Global stability in difference equations

János Dudás
Bolyai Institute, University of Szeged, Hungary

We consider 2- and 3-dimensional maps depending on a parameter. Local stability of a fixed point is known up to a critical parameter value where Neimark-Sacker bifurcation takes place. The aim is to show global stability for all parameter values where local stability holds. Near the fixed point analytical tools are used to construct a neighbourhood $\mathcal{N}$ belonging to the domain of attraction of the fixed point. The size of the neighbourhood $\mathcal{N}$ is crucial since outside $\mathcal{N}$ rigorous, computer-aided calculations are applied to show that each point enters into $\mathcal{N}$ after finite number of iterations. The 3-dimensional case is technically more complicated as it requires a center manifold reduction, and in particular, an estimation for the size of the center manifold is important. Among others, the difference equations

$$x_{k+1} = ax_k(1 - x_{k-d})$$

and

$$x_{k+1} = x_k e^{a - x_{k-d}}$$

where a is a positive parameter and $d = 1$ or $d = 2$, can be handled with our technique.

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