Thue equations and monogenity of algebraic number fields

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An algebraic number field K of degree n is called **monogene** if its ring of integers $\mathbb{Z}_K$ is a simple ring extension of $\mathbb{Z}$, that is $\mathbb{Z}_K = \mathbb{Z}[\alpha]$. In this case $\{1, \alpha, \ldots, \alpha^{n-1}\}$ is an integral basis of K called **power integral basis**.

To decide if a number field is monogene and to determine all generators of power integral bases are classical problems of algebraic number threory. These problems lead to a special types of Diophantine equations, called **index form equations**, that can often be reduced to various types of Thue equations. Therefore the effective methods for solving Thue equations and related equations can be very well applied in monogenity problems.

Starting from cubic and quartic number fields we describe several results on monogenity of number fields, involving some very recent results on infinite parametric families of pure fields and simpest sextic fields.

[1] István Gaál, *Diophantine equations and power integral bases*, Birkhäuser Basel, Boston, 2002.

[2] István Gaál, László Remete, Integral bases and monogenity of pure fields, *Journal of Number Theory* **173** (2017), 129–146.

[3] István Gaál, László Remete, Integral bases and monogenity of the simplest sextic fields, *Acta Arithmetica* (accepted)