4 + 119122958352\, \beta^5 - 2626513452\, \beta^6 + 9422160\, \beta^7 + 157531\, \beta^8)\big)\big)
\end{aligned}
$$

(* In particular,

eingegebenes Objekt

the coefficient at the monomial x^9 still depends on α and β and reads as follows: *)

Out[∘]=

$$-\frac{11}{16}\left(-3164+\alpha^2-2\,\alpha\,(-12+\beta)+24\,\beta+\beta^2\right)$$

(* The parameter α=Root[$F_{m(11),6}(\alpha)$,10] reads as follows: *)

Nullstelle

In[∘]:= Root[- 288 957 398 790 608 106 172 981 582 328 552 343 614 105 502 875 648 -

Nullstelle

307 064 478 855 856 313 019 393 437 101 182 656 801 933 226 934 272 z +
539 830 444 076 708 310 691 526 476 599 452 472 415 987 472 269 312 z^2 -
208 779 701 838 852 829 788 016 821 619 264 146 339 823 154 626 560 z^3 -
60 035 491 846 245 045 445 511 786 610 540 253 395 756 809 977 856 z^4 +
104 423 904 038 445 492 471 049 266 362 532 451 339 638 724 362 240 z^5 -
61 170 701 413 963 800 548 837 075 319 640 271 318 401 893 400 576 z^6 +
23 638 353 846 074 851 072 285 831 234 094 686 537 690 440 007 680 z^7 -
6 836 343 839 307 685 517 902 229 695 463 357 145 675 552 391 168 z^8 +
1 544 696 153 083 041 446 560 882 140 470 441 775 815 430 832 128 z^9 -
275 543 407 787 418 225 085 720 778 706 394 601 676 255 920 128 z^{10} +
38 192 073 882 883 823 791 867 535 903 685 711 472 545 595 392 z^{11} -
3 847 383 059 121 068 813 222 202 218 352 221 067 046 293 504 z^{12} +
207 661 091 969 661 497 349 277 928 987 216 122 751 877 120 z^{13} +
14 079 595 198 712 307 563 283 121 809 803 625 085 661 184 z^{14} -
5 425 990 191 700 818 052 531 651 696 942 533 634 424 832 z^{15} +
832 918 456 309 177 781 209 592 568 330 102 181 757 952 z^{16} -
90 613 203 793 950 119 811 022 338 318 459 586 833 408 z^{17} +
7 750 439 036 242 804 656 893 526 729 686 883 843 840 z^{18} -
540 392 289 771 173 113 505 538 930 138 624 700 416 z^{19} +
31 162 786 628 689 850 521 413 584 819 909 924 352 z^{20} -
1 492 919 943 724 551 945 334 172 528 928 208 896 z^{21} +
59 311 086 840 644 951 717 332 297 164 846 720 z^{22} -
1 940 833 411 277 138 342 357 264 528 988 672 z^{23} + 51 695 502 160 119 137 316 152 512 438 320 z^{24} -
1 100 850 887 553 086 586 040 223 387 904 z^{25} + 18 251 579 107 103 441 471 509 863 600 z^{26} -
226 349 436 089 754 569 260 746 000 z^{27} + 1 968 413 501 220 193 987 942 500 z^{28} -
10 660 506 439 099 558 387 500 z^{29} + 26 920 470 805 806 965 625 z^{30}, 10]

(* Its numerical approximation is α=
66.00757532283699935776634709856557525284220804351579265582233014736701676301044075l\
4105627001073786003478829697273782`99. *)

(* The parameter β=Root[$G_{m(11),6}(\beta)$,10]reads as follows: *)

