# Extension operators preserving Janowski classes of univalent functions

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Our main interest is devoted to study the extension operator $\Phi_{n,\alpha,\beta} : \mathcal{L}S \to \mathcal{L}S_n$ given by $\Phi_{n,\alpha,\beta}(f)(z) = \left(f(z_1), \tilde{z}\left(\frac{f(z_1)}{z_1}\right)^{\alpha}(f'(z_1))^{\beta}\right)$, $z = (z_1, \tilde{z}) \in \mathbf{B}^n$, where $\alpha, \beta \geq 0$. We shall prove that the extension operator $\Phi_{n,\alpha,\beta}$ preserves the notion of $g$-parametric representation for $\alpha \in [0,1]$, $\beta \in [0,1/2]$ and $\alpha+\beta \leq 1$, where the function $g$ is given by $g(\zeta) = \frac{1+A\zeta}{1+B\zeta}$, $\zeta \in U$ with $-1 \leq B < A \leq 1$. The notion of $g$-parametric representation was introduced in [3].

**Theorem 1.** *Let $g : U \to \mathbb{C}$ be the function given by $g(\zeta) = \frac{1+A\zeta}{1+B\zeta}$, $\zeta \in U$, where $-1 \leq B < A \leq 1$. If $f \in S$ has $g$-parametric representation, then $F = \Phi_{n,\alpha,\beta}(f)$ also has $g$-parametric representation on $\mathbf{B}^n$ for $\alpha \in [0,1]$, $\beta \in [0,1/2]$ and $\alpha + \beta \leq 1$.*

The following classes of Janowski starlike mappings and Janowski almost starlike mappings on $\mathbf{B}^n$ were introduced by Curt (see [2]).

**Definition 2.** (see [2]) Let be $a, b \in \mathbb{R}$ be such that $|1-a| < b \leq a$. Also, let $S^*(a,b,\mathbf{B}^n) = \left\{f \in \mathcal{L}S_n : \left|\frac{\|z\|^2}{\langle [Df(z)]^{-1}f(z), z\rangle} - a\right| < b,\ z \in \mathbf{B}^n \backslash \{0\}\right\}$ and $\mathcal{A}S^*(a,b,\mathbf{B}^n) = \left\{f \in \mathcal{L}S_n : \left|\frac{\langle [Df(z)]^{-1}f(z), z\rangle}{\|z\|^2} - a\right| < b,\ z \in \mathbf{B}^n \backslash \{0\}\right\}$.

As a consequence of the previous result, the operator $\Phi_{n,\alpha,\beta}$ preserves the notions of Janowski starlikeness on $\mathbf{B}^n$ and Janowski almost starlikeness on $\mathbf{B}^n$.

**Corollary 3.** *Let $a, b \in \mathbb{R}$ be such that $|1-a| < b \leq a$ and let $f \in S^*(a,b)$. Then $F = \Phi_{n,\alpha,\beta}(f) \in S^*(a,b,\mathbf{B}^n)$, with $\alpha \in [0,1]$, $\beta \in [0,1/2]$ and $\alpha + \beta \leq 1$.*

**Corollary 4.** *Let $a, b \in \mathbb{R}$ be such that $|1-a| < b \leq a$ and let $f \in \mathcal{A}S^*(a,b)$. Then $F = \Phi_{n,\alpha,\beta}(f) \in \mathcal{A}S^*(a,b,\mathbf{B}^n)$, with $\alpha \in [0,1]$, $\beta \in [0,1/2]$ and $\alpha + \beta \leq 1$.*

The above results continue the work in [3], [4] and [1] concerning extension operators and $g$-parametric representation in $\mathbb{C}^n$.

On the other hand, we are also concerned about some radius problems related to the operator $\Phi_{n,\alpha,\beta}$ and the Janowski class $S^*(a,b)$. We compute the radius $S^*(a,b)$ of the class $S$ (respectively $S^*$).

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[2] P. Curt, Janowski starlikeness in several complex variables and complex Hilbert spaces, *Taiwan. J. Math.* **18** (2014), no. 4, 1171–1184.

[3] I. Graham, H. Hamada, G. Kohr, Parametric representation of univalent mappings in several complex variables, *Canadian J. Math.* **54** (2002), 324–351.

[4] I. Graham, H. Hamada, G. Kohr, T.J. Suffridge, Extension operators for locally univalent mappings, *Michigan Math. J.* **50** (2002), 37–55.