

ON A FUNCTIONAL EQUATION INVOLVING THE ARITHMETIC MEAN

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Let I stand for a nonempty open subinterval of the reals and consider the functional equation

$$\varphi\left(\frac{x+y}{2}\right)(f(x) - f(y)) = G(x) - G(y), \quad (x, y \in I)$$

concerning the unknown functions $\varphi : I \rightarrow \mathbb{R}$, $f : I \rightarrow \mathbb{R}$ and $G : I \rightarrow \mathbb{R}$.

This equation is strongly related to the generalized *Matkowski–Sutô problem*, more precisely, to the *invariance problem of generalized quasi-arithmetic means*.

The aim of the talk is to describe the differentiable solutions assuming only weak regularity conditions for the functions. Solutions having arbitrary – even pathological – parts will be also discussed.

- [1] Sz. Baják, Zs. Páles, *Invariance equation for generalized quasi-arithmetic means*, *Aequationes Mathematicae* 77 (2009), p. 133-145
- [2] Sz. Baják, Zs. Páles, *Computer aided solution of the invariance equation for two-variable Gini means*, *Computers and Mathematics with Applications* 58 (2009), p. 334-340
- [3] Z. Daróczy, Zs. Páles, *Gauss-composition of means and the solution of the Matkowski–Sutô problem*, *Publ. Math. Debrecen* 61/1-2 (2002), p. 157-218
- [4] T. Kiss, Zs. Páles, *On a functional equation related to two variable weighted quasi-arithmetic means*, *Journal of Difference Equations and Applications* (2017), p. 107-126