Nullstelle

```
Root[- 32 931 993 845 048 601 740 560 928 269 925 700 312 889 746 352 315 039 744 +
Nullstelle
  14 504 273 247 361 956 373 110 629 948 617 138 310 471 703 486 718 279 680 z +
  7 498 874 513 680 640 835 743 347 823 618 598 708 228 657 891 601 547 264 z^2 -
  15 256 036 226 611 378 886 199 070 563 444 303 326 599 079 006 312 595 456 z^3 +
  13 502 955 515 092 848 579 189 839 488 409 260 269 073 064 611 984 965 632 z^4 -
  7 503 337 088 066 741 974 478 413 019 729 617 422 514 549 978 648 018 944 z^5 +
  2 633 099 975 117 016 793 756 167 673 535 938 989 704 889 006 378 582 016 z^6 -
  502 141 513 375 005 597 585 115 820 573 940 886 804 524 829 708 910 592 z^7 -
  17 628 518 946 070 813 989 648 627 951 160 413 310 879 570 396 184 576 z^8 +
  49 046 493 945 665 170 733 919 127 448 518 016 574 641 475 541 204 992 z^9 -
  19 564 791 955 859 058 134 446 613 510 507 510 255 029 259 437 309 952 z^10 +
  5 051 555 393 095 496 775 505 835 345 119 434 138 327 502 341 242 880 z^11 -
  988 876 133 233 060 945 945 471 713 627 076 734 890 067 837 038 592 z^12 +
  155 145 039 794 924 430 638 679 391 542 895 523 373 196 783 190 016 z^13 -
  20 018 979 796 635 544 379 314 790 757 156 754 593 401 733 279 744 z^14 +
  2 151 984 160 005 959 617 605 160 352 744 693 717 521 519 312 896 z^15 -
  193 761 827 821 523 377 491 970 574 223 347 467 999 731 314 688 z^16 +
  14 610 747 764 568 880 206 779 414 396 799 043 140 553 331 712 z^17 -
  917 632 125 636 111 873 918 600 986 180 273 433 552 126 720 z^18 +
  47 407 818 834 844 310 188 277 598 652 187 086 163 017 728 z^19 -
  1 966 052 081 390 652 827 434 390 740 123 841 315 347 456 z^20 +
  62 119 985 495 989 655 133 015 200 779 586 543 612 928 z^21 -
  1 288 441 110 631 450 401 043 815 805 311 640 505 472 z^22 +
  4 772 493 754 198 120 189 409 068 931 725 592 064 z^23 +
  871 980 172 767 961 145 934 616 348 329 060 432 z^24 -
  40 511 400 060 052 313 165 116 979 840 723 904 z^25 +
  1 055 284 182 688 553 311 613 119 627 611 600 z^26 -
  18 193 615 814 288 220 409 185 876 486 000 z^27 + 206 507 192 743 386 378 005 784 930 000 z^28 -
  1 404 724 272 973 709 709 162 487 500 z^29 + 4 335 568 743 746 017 620 871 875 z^30, 10]

(* Its numerical approximation is β=
 66.00757532283699935793062928059295048186274709523236613805558209285784147471169003
  0125464676285447697752579079618072`99. *)
```

(* After insertion,
a numerical approximation of $Z_{11,6}(x)=\sum_{k=0}^{11} a^{*}_{k,11}(6)\, x^{k}$ reads as follows: *)

Out[•]=

0.128898853072793683566581976236562407891169516195260194872559151972723546956252492789374009925142435 −
0.019530353389900685045297976452476419387186972445266119952649155281556137107969336419715682398620452 5 x −
6.4450757923869706613028039800685985762413100607629294085066812084260280149538376709094717108517843 x^{2} +
0.3320213871088280789385197429153140764677908366168218523790353302795395617678375737894217581442188 41 x^{3} +
51.5613164441065973718562693094103502121078704452629000896307071389079181770534307086955757471638 14 x^{4} −
1.62496951435621419841305638269985601496106012619396604667972964015780611597749821815346586835867883 x^{5} −
144.373106291553229795312723123460475137349677322408240716216576372101938496072745159026345734267685 x^{6} +
3.3124641342582656135199448723526929805307707381575574217292459761144715148151791438290406742238050 6 x^{7} +
164.999053138984601394007276903634525094698048068593741666433099237210035064292867718068557590919227 x^{8} −
2.9999856536209788090001102561156746226503144761351471074759025109546488234975491630452808816107246 1 x^{9} −
66.000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000 x^{10} +
1.0000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000 0 x^{11}

(* To obtain a non-numerical explicit representation of $Z_{11,6}(x) = \sum_{k=0}^{11} a^{*}_{k,11}(6)\, x^{k}$ would require to insert into $(R_{11,\alpha,\beta}(x))$ /. s→6 the parameters α and β *)
(* In particular, inserting the parameters α and β into the coefficient at the
monomial x^9 $\left(\text{i.e., into } -\frac{11}{16}\left(-3164+\alpha^{2}-2\,\alpha\,(-12+\beta)+24\,\beta+\beta^{2}\right)\right)$ would yield,
after simplification with RootReduce or FullSimplify,
the following algebraic representation of $a^{*}_{9,11}(6)$ as a root object: *)

