

```

> restart: with(linalg):

print('EMLEKEZTETO_ALT_INV_FROBENIUS_FELBONTASSAL');
A:=F0 *F1;
A:=matrix([[2, 4, 8, 16], [1, 2, 7, 15], [0, 0, 6, 14]]):
F0:=matrix([[2, 8], [1, 7], [0, 6]]):
F1:=matrix([[1, 2, 0, -4/3], [0, 0, 1, 7/3]]):
evalm(A)=evalm(F0)&*evalm(F1);
print('ELMELET___',Ainv_=F1inv_*F0inv);


$$EMLEKEZTETO\_ALT\_INV\_FROBENIUS\_FELBONTASSAL$$


$$A = F0 \cdot F1$$


$$\begin{bmatrix} 2 & 4 & 8 & 16 \\ 1 & 2 & 7 & 15 \\ 0 & 0 & 6 & 14 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 1 & 7 \\ 0 & 6 \end{bmatrix} \&* \begin{bmatrix} 1 & 2 & 0 & -\frac{4}{3} \\ 0 & 0 & 1 & \frac{7}{3} \end{bmatrix}$$


$$ELMELET___, Ainv_ = F1inv \cdot F0inv$$
 (1)

> print(F1,'SVD_FELBONTASA');
print('CELSZERUBB__',F1trp,'__SVD_MAJD_ENNEK_TRANSZPONALTJA');
F1trp:=evalm(transpose(F1));
B:=evalm(F1&*transpose(F1));
print(B=F1&*F1trp,'____',B_=evalm(F1)&*evalm(F1trp),'____',
evalm(B));
lambda:=array(1..2): R:=matrix(2,2):
print(B_, '__FOTENGELY_ALAKJA__',B_=R_*Lambda_*Rtrp_);
Lambda:=diag(lambda[1],lambda[2]):
print(evalm(B)=[r1,r2]*evalm(Lambda)*matrix(2,1,[r1trp,r2trp]),
'____',r1_,r2_,'OSZL_VEKT');
print(lambda[1],lambda[2],'__',B_,'KARAKT_POL_GYOKEI');
det_(evalm(B-diag(lambda[1],lambda[2])))=charpoly(B,lambda);
solve(lambda^2-(119/9)*lambda+34=0,lambda);
print(lambda[1]=119/18+(1/18)*sqrt(3145),'____',lambda[2]=
119/18-(1/18)*sqrt(3145));
print(r1_,'SAJAT_EGYSEG_VEKTORA_',B_,'NEK_',lambda[1],
'SAJATERTEKKEL');
r1:=array(1..2):
print(evalm(B)&*matrix(2,1,[r1[1],r1[2]])=(119/18+(1/18)*sqrt
(3145))*matrix(2,1,[r1[1],r1[2]]), '____',r1[1]^2+r1[2]^2=1);
evalm(B&*matrix(2,1,[r1[1],r1[2]]))=evalm((119/18+(1/18)*sqrt
(3145))*matrix(2,1,[r1[1],r1[2]]));
print((61/9)*r1[1]-(28/9)*r1[2]=(119/18+(1/18)*sqrt(3145))*r1
[1], '____',r1[1]^2+r1[2]^2=1);
solve({(61/9)*r1[1]-(28/9)*r1[2]=(119/18+(1/18)*sqrt(3145))*r
[1],r1[1]^2+r1[2]^2=1}, {r1[1],r1[2]});
r1[2]=solve((61/9)*r1[1]-(28/9)*r1[2]=(119/18+(1/18)*sqrt(3145))
*r1[1],r1[2]);
r1[1]^2+( (3/56)*r1[1]-(1/56)*r1[1]*sqrt(3145) )^2=1;
solve(r1[1]^2+( (3/56)*r1[1]-(1/56)*r1[1]*sqrt(3145) )^2=1,r1[1]
);
print('VALASZTAS___',r1[1]=28*sqrt(6290-6*sqrt(3145))/(-3145+3*sqrt(3145)));
print(r1[2]^2=1-r1[1]^2,'____',r1[2]=sqrt(1-(28*sqrt(6290-6*sqrt
(3145))/(-3145+3*sqrt(3145)))^2));

