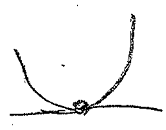
$f: \mathbb{R}^N \to \mathbb{R}$ $e^*[\nabla^2 f(x)]e$ $\frac{d^2}{dt^2}\Big|_{t=0} f(x+te)$



$\|e\|=1$ $mI \le \nabla^2 f \le MI$

$$m\|x-x_0\|^2 \le f(x) - [f(x_0) + \nabla f(x_0)(x-x_0)] \le M\overbrace{\|x-x_0\|^2}^{d(x,x_0)^2}$$

$m>0 \Rightarrow f(x) \ge f(x_0) - \|\nabla f(x_0)\|\|x-x_0\| + m\|x-x_0\|^2$

$\|x\|\to\infty \Rightarrow f(x)\to\infty$

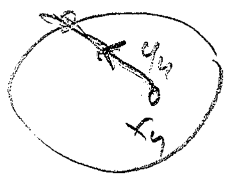
$\{x : f(x)\le c\}$ korl. $\forall c$

konv

$\exists!\, x_*$ $f(x_*) = \min f$

I. $f: K$ konv $\to \mathbb{R}$ $mI \le \nabla^2 f \le MI$ $m>0$

$x_0 \in K$

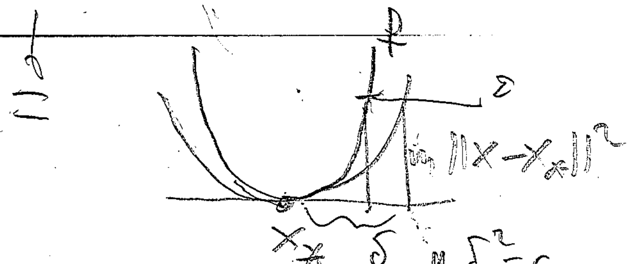


$x_{n+1} := [f$ min-hely $x_n + \mathbb{R}_+ y_n]$-en

y_n ereszkedésre $\measuredangle(y_n, \nabla f(x_n)) < \alpha < 90°$

Akkor $x_n \to x_*$ $(n\to\infty)$

II.



MA $f(x) - f(x_*) = \varepsilon$,

Akkor $\|x-x_*\| \le \sqrt{\varepsilon/m}$

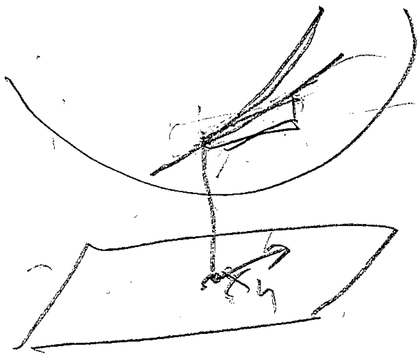
$m\delta^2 = \varepsilon$

$\delta = \sqrt{\varepsilon/m}$

Első lépés: $f(x_n) \searrow f(x_*)$

Mennyit csökken $f(x_{k+1})$ $f(x_k)$-hoz képest?

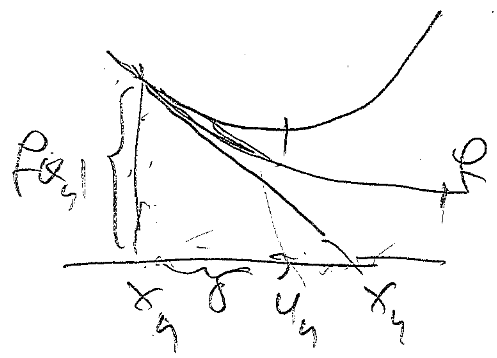
v_k szöge $\nabla f(x_k)$-val $< \alpha$



$\lambda_k := [f$ iránymenti meredeksége v_k irányában$]$

$\leq -\cos\alpha \cdot \|\nabla f(x_k)\| \quad (<0)$

f ALATTA VAN



δ-val v_k irányban menve

$\delta \mapsto f(x_k) - \lambda_k \delta + M \delta^2$ min-je

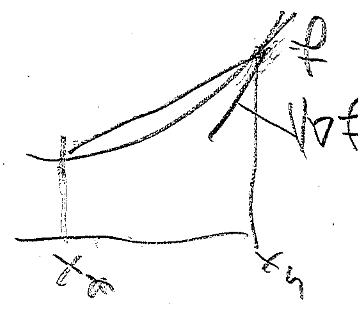
$-\lambda_k + M \cdot 2\delta_k = 0$

$\delta_k = \frac{\lambda_k}{2M}$

csökkenés $= -\lambda_k \delta_k + M \delta_k^2 = -\frac{\lambda_k^2}{2M} + \frac{M \lambda_k^2}{(2M)^2} =$

$= -\frac{\lambda_k^2}{4M}$

Mennyire lehet kicsi $\|\nabla f(x_k)\|$? (s így λ_k)



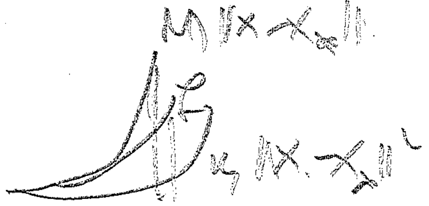
$\|\nabla f(x_k)\|$ egyik meredeksége

$\|\nabla f(x_k)\| \geq \frac{f(x_k) - f(x_*)}{\|x_k - x_*\|}$

[illegible]

$$\frac{\lambda_k^2}{2M} \ge \frac{1}{2M}\cos^2\alpha\,\|\nabla f(x_k)\|^2 \ge \frac{1}{2M}\left(\frac{f(x_k)-f(x_*)}{\|x_k-x_*\|}\right)^2\cos^2\alpha \quad \text{вып}$$

$$\|x_k-x_*\| \ge \sqrt{(f(x_k)-f(x_*))/M}$$



[illegible] $\ge \frac{1}{2M}\left(\frac{f(x_k)-f(x_*)}{\sqrt{(f(x_k)-f(x_*))/m}}\right)^2\cos^2\alpha =$

$$= \frac{m}{2M}\cos^2\alpha\,(f(x_k)-f(x_*))$$

[illegible] $\varepsilon_k := f(x_k)-f(x_*)$

[illegible] $\|x_k-x_*\| \le \sqrt{\varepsilon_k/M}$

$$\varepsilon_{k+1} \le \varepsilon_k - \frac{m}{2M}\cos^2\alpha\,\varepsilon_k = \left(1-\frac{m}{2M}\cos^2\alpha\right)\varepsilon_k$$

$$\varepsilon_k \le \left(1-\frac{m}{2M}\cos^2\alpha\right)^k \varepsilon_0$$