Root[

6 406 505 116 237 288 699 302 300 764 313 639 697 830 007 281 688 034 470 222 110 777 305 385 057 619 921 232 522 456 202 696 854 925 436 816 843 601 477 274 921 875 +
2 036 957 520 203 046 766 508 047 480 483 042 315 003 764 725 905 288 143 892 364 959 975 299 930 679 134 217 187 925 376 738 973 681 749 368 547 425 276 944 625 000 z −
32 122 549 932 784 548 824 852 897 349 353 308 999 259 093 218 469 427 740 119 151 485 904 836 129 768 766 641 838 585 185 712 138 771 649 091 096 502 167 700 000 z^{2} +
239 573 290 288 019 936 995 281 133 903 831 963 436 081 306 553 735 769 761 776 400 825 191 814 984 001 215 177 744 725 308 336 830 999 987 000 616 976 450 000 z^{3} −
1 138 992 319 564 003 156 965 875 271 559 980 815 366 090 588 409 792 288 863 231 790 367 815 117

```
   894 667 623 652 440 779 549 360 899 820 655 436 442 520 000 z^4 +
3 885 528 268 120 526 940 011 723 247 134 569 552 862 681 871 516 717 040 025 233 352 335 292 107
   766 214 071 518 821 645 050 706 898 485 608 435 809 600 z^5 -
10 131 612 165 805 115 763 282 434 619 519 779 564 918 906 218 377 355 566 406 533 435 102 917 562
   834 175 473 420 724 035 145 424 118 970 361 765 920 z^6 +
20 990 998 098 439 747 705 027 682 015 784 512 995 017 674 655 283 332 933 694 458 363 670 731 200
   035 578 715 783 781 869 784 066 848 924 002 816 z^7 -
35 450 046 651 939 297 599 466 665 635 492 811 001 530 824 345 567 378 478 238 426 787 098 650 022
   216 279 834 074 924 336 072 109 346 174 592 z^8 +
49 661 741 679 747 488 210 713 992 383 395 485 940 077 397 499 326 945 013 576 568 361 437 786 552
   590 457 656 314 654 260 403 318 092 672 z^9 -
58 404 456 562 589 459 008 737 709 107 392 664 250 700 035 723 387 385 163 386 681 671 194 642 701
   197 119 284 029 167 495 075 675 648 z^10 +
58 112 652 238 681 748 931 650 970 157 869 073 563 813 888 085 550 824 067 335 303 959 473 282 046
   552 022 861 574 664 594 404 352 z^11 -
49 126 791 914 145 438 450 126 144 169 812 282 070 502 263 102 124 263 742 388 577 833 459 519 150
   238 660 368 753 382 834 944 z^12 +
35 305 566 680 813 153 890 867 943 623 559 953 758 244 275 602 351 282 693 131 001 645 400 405 922
   425 049 936 599 398 400 z^13 -
21 487 504 109 615 725 098 690 498 677 513 434 372 153 156 225 274 598 099 656 437 170 362 885 805
   477 759 799 160 832 z^14 +
10 959 452 726 765 333 821 806 005 007 764 485 293 614 034 633 408 130 127 470 326 635 890 553 195
   276 851 273 728 z^15 -
4 576 645 128 746 743 076 452 556 656 973 496 377 106 024 079 950 789 258 584 844 545 762 233 949
   238 448 128 z^16 +
1 480 269 555 993 782 975 305 270 553 965 070 787 073 188 793 286 156 773 547 720 153 272 330 083
   311 616 z^17 -
308 865 980 791 242 351 797 770 012 828 157 885 462 162 806 778 205 879 531 278 422 144 037 187 584
 z^18 - 5 385 521 530 592 347 709 287 456 325 807 289 432 716 095 357 401 619 032 406 782 422 286 336
 z^19 + 40 625 074 572 118 503 555 676 687 304 215 262 853 059 373 864 450 053 144 718 624 751 616 z^20 -
22 179 772 695 629 105 465 892 586 025 833 496 461 388 745 547 739 805 953 550 516 224 z^21 +
7 894 234 325 018 103 996 305 709 466 617 394 470 783 311 662 392 569 526 878 208 z^22 -
2 126 135 337 796 357 669 083 273 044 425 265 519 259 487 051 871 576 129 536 z^23 +
449 821 132 426 798 836 345 714 364 038 103 863 460 755 481 434 062 848 z^24 -
75 077 268 650 628 737 590 270 228 872 808 869 943 984 755 572 736 z^25 +
9 747 125 103 175 566 954 227 098 973 216 607 975 912 243 200 z^26 -
953 506 489 233 689 086 862 863 428 088 518 475 776 000 z^27 +
66 304 498 382 682 601 863 832 638 225 776 640 000 z^28 -
2 926 855 640 543 956 955 556 839 424 000 000 z^29 + 61 734 985 035 729 950 549 606 400 000 z^30, 2]

(* Its numerical value is indeed *)

In[•]:= N[Root[
         Nullstelle
  6 406 505 116 237 288 699 302 300 764 313 639 697 830 007 281 688 034 470 222 110 777 305 385 057
     619 921 232 522 456 202 696 854 925 436 816 843 601 477 274 921 875 +
   2 036 957 520 203 046 766 508 047 480 483 042 315 003 764 725 905 288 143 892 364 959 975 299 930
      679 134 217 187 925 376 738 973 681 749 368 547 425 276 944 625 000 z -
   32 122 549 932 784 548 824 852 897 349 353 308 999 259 093 218 469 427 740 119 151 485 904 836 129
      768 766 641 838 585 185 712 138 771 649 091 096 502 167 700 000 z^2 +
```