```

$$\begin{bmatrix} 1 & 2 & 0 & -\frac{4}{3} \\ 0 & 0 & 1 & \frac{7}{3} \end{bmatrix}, SVD_FELBONTASA$$

CELSZERUBB, *F1trp*, *SVD_MAJD_ENNEK_TRANSZPONALTJA*

$$F1trp := \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \\ -\frac{4}{3} & \frac{7}{3} \end{bmatrix}$$

$$B_1 = F1 \&* F1trp, \dots, B_2 = \begin{bmatrix} 1 & 2 & 0 & -\frac{4}{3} \\ 0 & 0 & 1 & \frac{7}{3} \end{bmatrix} \&* \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \\ -\frac{4}{3} & \frac{7}{3} \end{bmatrix}, \dots, \begin{bmatrix} \frac{61}{9} & -\frac{28}{9} \\ -\frac{28}{9} & \frac{58}{9} \end{bmatrix}$$

*B*_2, *FOTENGELY_ALAKJA*, *B*_2 = *R_Lambda_Rtrp*

$$\begin{bmatrix} \frac{61}{9} & -\frac{28}{9} \\ -\frac{28}{9} & \frac{58}{9} \end{bmatrix} = [r1, r2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} r1trp \\ r2trp \end{bmatrix}, \dots, r1, r2, OSZL_VEKT$$

$\lambda_1, \lambda_2, \dots, B_2, KARAKT_POL_GYKEI$

$$det - \left(\begin{array}{cc} \frac{61}{9} - \lambda_1 & -\frac{28}{9} \\ -\frac{28}{9} & \frac{58}{9} - \lambda_2 \end{array} \right) = \lambda^2 - \frac{119}{9} \lambda + 34$$

$$\frac{119}{18} + \frac{1}{18} \sqrt{3145}, \frac{119}{18} - \frac{1}{18} \sqrt{3145}$$

$$\lambda_1 = \frac{119}{18} + \frac{1}{18} \sqrt{3145}, \dots, \lambda_2 = \frac{119}{18} - \frac{1}{18} \sqrt{3145}$$

r1, *SAJAT_EGYSEG_VEKTORA*, *B*_2, *NEK*, λ_1 , *SAJATERTEKKEL*

$$\begin{bmatrix} \frac{61}{9} & -\frac{28}{9} \\ -\frac{28}{9} & \frac{58}{9} \end{bmatrix} \&* \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \left(\frac{119}{18} + \frac{1}{18} \sqrt{3145} \right) \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}, \dots, r1^2 + r2^2 = 1$$

