On the volume bound in the Dvoretzky-Rogers lemma

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In this talk we investigate the volume of parallelotopes spanned by d vectors that are selected from a finite set of isotropic vectors in Euclidean d-space. The Dvoretzky–Rogers lemma [2] guarantees the existence of such a parallelotope with not too small volume. Recently, Pivovarov [6] has determined the expectation of the square of the volume of parallelotopes spanned by d independent random vectors, where each vector is distributed according to an isotropic measure in $\mathbb{R}^d$.

We extend Pivovarov's result to a class of more general probability measures, and then apply this to improve the lower bound of the Dvoretzky–Rogers lemma on the maximal volume of parallelotopes [3]. The improvement is significant in the case when the support of the discrete isotropic measure is small compared to the dimension, and in certain cases, the bound we obtain is optimal. By John's theorem [5], cf. also Ball [1], this result can be used, for example, in the study of polytopes with few facets. González and Schymura [4] have also ivestigated this problem recently and have reached similar conclusions using linear algebraic methods. Our approach in [3] uses probabilistic methods and it also provides a new interpretation of the volume bound in the Dvoretzky–Rogers lemma as an expectation.

This talk is based on joint work [3] with F. Fodor from the University of Szeged, Hungary, and M. Naszódi from Eötvös Lóránd University, Hungary.

[1] K. Ball, Ellipsoids of maximal volume in convex bodies, *Geom. Dedicata* **41** (1992), no. 2, 241–250.

[2] A. Dvoretzky, C. A. Rogers, Absolute and unconditional convergence in normed linear spaces, Proc. Nat. Acad. Sci. U. S. A. **36** (1950), 192–197. *J. Math. Anal. Appl.* **176** (1993), 261–281.

[3] F. Fodor, M. Naszódi, T. Zarnócz, *On the volume bound in the Dvoretzky-Rogers lemma*, arXiv:1804.03444 (2018), submitted.

[4] B. González Merino, M. Schymura, *On the reverse isodiametric problem and Dvoretzky-Rogers-type volume bounds*, arXiv:1804.05009 (2018).

[5] F. John, *Extremum problems with inequalities as subsidiary conditions*, Studies and Essays Presented to R. Courant on his 60th Birthday, January 8, 1948, Interscience Publishers, Inc., New York, N. Y., 1948, pp. 187–204.

[6] P. Pivovarov, On determinants and the volume of random polytopes in isotropic convex bodies, *Geom. Dedicata* **149** (2010), 45–58.