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239 573 290 288 019 936 995 281 133 903 831 963 436 081 306 553 735 769 761 776 400 825 191 814
  984 001 215 177 744 725 308 336 830 999 987 000 616 976 450 000 z^3 -
1 138 992 319 564 003 156 965 875 271 559 980 815 366 090 588 409 792 288 863 231 790 367 815 117
  894 667 623 652 440 779 549 360 899 820 655 436 442 520 000 z^4 +
3 885 528 268 120 526 940 011 723 247 134 569 552 862 681 871 516 717 040 025 233 352 335 292 107
  766 214 071 518 821 645 050 706 898 485 608 435 809 600 z^5 -
10 131 612 165 805 115 763 282 434 619 519 779 564 918 906 218 377 355 566 406 533 435 102 917 562
  834 175 473 420 724 035 145 424 118 970 361 765 920 z^6 +
20 990 998 098 439 747 705 027 682 015 784 512 995 017 674 655 283 332 933 694 458 363 670 731 200
  035 578 715 783 781 869 784 066 848 924 002 816 z^7 -
35 450 046 651 939 297 599 466 665 635 492 811 001 530 824 345 567 378 478 238 426 787 098 650 022
  216 279 834 074 924 336 072 109 346 174 592 z^8 +
49 661 741 679 747 488 210 713 992 383 395 485 940 077 397 499 326 945 013 576 568 361 437 786 552
  590 457 656 314 654 260 403 318 092 672 z^9 -
58 404 456 562 589 459 008 737 709 107 392 664 250 700 035 723 387 385 163 386 681 671 194 642 701
  197 119 284 029 167 495 075 675 648 z^10 +
58 112 652 238 681 748 931 650 970 157 869 073 563 813 888 085 550 824 067 335 303 959 473 282 046
  552 022 861 574 664 594 404 352 z^11 -
49 126 791 914 145 438 450 126 144 169 812 282 070 502 263 102 124 263 742 388 577 833 459 519 150
  238 660 368 753 382 834 944 z^12 +
35 305 566 680 813 153 890 867 943 623 559 953 758 244 275 602 351 282 693 131 001 645 400 405 922
  425 049 936 599 398 400 z^13 -
21 487 504 109 615 725 098 690 498 677 513 434 372 153 156 225 274 598 099 656 437 170 362 885 805
  477 759 799 160 832 z^14 +
10 959 452 726 765 333 821 806 005 007 764 485 293 614 034 633 408 130 127 470 326 635 890 553 195
  276 851 273 728 z^15 -
4 576 645 128 746 743 076 452 556 656 973 496 377 106 024 079 950 789 258 584 844 545 762 233 949
  238 448 128 z^16 +
1 480 269 555 993 782 975 305 270 553 965 070 787 073 188 793 286 156 773 547 720 153 272 330 083
  311 616 z^17 -
308 865 980 791 242 351 797 770 012 828 157 885 462 162 806 778 205 879 531 278 422 144 037 187 584
 z^18 - 5 385 521 530 592 347 709 287 456 325 807 289 432 716 095 357 401 619 032 406 782 422 286 336
 z^19 + 40 625 074 572 118 503 555 676 687 304 215 262 853 059 373 864 450 053 144 718 624 751 616
 z^20 - 22 179 772 695 629 105 465 892 586 025 833 496 461 388 745 547 739 805 953 550 516 224 z^21 +
7 894 234 325 018 103 996 305 709 466 617 394 470 783 311 662 392 569 526 878 208 z^22 -
2 126 135 337 796 357 669 083 273 044 425 265 519 259 487 051 871 576 129 536 z^23 +
449 821 132 426 798 836 345 714 364 038 103 863 460 755 481 434 062 848 z^24 -
75 077 268 650 628 737 590 270 228 872 808 869 943 984 755 572 736 z^25 +
9 747 125 103 175 566 954 227 098 973 216 607 975 912 243 200 z^26 -
953 506 489 233 689 086 862 863 428 088 518 475 776 000 z^27 +
66 304 498 382 682 601 863 832 638 225 776 640 000 z^28 -
2 926 855 640 543 956 955 556 839 424 000 000 z^29 +
61 734 985 035 729 950 549 606 400 000 z^30, 2], 99]
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Out[∘]=

```
-2.999985653620978809000110256115674622650314476135147107475902510954648823497549163 0
  4528088161072461
```