$$\begin{aligned}
& \left[\begin{array}{c} \frac{61}{9} r_1 - \frac{28}{9} r_2 \\ -\frac{28}{9} r_1 + \frac{58}{9} r_2 \end{array} \right] = \left[\begin{array}{c} \left(\frac{119}{18} + \frac{1}{18} \sqrt{3145} \right) r_1 \\ \left(\frac{119}{18} + \frac{1}{18} \sqrt{3145} \right) r_2 \end{array} \right] \\
& \frac{61}{9} rI_1 - \frac{28}{9} rI_2 = \left(\frac{119}{18} + \frac{1}{18} \sqrt{3145} \right) rI_1, \quad rI_1^2 + rI_2^2 = 1 \\
& \left\{ rI_1 = \text{RootOf} \left(- (2293045 - 31535 \sqrt{3145}) Z^2 - (19764 r_1 \sqrt{3145} - 2351916 r_1) Z \right. \right. \\
& \quad \left. \left. - 892296 r_1^2 + 399056 - 5488 \sqrt{3145} \right), rI_2 = \frac{61}{28} \text{RootOf} \left((-2293045 \right. \right. \\
& \quad \left. \left. + 31535 \sqrt{3145}) Z^2 + (-19764 r_1 \sqrt{3145} + 2351916 r_1) Z - 892296 r_1^2 + 399056 \right. \\
& \quad \left. \left. - 5488 \sqrt{3145} \right) - \frac{17}{8} r_1 - \frac{1}{56} r_1 \sqrt{3145} \right\} \\
& rI_2 = \frac{3}{56} rI_1 - \frac{1}{56} rI_1 \sqrt{3145} \\
& rI_1^2 + \left(\frac{3}{56} rI_1 - \frac{1}{56} rI_1 \sqrt{3145} \right)^2 = 1 \\
& \frac{28 \sqrt{6290 - 6 \sqrt{3145}}}{-3145 + 3 \sqrt{3145}}, - \frac{28 \sqrt{6290 - 6 \sqrt{3145}}}{-3145 + 3 \sqrt{3145}} \\
& \text{VALASZTAS}, rI_1 = \frac{28 \sqrt{6290 - 6 \sqrt{3145}}}{-3145 + 3 \sqrt{3145}} \\
& rI_2^2 = 1 - rI_1^2, \quad r_2 = \sqrt{1 - \frac{28 \sqrt{6290 - 6 \sqrt{3145}}}{(-3145 + 3 \sqrt{3145})^2}} \tag{2}
\end{aligned}$$

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> print('HORROR____KEZI_SOT_SZIMBOLIKUS_SZAMITASSAL');
print
('TANULSAZ____SVD_ES_FOTENGELY____CSAK_GEPI_NUM_MODSZEREKKEL');
print('');
print('DE_EZ_NEM_IS_KELL ',F1inv,'_HEZ');
print('ELEG_ANNYI ',B=F1_*Fltrp,'_INVERTALHATO_MERT_',
lambda[1],lambda[2]<>0);
print('EHHEZ_NEM_IS_KELL_SZAMOLNI_HISZEN ',rank(F1)=2,'',
Matrix(2, 4, {(1, 1) = 1, (1, 2) = 2, (1, 3) = 0, (1, 4) = -4/3,
(2, 1) = 0, (2, 2) = 0, (2, 3) = 1, (2, 4) = 7/3}),
'_BEN_VAN_A_KET_EGYSEGV');
print('');
print('AZ_SVD_PUSZTALETEZESENNEK_FELHASZNALASA');
Fltrp=Q_*Lambda*Rtrp;
F1_*Fltrp=R*Lambda*Qtrp*(Qtrptrp_*Lambda_trp_*Rtrp);
F1_*Fltrp=R*Lambda*Qtrp*Q_*Lambda_trp*Rtrp;
F1_*Fltrp=R_*Lambda_*Lambda_trp*Rtrp;
F1_*Fltrp=R_*diag(lambda[1]^2,lambda[2]^2)*Rtrp;
print('EZ_INVERTALHATO_MERT_',lambda[1],lambda[2]<>0);
print('ESZREVETEL');

