LAGRANGE INTERPOLÁCIÓ

Cél: $1 \mapsto 1,\ 2 \mapsto 2, \ldots, 100 \mapsto 100$ DE $101 \mapsto 31$

$P: x_1 \mapsto y_1, \ldots, \cancel{x_r} \mapsto y_r \qquad x_1, \ldots, x_r$ különbözők

T $\exists!\, P \in Pol_{r-1}$ S

B $P = \sum_{k=1}^{r} y_k \prod_{j: j \neq k} \frac{x - x_j}{x_k - x_j}$ MEGFELEL

Tf. $P, \tilde{P}: x_1 \mapsto y_1, \ldots, \cancel{x_r} \mapsto y_r$

$Q := P - \tilde{P} \qquad Q: x_1, \ldots, x_r \mapsto 0 \qquad Q \in Pol_{r-1}$ r gyökkel $\equiv 0$

K $P := \alpha_0 + \alpha_1 x + \cdots + \alpha_{r-1} x^{r-1} \qquad \alpha_0, \ldots, \alpha_{r-1}$ ismeretlenek

$\alpha_0 + \alpha_1 x_k + \cdots + \alpha_{r-1} x_k^{r-1} = y_k \ (k = 1, \ldots, r)$ egyenletrendszer $\det \neq 0$

HERMITE INTERPOLÁCIÓ

Feladat: $P(x_1) = y_1^{(0)},\ P'(x_1) = y_1^{(1)},\ P^{(2)}(x_1) = y_1^{(2)}, \ldots, P^{(m_1)}(x_1) = y_1^{(m_1)}$

$P(x_2) = y_2^{(0)},\ P'(x_2) = y_2^{(1)},\ P^{(2)}(x_2) = y_2^{(2)}, \ldots, P^{(m_2)}(x_2) = y_2^{(m_2)}$

$\ldots$

$P(x_r) = y_r^{(0)},\ P'(x_r) = y_r^{(1)},\ P^{(2)}(x_r) = y_r^{(2)}, \ldots, P^{(m_r)}(x_r) = y_r^{(m_r)}$

$m_1 + \cdots + m_r$ ADAT

VAN-E ILYEN $(m_1 + \cdots + m_r - 1)$-FOKÚ POL ?

T (Hermite) IGEN $\quad P = \alpha_0 + \alpha_1 x + \cdots + \alpha_{m_1 + \cdots + m_r - 1 + r} x^{m_1 + \cdots + m_r - 1 + r}$

$\alpha_0, \ldots, \alpha_{m_1 + \cdots + m_r - 1}$ ismeretlenek

$\frac{d^n}{dx^n}\Big|_{x = x_k} = y_k^{(n)} \quad (k = 1, \ldots, r;\ n = 0, \ldots, m_k)$

egyenletrendszer megoldható

$(x-x_1)^{m_1}\cdots(x-x_r)^{m_r} =: q(x)$

$$\frac{d^m}{dx^m} q(x_\ell) \underset{\uparrow}{=} \frac{d^m}{dt^m}\Big|_{t=0} t^{m_k}\prod_{j:\, j\neq k}[t+(x_k-x_j)]^{m_j} =$$

$t := x - x_k$

$x - x_j = t + (x_k - x_j)$

$$= \frac{d^m}{dt^m}\Big|_{t=0}\Big\{ t^{m_k}\prod_{j:\, j\neq k}(x_k-x_j) + [t \text{ magasabb hatványai}]\Big\} =$$

$$= \begin{cases} 0 & \text{ha } m \neq m_k \\ m_k!\prod_{j:\, j\neq k}(x_k-x_j) & \text{ha } m = m_k \quad [\text{EZ} \neq 0] \end{cases}$$

