Definability in the substructure ordering of finite directed graphs

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In 2009–2010 J. Ježek and R. McKenzie published a series of papers in which they examined the first-order definability in the substructure orderings of finite mathematical structures with a given type. They considered finite semilattices, ordered sets, distributive lattices, and lattices. Their results were analogous for all these structure types. Among others, they determined the minimal first-order definable subsets of these orderings. Similar investigations have been emerging since by a number of authors, including the speaker.

The papers [1] and [2] considered finite directed graphs, i.e. binary relations on finite sets. However, instead of the substructure ordering, proposed by Ježek and McKenzie, the order relation between directed graphs was embeddability. The embeddability and the substructure orderings are completely different, even their automorphism groups differ, as we will see in the talk.

Now we examine first-order definability in the substructure ordering of finite directed graphs. We aim to prove analogue results to the existing ones. As tempting as it may seem to use the results of [1] and [2], such a reduction is worth it only if it is easier than building from the ground up. We consider both approaches in the talk.

[1] Á. Kunos, Definability in the embeddability ordering of finite directed graphs, *Order* **32/1** (2015), 117-133.

[2] Á. Kunos, Definability in the embeddability ordering of finite directed graphs, II, *submitted*