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inv(F1_F1trp)=R_diag(1/lambda[1]^2,1/lambda[2]^2)*Rtrp;
print(F1=R_Lambda_trp*Qtrp,'_____',F1trp=Q_Lambda_trp*Rtrp)
;
print(F1trp*inv(F1_F1trp)=
Q_diag(1/lambda[1],1/lambda[2])*Rtrp,'_____EZ_EPPEN__',Flinv)
;
print(' ');
Flinv=F1trp*inv(Matrix([[61/9,-28/9],[-28/9,58/9]]));
Flinv=evalm(transpose(F1))*evalm(inverse(Matrix([[61/9,-28/9],
[-28/9,58/9]])));
Flinv:=evalm(transpose(F1)&*inverse(Matrix([[61/9,-28/9],[-28/9,
58/9]])));

```

HORROR KEZI_SOT_SZIMBOLIKUS_SZAMITASSAL
 TANULSAG_SVD_ES_FOTENGELY_CSAK_GEPI_NUM_MODSZEREKKEL

DE_EZ_NEM_IS_KELL_, Flinv,_HEZ
 ELEG_ANNYI_____, B=F1_F1trp,_INVERTALHATO_MERT_, $\lambda_1, \lambda_2 \neq 0$

$$EHHEZ_NEM_IS_KELL_SZAMOLNI_HISZEN, 2=2, \begin{bmatrix} 1 & 2 & 0 & -\frac{4}{3} \\ 0 & 0 & 1 & \frac{7}{3} \end{bmatrix},$$

_BEN_VAN_A_KET_EGYSEGV

AZ_SVD_PUSZTA_LETEZESENEK_FELHASZNALASA
 F1trp_=Q_Lambda_Rtrp_

F1_F1trp_=R_Lambda_Qtrp_Qtrptrp_Lambda_trp_Rtrp

F1_F1trp_=R_Lambda_Qtrp_Q_Lambda_trp_Rtrp

F1_F1trp_=R_Lambda_Lambda_trp_Rtrp_

$$F1_F1trp_=R_{\begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix}} Rtrp$$

EZ_INVERTALHATO_MERT_, $\lambda_1, \lambda_2 \neq 0$

ESZREVETEL

$$inv(F1_F1trp)=R_{\begin{bmatrix} \frac{1}{\lambda_1^2} & 0 \\ 0 & \frac{1}{\lambda_2^2} \end{bmatrix}} Rtrp$$

F1_=R_Lambda_trp_Qtrp, _____, F1trp=Q_Lambda_trp_Rtrp

$$\begin{aligned}
F1trp_inv(F1_F1trp_)=Q_-\left[\begin{array}{cc} \frac{1}{\lambda_1} & 0 \\ 0 & \frac{1}{\lambda_2} \end{array}\right] Rtrp_ , \quad EZ_EPSEN_ , F1inv \\
F1inv_ = F1trp_ inv \overline{\left(\begin{bmatrix} \frac{61}{9} & -\frac{28}{9} \\ -\frac{28}{9} & \frac{58}{9} \end{bmatrix}\right)} \\
F1inv_ = \left[\begin{array}{cc} 1 & 0 \\ 2 & 0 \\ 0 & 1 \\ -\frac{4}{3} & \frac{7}{3} \end{array}\right] \left[\begin{array}{cc} \frac{29}{153} & \frac{14}{153} \\ \frac{14}{153} & \frac{61}{306} \end{array}\right] \\
F1inv := \left[\begin{array}{cc} \frac{29}{153} & \frac{14}{153} \\ \frac{58}{153} & \frac{28}{153} \\ \frac{14}{153} & \frac{61}{306} \\ -\frac{2}{51} & \frac{35}{102} \end{array}\right] \tag{3}
\end{aligned}$$

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>
F0:=Matrix(3, 2, {(1, 1) = 2, (1, 2) = 8, (2, 1) = 1, (2, 2) =
7, (3, 1) = 0, (3, 2) = 6});
print(A = evalm(A), 'ALT_INV_HEZ_KELL_MEG ', F0inv);
print('HAZI_FELADAT ', F0inv = F0trp_*inv(F0 * F0trp_), ' ');
evalm(inverse(transpose(F0) &* F0)) &* transpose(F0), ' ');
evalm(inverse(transpose(F0) &* F0) &* transpose(F0)));
print('ELMELET_ALAPJAN ', Ainv = F1inv * F0inv);
print(Ainv=evalm(F1inv)&*Matrix(2, 3, {(1, 1) = 19/36, (1, 2) =
-1/18, (1, 3) =
-23/36, (2, 1) = -1/36, (2, 2) = 1/18, (2, 3) = 5/36}), ' ');
evalm(F1inv&*Matrix(2, 3, {(1, 1) = 19/36, (1, 2) = -1/18, (1,
3) =
-23/36, (2, 1) = -1/36, (2, 2) = 1/18, (2, 3) = 5/36})));

