T: lineárisan rendezett test

e: multiplikatív egységelem

$e+e=2e \qquad e+e+e=3e$

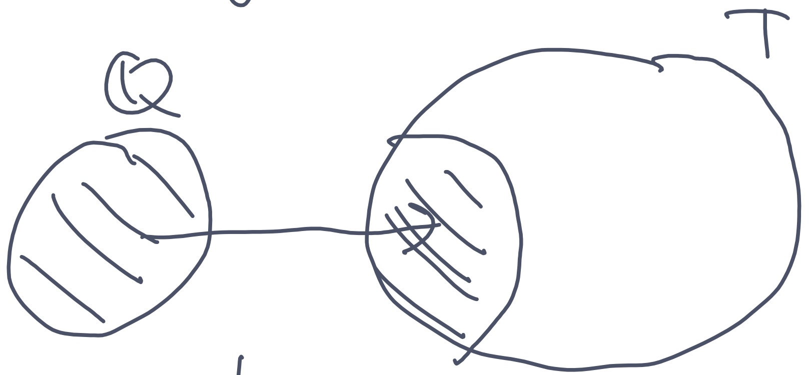
$2e+3e=(e+e)+(e+e+e)=5e$

$2e\cdot 3e=(e+e)\cdot(e+e+e)=6\cdot ee=6e$

$\mathbb{N}_0\to T,\ n\mapsto ne$ beágyazás

$\mathbb{Z}\to T,\ n\mapsto ne$

$\mathbb{Q}\to T,\ \frac{a}{b}\mapsto ae\cdot(be)^{-1}$



$ae\cdot(be)^{-1}=\frac{a}{b} \qquad \mathbb{Q}\le T$

DEF T arkhimédeszi, ha $\forall a\in T\ \exists n\in\mathbb{N}: n>a$

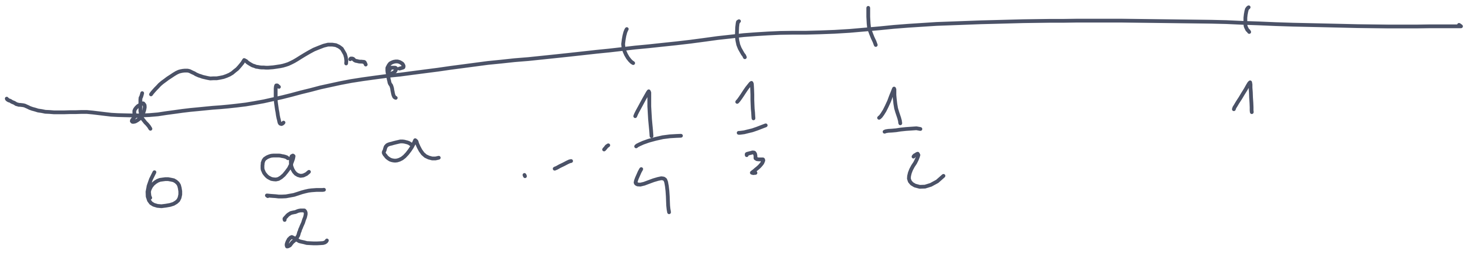
T nem arkh. $\Rightarrow \exists a\in T\ \forall n\in\mathbb{N}: n\le a$

$$\frac{1}{a} \in I^+ \quad \frac{1}{a} \le \frac{1}{n}$$

$$a \in I^+,\ b \in F^+ \overset{?}{\Longrightarrow} a \cdot b \in I^+$$

Cél: $\forall n \in \mathbb{N}: ab \overset{?}{<} \frac{1}{n}$

$$\left.\begin{array}{l} b \in F^+ \Rightarrow \exists m \in \mathbb{N}: b < m \\ a \in I^+ \underbrace{\Longrightarrow}_{\forall n \in \mathbb{N}} a < \frac{1}{m \cdot n} \end{array}\right\} \Longrightarrow a \cdot b < m \cdot \frac{1}{mn} = \frac{1}{n}$$



$\varepsilon \in I^+$

$$a = \frac{1}{\varepsilon^2},\ b = 5 + 100\varepsilon^2,\ c = \frac{1+\varepsilon}{\varepsilon^2 + 2\varepsilon},\ d = 6 - 37\varepsilon$$

$$a \approx \frac{1}{\varepsilon^2},\ b \approx 5,\ c \approx \frac{1}{2\varepsilon},\ d \approx 6$$

$$5 < 6 \ll \frac{1}{2\varepsilon} \ll \frac{1}{\varepsilon^2}$$

$$b \underset{?}{<} d \underset{?}{\ll} c \underset{?}{\ll} a$$

$$b \overset{?}{<} d \qquad 5 + 100\varepsilon^2 \overset{?}{<} 6 - 37\varepsilon$$

$$100\varepsilon^2 + 37\varepsilon \overset{?}{<} \underline{1}$$

$$\wedge$$

$$100\varepsilon + 37\varepsilon \overset{?}{<} 1$$

$$137\varepsilon \overset{?}{<} 1$$

$$\varepsilon \overset{?}{<} \frac{1}{137}$$ ✓ mert $\varepsilon \in I^+$

$b \overset{?}{<} d \qquad 2b \overset{?}{>} d$

$$10 + 200\varepsilon^2 \overset{?}{>} 6 - 37\varepsilon$$

$$4 \overset{?}{>} -200\varepsilon^2 - 37\varepsilon$$ ✓

0,2

$q \in \mathbb{H} \quad q^2 = -1$

$$q = a + bi + cj + dk = \lambda + \vec{v}$$

$$q^2 = (\lambda^2 - \underbrace{\langle \vec{v}, \vec{v} \rangle}_{|\vec{v}|^2}) + (\lambda\vec{v} + \lambda\vec{v} + \underbrace{\vec{v} \times \vec{v}}_{0})$$

$$q^2 = \underbrace{(\lambda^2 - |\vec{v}|^2)}_{-1} + \underbrace{2\lambda\vec{v}}_{0} = -1 + \underline{0}$$

$$2\lambda\vec{v} = 0 \iff \lambda = 0 \quad \vee \quad \vec{v} = \underline{0}$$

$$\lambda^2 - |\vec{v}|^2 = -1 \qquad \qquad \lambda^2 - |\vec{v}|^2 = \lambda^2 \neq -1$$

$$|\vec{v}|^2 = 1$$

$$q^2 = -1 \iff \lambda = 0 \text{ és } |\vec{v}| = 1$$

$$\iff q = bi + cj + dk \quad b^2 + c^2 + d^2 = \underline{1}$$

$$q = a + bi + cj + dk = a + h\left(\frac{b}{h} i + \frac{c}{h} j + \frac{d}{h} k\right)$$

$$\sqrt{b^2 + c^2 + d^2} = h \qquad = a + h(\; \ldots \;)$$

négyzete $-\underline{1}$