KONJUGÁLT GRADIENS (Fletcher) MÓDSZER

$f \approx \frac{1}{2}\langle Ax, x\rangle - \langle b, x\rangle = f_*(x) \qquad \nabla f \approx Ax - b \qquad \nabla^2 f \approx A$

$x_0 \in \mathbb{R}^N$ TETSZ. $\quad S_0 := \langle x_0 \rangle \qquad e_1 := -\nabla f_*(x_0)$

$S_1 := x_0 + \mathbb{R}e_1 \qquad x_1 := [f_*$ MIN-HELYE S_1-en$]$

$e_{k+1} := -\nabla f_*(x_k)$

$S_{k+1} := S_k + \mathbb{R}e_{k+1}$

$x_{k+1} := [f_*$ MIN-HELYE S_{k+1}-en$]$

$v_{k+1} := x_{k+1} - x_k$

$\nabla f_*(x_k) = e_{k+1} \perp [S_k$ érintőtere$] = \mathrm{Span}\langle e_1, \dots, e_k \rangle$

$\Rightarrow \langle e_1, \dots, e_N \rangle$ TON rsz $\subset \mathbb{R}^N$

$e_{k+2} = -\nabla f_*(x_{k+1}) \perp e_1, \dots, e_{k+1}$

$\qquad = -Ax_{k+1} + b$

$e_{k+1} = -\nabla f_*(x_k) \perp e_1, \dots, e_k$

$\qquad = -Ax_k + b$

$e_{k+1} - e_{k+2} = A(x_{k+2} - x_{k+1}) \perp e_1, \dots, e_k$

$\qquad = A v_{k+1}$

$\mathrm{Span}\langle e_1, \dots, e_k \rangle = \mathrm{Span}\langle v_1, \dots, v_k \rangle \qquad A v_{k+1} \perp v_1, \dots, v_k$

$\langle x,y\rangle_A := \langle Ax,y\rangle$

$v_{n+1} \perp^A v_1,\dots,v_n$ $\quad$ $v_1,\dots,v_N$ $\langle,\rangle_A$-ONB von $\mathbb{R}^N$

$$v_{n+1} = \alpha_{n+1} e_{n+1} + \sum_{k=1}^{n} \alpha_k^{(n+1)} v_k$$

$[v_{n+1} \in S_{n+1} - x_0 =$
$= \operatorname{span}\langle e_{n+1},\dots,e_1\rangle =$
$= \operatorname{span}\langle e_{n+1}, v_1,\dots,v_n\rangle]$

$\langle Av_{n+1}, v_k\rangle = 0 \quad k=1,\dots,n$

$$\alpha_{n+1}\langle Ae_{n+1}, v_k\rangle + \alpha_k^{(n+1)}\langle Av_k, v_k\rangle = 0 \quad k\le n$$

$\Downarrow$

$\frac{\alpha_1^{(n+1)}}{\alpha_{n+1}},\dots,\frac{\alpha_n^{(n+1)}}{\alpha_{n+1}}$ eindeutig bestimmt $\quad [e_{n+1}, v_1,\dots,v_n$ linear$]$

α_{n+1} eindeutig bestimmt

$t \mapsto f_*(x_n + t v_{n+1})$ Min-Stelle $t=1$

$$0 = \frac{d}{dt}\Big|_{t=1} f_*(x_n + t v_{n+1}) \iff \nabla f_*(\underbrace{x_n + v_{n+1}}_{x_{n+1}}) \perp v_1,\dots,v_n, e_{n+1}$$

$$0 = \langle \nabla f_*(x_n+v_{n+1}), e_{n+1}\rangle = \langle A(x_n+v_{n+1}) - b, e_{n+1}\rangle =$$

$$= \langle \underbrace{Ax_n - b}_{\nabla f_*(x_n) = -e_{n+1}}, e_{n+1}\rangle + \langle A\underbrace{v_{n+1}}_{\alpha_{n+1} e_{n+1} + \sum_{k\le n}\alpha_k^{(n+1)} v_k}, e_{n+1}\rangle =$$

$\perp e_{n+1}$

$$= -\langle e_{n+1}, e_{n+1}\rangle + \alpha_{n+1}\langle Ae_{n+1}, e_{n+1}\rangle \rightsquigarrow \alpha_n$$