```

$$F0 := \begin{bmatrix} 2 & 8 \\ 1 & 7 \\ 0 & 6 \end{bmatrix}$$

$$A_-= \begin{bmatrix} 2 & 4 & 8 & 16 \\ 1 & 2 & 7 & 15 \\ 0 & 0 & 6 & 14 \end{bmatrix}, \text{ALT_INV_HEZ_KELL_MEG}, F0inv_-$$

HAZI_FELADAT, $F0inv_- = F0trp_- inv(F0_- F0trp_-)$,

$$\begin{bmatrix} \frac{149}{216} & -\frac{23}{216} \\ -\frac{23}{216} & \frac{5}{216} \end{bmatrix} & * \begin{bmatrix} 2 & 1 & 0 \\ 8 & 7 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{19}{36} & -\frac{1}{18} & -\frac{23}{36} \\ -\frac{1}{36} & \frac{1}{18} & \frac{5}{36} \end{bmatrix}$$

ELMELET_ALAPJAN, $Ainv_- = F1inv_- F0inv$

$$Ainv = \begin{bmatrix} \frac{29}{153} & \frac{14}{153} \\ \frac{58}{153} & \frac{28}{153} \\ \frac{14}{153} & \frac{61}{306} \\ -\frac{2}{51} & \frac{35}{102} \end{bmatrix} & * \begin{bmatrix} \frac{19}{36} & -\frac{1}{18} & -\frac{23}{36} \\ -\frac{1}{36} & \frac{1}{18} & \frac{5}{36} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{179}{1836} & -\frac{5}{918} & -\frac{199}{1836} \\ \frac{179}{918} & -\frac{5}{459} & -\frac{199}{918} \\ \frac{157}{3672} & \frac{11}{1836} & -\frac{113}{3672} \\ -\frac{37}{1224} & \frac{13}{612} & \frac{89}{1224} \end{bmatrix}$$

(4)

```
>
print
('
');
print('ALTALANOS_EGYENLETMEGOLDAS_GAUSS_JORDAN_ELIMINACIOVAL');
print
('
');
X:=matrix(4,1,[x,y,z,u]): b:=matrix(3,1,[p,q,r]);
A_x_=b;
evalm(A&*x=b);
print(1,'SOR_MINUS_2x ',3,'SOR');
L1:=matrix(3,3,[1,-2,0,0,1,0,0,0,1]);
print(L1 *A=L1 *b,' ',A1=evalm(A1));
A1:=evalm(L1&*A): b1:=evalm(L1&*b):
evalm(A1&*X)=evalm(b1);
print(1,2,'SORCSERE');
L2:=matrix(3,3,[0,1,0,1,0,0,0,0,1]);
A2:=evalm(L2&*A1); b2:=evalm(L2&*b1):
evalm(A2&*X=evalm(b2));
print(3,'SOR_PLUS',2,'SOR');
L3:=matrix(3,3,[1,0,0,0,1,0,0,1,1]);
A3:=evalm(L3&*A2); b3:=evalm(L3&*b2):
evalm(A3&*X=evalm(b3));
print('PONTOS_MEGOLDAS_FELTETELE_____ ',p-2*q+r=0);
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```

print(2,'SOR PER',-6);
L4:=matrix(3,3,[1,0,0,0,(-1/6),0,0,0,1]);
A4:=evalm(L4&*A3); b4:=evalm(L4&*b3):
evalm(A4&*X=evalm(b4));
print(1,'SOR_MINUS_7x_',2,'SOR');
L5:=matrix(3,3,[1,-7,0,0,1,0,0,0,1]);
A5:=evalm(L5&*A4); b5:=evalm(L5&*b4):
evalm(A5&*X=evalm(b5));
print('EGYSEGVEKTOROK_JO_SORRENDBEN_',1,3,'_OSZLOP');
print('TELJES_MEGOLDAS');
print(1,3,'VALTOZORA ',2,4,'VALT_TETSZ');
print(p-2*q+r=0,' ESETEN');
x=(7/6)*p-(4/3)*q-2*y+(4/3)*u;
z=(-1/6)*p+(1/3)*q-(7/3)*u;

