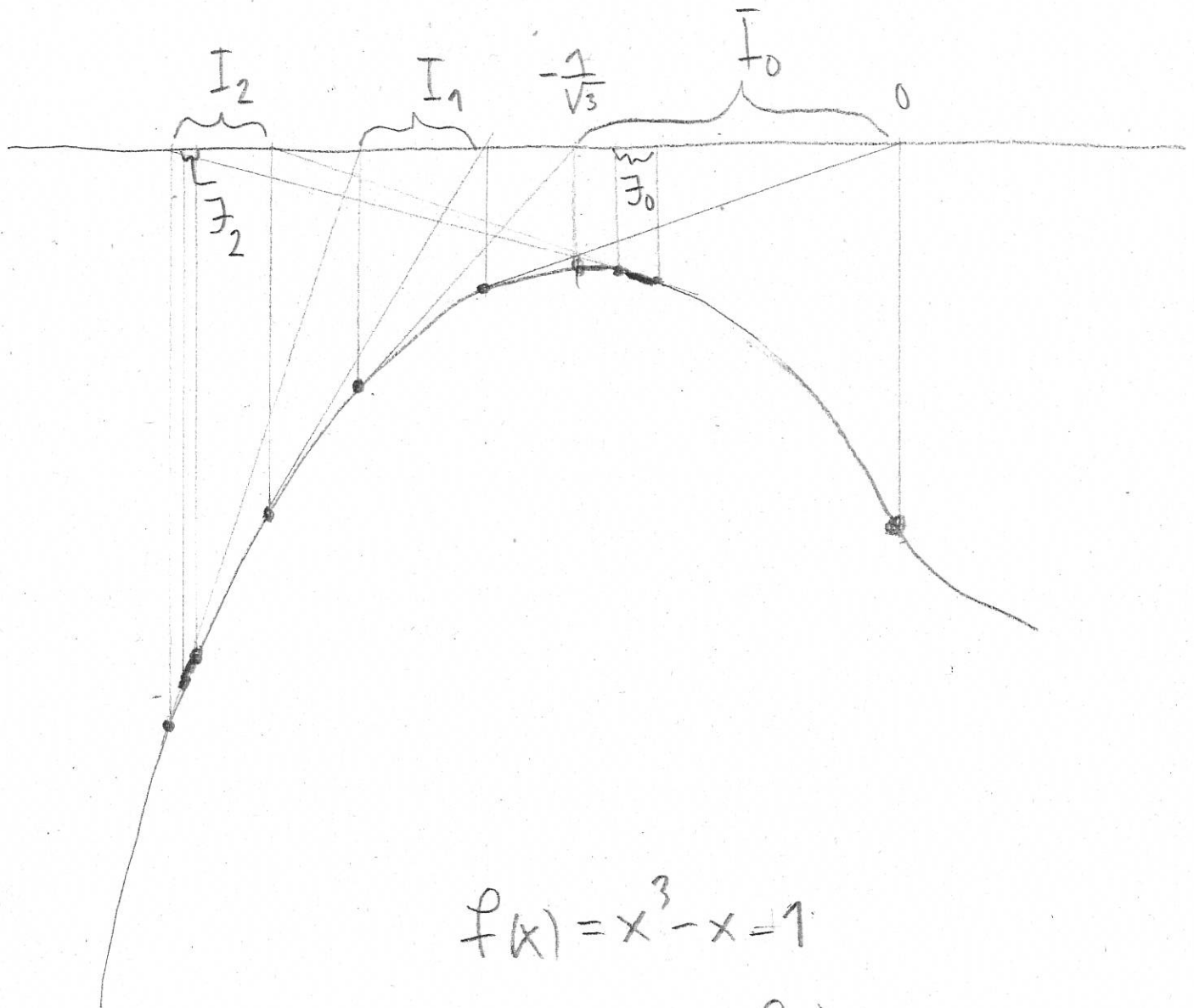




$$f(x) = x^3 - x - 1$$

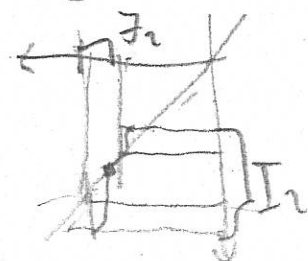
$$N(x) = x - \frac{f(x)}{f'(x)} \quad \text{Newton-iterate}$$

$$I_0 = [-\frac{1}{\sqrt{3}} 10]$$

$$N: I_{-2} \leftrightarrow I_{-1} \leftrightarrow I_0 \leftrightarrow (-\infty, -1]$$

$$J_0 := N^{-1}(I_2), \quad J_2 := N^{-2}(J_0)$$

$$N^3: J_2 \leftrightarrow I_2$$



3-FIXPOINT

$$\exists x_* \in J_2 \quad N^3(x_*) = x_*$$

$$I_0 = \left[-\frac{1}{\sqrt{3}}, 0\right] \quad I_1 = N^{-1}(I_0) = \left[-\frac{\sqrt{3}}{2}, -\frac{1}{3\sqrt{3}}\right] \quad \text{b/c } I_0 \text{ is } \langle 1, 0 \rangle$$

$$I_2 = N^{-1}(I_1) = \left[-1.15856, -0.54525\right] \quad \text{b/c } I_1 \text{ is } \langle 1, 0 \rangle$$

$$\text{Root}(78x^6 - 81x^4 + 18x^2 - 1)$$

$$\text{Root}(6512x^{18} - 15553x^{12} + 2786x^{10} - 17823x^8 + 5112x^6 - 851x^4 + 57x^2 - 1)$$

