Asymptotic behaviour of infinite coupled map systems

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In this talk we study coupled map systems of infinite size. Such system arises as the limit of systems describing the behavior of finitely many interacting units. The phase space of our system is the (flat) circle $\mathbb{R}/\mathbb{Z}$, and the system state is represented by a density function denoted by f. The dynamics is a composition of two circle maps, an expanding map T and a coupling map of meanfield-type, which we shall denote by Φ_f^ε. The parameter ε is called the coupling strength, and the characteristics of the system are expected to depend dramatically on its magnitude. Given a density f_i, the density after one instance of time, denoted by f_{i+1}, can be computed by applying to f_i the transfer (or Perron–Frobenius) operator associated to the dynamics $T \circ \Phi_{f_i}$. Our main objective is to study the asymptotic behavior of such sequences of densities for various values of ε.

Our model was first introduced in [2] for $T(x) = 2x \mod 1$ as the limit of the finite coupled map systems defined in [4], [1]. We expanded our model to a more general setting in [3], and showed that if we assume some regularity conditions on T, the system has a unique invariant density (in a suitable class of functions) for *sufficiently weak coupling*. Furthermore, all initial densities from this suitable class converge to the unique invariant density with exponential speed. We have also showed that a different behavior is possible for *sufficiently strong coupling*: some initial distributions approach a moving point mass in any sensible metric on spaces of measures. This can be interpreted as the perfect synchronization of the coupled map system.

In this talk we discuss the special case when the unique invariant measure of T is the Lebesgue measure. What makes this setup particularly nice and interesting is that the unique invariant density of the coupled map system (for small enough ε) can be computed explicitly, and the proof of the convergence results are surprisingly brief and elementary, as opposed to the techniques of [3]. Yet they require some subtle details in addition to the methods used in the case of the doubling map in [2]. Studying this simple case also makes it possible to generalize our results to a wider class of meanfield-type coupling maps. Results of this type will be also stated.

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