```

ALTALANOS_EGYENLETMEGOLDAS_GAUSS_JORDAN_ELIMINACIOVAL

$$b := \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$A_x = b$$

$$\begin{bmatrix} 2x_{1,1} + 4x_{2,1} + 8x_{3,1} + 16x_{4,1} \\ x_{1,1} + 2x_{2,1} + 7x_{3,1} + 15x_{4,1} \\ 6x_{3,1} + 14x_{4,1} \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

1, SOR_MINUS_2x_, 3, SOR

$$LI := \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$LI \cdot A = LI \cdot b, \quad AI = \begin{bmatrix} 0 & 0 & -6 & -14 \\ 1 & 2 & 7 & 15 \\ 0 & 0 & 6 & 14 \end{bmatrix}$$

$$\begin{bmatrix} -6z - 14u \\ x + 2y + 7z + 15u \\ 6z + 14u \end{bmatrix} = \begin{bmatrix} p - 2q \\ q \\ r \end{bmatrix}$$

1, 2, SORCSERE

$$L2 := \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A2 := \begin{bmatrix} 1 & 2 & 7 & 15 \\ 0 & 0 & -6 & -14 \\ 0 & 0 & 6 & 14 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + 7z + 15u \\ -6z - 14u \\ 6z + 14u \end{bmatrix} = \begin{bmatrix} q \\ p - 2q \\ r \end{bmatrix}$$

3, SOR_PLUS, 2, SOR

$$L3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A3 := \begin{bmatrix} 1 & 2 & 7 & 15 \\ 0 & 0 & -6 & -14 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + 7z + 15u \\ -6z - 14u \\ 0 \end{bmatrix} = \begin{bmatrix} q \\ p - 2q \\ p - 2q + r \end{bmatrix}$$

PONTOS_MEGOLDAS_FELTETELE, $p - 2q + r = 0$
2, SOR_PER, -6

$$L4 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{6} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A4 := \begin{bmatrix} 1 & 2 & 7 & 15 \\ 0 & 0 & 1 & \frac{7}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + 7z + 15u \\ z + \frac{7}{3}u \\ 0 \end{bmatrix} = \begin{bmatrix} q \\ -\frac{1}{6}p + \frac{1}{3}q \\ p - 2q + r \end{bmatrix}$$

1, SOR_MINUS_7x_, 2, SOR

$$L5 := \begin{bmatrix} 1 & -7 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A5 := \begin{bmatrix} 1 & 2 & 0 & -\frac{4}{3} \\ 0 & 0 & 1 & \frac{7}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y - \frac{4}{3}u \\ z + \frac{7}{3}u \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{4}{3}q + \frac{7}{6}p \\ -\frac{1}{6}p + \frac{1}{3}q \\ p - 2q + r \end{bmatrix}$$

*EGYSEGVEKTOROK_JO_SORRENDBEN_, 1, 3, _OSZLOP
TELJES_MEGOLDAS*

1, 3, VALTOZORA_, 2, 4, VALT_TETSZ

$$p - 2q + r = 0, \text{ _ESETEN}$$

$$x = \frac{7}{6}p - \frac{4}{3}q - 2y + \frac{4}{3}u$$

$$z = -\frac{1}{6}p + \frac{1}{3}q - \frac{7}{3}u$$

(5)

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