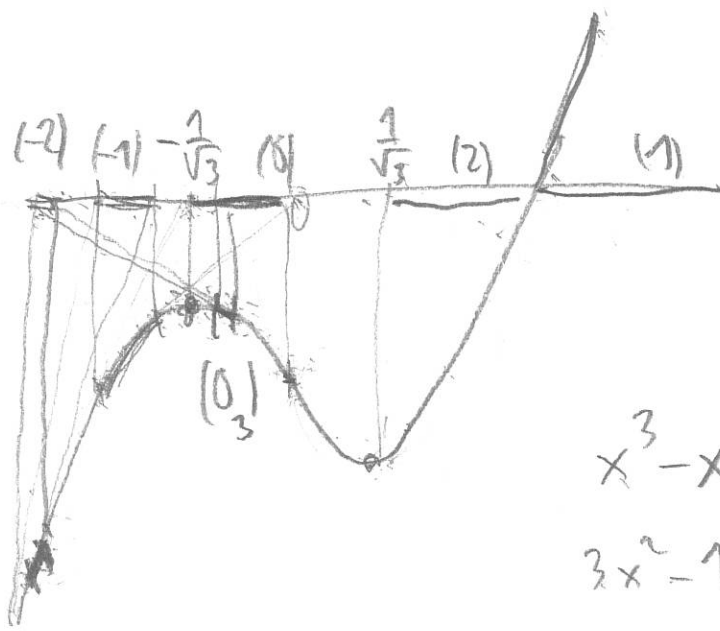
$$J_0 = N^{-1}(I_2) \quad \text{golden ratio} = \left[-0.956726, -0.731702\right]$$

$$J_1 = N^{-1}(J_0) \quad \text{b/c } J_0 \text{ is } \langle 1, 0 \rangle = \left[-0.800755, -0.770285\right]$$

$$J_2 = N^{-1}(J_1) = \left[-1.11056, -1.07139\right]$$

$$X_* = -1.09751 \quad 3\text{-ADREVEN FIXPOINT}$$

$$N^3: J_2 \nearrow I_2$$



$$x^3 - x + \frac{1}{\sqrt{3}} = f(x)$$

$$3x^2 - 1 = f'(x)$$

$$N(x) := x - \frac{f(x)}{f'(x)}$$

$$N: (1) \rightarrow (1) \rightsquigarrow \text{Root}$$

$$(2) \rightarrow (1) \rightsquigarrow \text{Root}$$

$$x_0 \in (-2)$$

$$x_1 \in (-1)$$

$$x_2 \in (0)$$

$$N: (-2) \xrightarrow{N} (-1) \xrightarrow{N} (0) \xrightarrow{N} (-\infty)$$

$$N: \bigcirc \rightarrow \bigcirc \quad \text{Flyt}$$

$$0_3 := N^{-1}(-2)$$

$$N^2: (-2) \leftrightarrow (0)$$

$$|x| \rightarrow \infty \quad |N(x)| = \left| \frac{x(3x^2 - 1) - x^3 + \frac{1}{\sqrt{3}}}{3x^2 - 1} \right|$$

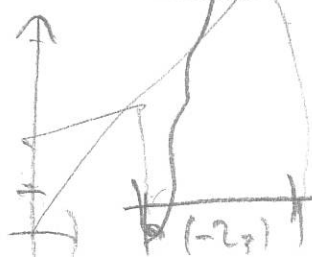
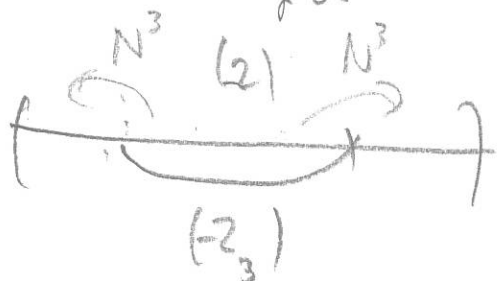
$$= \left| \frac{2x^3 + 3}{3x^2 - 1} \right| \rightarrow \infty$$

$$(-2_3) := N^2(0_3)$$

$$N^3: (-2_3) \xrightarrow{N^2} N^2(0_3) \leftrightarrow N^3 N^2(0_3) = N(0_3) = (-2)$$

$$N^2: (-2_3) \text{ b.p.} \rightarrow (0_3) \text{ b.p.}$$

$$N^3: (-2_3) \text{ b.p.} \rightarrow (2) \text{ b.p.}$$



$$\exists x_* \in (-2_3) \quad N^3(x_*) = x_*$$

$$N^3(x_*) = x_*$$