HA f ADOTT SIMA FÜGGVÉNY, AKKOR ADOTT s-hez

$\exists!\, \alpha$ $\quad \frac{d^m}{dx^m}\Big|_{x=x_k} f(x) + \alpha(x-x_1)^{m_1}\cdots(x-x_r)^{m_r} = \frac{d^m}{dx^m}\Big|_{x=x_k} f(x)$

tetszőleges x_k helyen és

$m=0,\dots,m_k-1$ deriválásrendekre

továbbá $m=m_s$-re

ADOTT: $x_1,\dots,x_r$ (különbözőek) $\in \mathbb{R}$

$m_1,\dots,m_r \geq 0$ egészek

$(y_1^{(0)}, y_1^{(1)},\dots, y_1^{(m_1)}), (y_2^{(0)},\dots,y_2^{(m_2)}),\dots,(y_r^{(0)},\dots,y_r^{(m_r)})$

EKKOR $\exists\, \alpha_1^{(0)},\dots,\alpha_1^{(m_1)},\dots,\alpha_r^{(0)},\dots,\alpha_r^{(m_r)}$

$$P(x) := \sum_{k=1}^{r}\sum_{j=0}^{m_k}\alpha_k^{(j)}\prod_{\ell:\,\ell\neq k}(x-x_\ell)^{m_\ell+1}\cdot(x-x_k)^{j}$$

derivált(?)

$$\frac{d^n}{dx^n}\Big|_{x=x_k} P(x) = y_k^{(n)} \qquad (0 \le n \le m_k, \; k=1,\dots,r)$$

Algorithmus

$$P_{x_1}^{[y_1^{(0)}]}(x) \equiv y_1^{(0)} \qquad \alpha_1^{(0)} := y_1^{(0)} \qquad P_{x_1}^{[y_1^{(0)}]} : x_1 \mapsto y_1^{(0)}$$

$$P_{x_1}^{[y_1^{(0)}, y_1^{(1)}]}(x) := y_1^{(0)} + \alpha_1^{(1)} \cdot (x - x_1) \qquad \frac{d^n}{dx^n} P_{x_1}^{[y_1^{(0)}, y_1^{(1)}]} : x_1 \mapsto y_1^{(n)} \quad n=0,1$$

$$\vdots$$

$$P_{x_1}^{[y_1^{(0)}, \dots, y_1^{(m_1)}]} := P_{x_1}^{[y_1^{(0)}, \dots, y_1^{(m_1-1)}]} + \alpha_1^{(m_1)} (x - x_1)^{m_1}$$

$$\frac{d^m}{dx^m} P_{x_1}^{[y_1^{(0)}, \dots, y_1^{(m_1)}]} : x_1 \mapsto y_1^{(m)} \quad m = 0, \dots, m_1$$

Allgemein Lépés:

$$P_{x_1 \;\; x_2 \;\; \dots \; x_k}^{[y_1^{(0)}, \dots, y_1^{(m_1)}] \; [y_2^{(0)}, \dots, y_2^{(m_2)}] \; [y_k^{(0)}, \dots, y_k^{(j+1)}]} :=$$

$$:= P_{x_1 \;\; \dots \; x_k}^{[y_1^{(0)}, \dots, y_1^{(m_1)}] \cdots [y_k^{(0)}, \dots, y_k^{(j)}]} + \alpha_k^{(j+1)} \prod_{l: l<k} (x - x_l)^{m_l} \, (x - x_k)^{j+1}$$

$1, (x - x_1), (x - x_1)^2, \dots, (x - x_1)^{m_1 - 1}$, ...

$(x - x_1)^{m_1}, (x - x_1)^{m_1}(x - x_2), (x - x_1)^{m_1}(x - x_2)^2, \dots, (x - x_1)^{m_1}(x - x_2)^{m_2 - 1}$

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$(x - x_1)^{m_1} \dots (x - x_{r-1})^{m_{r-1}}, \; (x - x_1)^{m_1} \dots (x - x_{r-1})^{m_{r-1}} (x - x_r), \dots,$
$(x - x_1)^{m_1} \dots (x - x_{r-1})^{m_{r-1}} (x - x_r)^{m_r - 1}$