```
> restart :
  with(linalg) :
  with(plots) :

  print("Mesh vertices");
  R := 4;
  print(R, "  points");
  print("3D-Coordinates of the ", R, " points in the rows");
  P := matrix(4, 3, [0, 0, 0,   1, 0, 0,   0, 1, 0,   0, 0, 1]);

  print("Mesh triangles");
 N := 4;
  print("Point Indices of  the ", N, " triangles,");
  print("i_star(n,i) = index of point i in triangle n");
  print("Lexicographic order");
 i_star := matrix(N, 3, [1, 2, 3,   1, 2, 4,   1, 3, 4,   2, 3, 4]);

  #Calculation for nuber of edges"
  M := 0 :
  j_star0 := matrix(R·(R − 1)·2^(−1), 2) :
  for i1 from 1 to R − 1 do:
  for i2 from i1 + 1 to R do:
  s := 0 :
  for n from 1 to N do:
  if i1 = i_star[n, 1] and i2 = i_star[n, 2] then s := 1 fi:
  if i1 = i_star[n, 1] and i2 = i_star[n, 3] then s := 1 fi:
  if i1 = i_star[n, 2] and i2 = i_star[n, 3] then s := 1 fi:
  od:#n
  if s = 1 then M := M + 1 : j_star0[M, 1] := i1 : j_star0[M, 2] := i2 : fi:
  od:od:#i1,i2

  print("M= ", M, "mesh edges");
  print("j_star(m,ell)= index of point ell in edge m");
  print("Lexicographic order");
  j_star := matrix(M, 2) :
  for m from 1 to M do: for ell from 1 to 2 do: j_star[m, ell] := j_star0[m, ell] : od:od:
  print(evalm(j_star));

   n_star := matrix(M, 2) :
  for m from 1 to M do:
  i1 := j_star[m, 1] :  i2 := j_star[m, 2] :
  s := 0 :
  for n from 1 to N do:
  if i1 = i_star[n, 1] and i2 = i_star[n, 2] then s := s + 1 : n_star[m, s] := n : fi:
  if i1 = i_star[n, 1] and i2 = i_star[n, 3] then s := s + 1 : n_star[m, s] := n :  fi:
   if i1 = i_star[n, 2] and i2 = i_star[n, 3] then s := s + 1 : n_star[m, s] := n : fi:
  od:#n
  if s = 1 then n_star[m, 2] := n_star[m, 1] : fi:
  od:#m
  print("n_star(m,ell)=index ell of neighboring triangle of edge m");
 print(evalm(n_star));
```

```
k_star := matrix(M, 2) :
for m from 1 to M do:
i1 := j_star[m, 1] : i2 := j_star[m, 2] : n1 := n_star[m, 1] : n2 := n_star[m, 2] :
k_star[m, 1] := i_star[n1, 1] + i_star[n1, 2] + i_star[n1, 3] - i1 - i2 :
k_star[m, 2] := i_star[n2, 1] + i_star[n2, 2] + i_star[n2, 3] - i1 - i2 :
od:#m
print("k_star(m,1), k_star(m,2)  indices of opposite vertices to edge m in neighboring
    triangles");
print(evalm(k_star));

m_star := matrix(N, 3) :
for n from 1 to N do:
i1 := i_star[n, 1] : i2 := i_star[n, 2] : i3 := i_star[n, 3] :
for m from 1 to M do:
j1 := j_star[m, 1] :  j2 := j_star[m, 2] :
if j1 = i1 and j2 = i2 then m_star[n, 3] := m : fi:
if j1 = i1 and j2 = i3 then m_star[n, 2] := m : fi:
if j1 = i2 and j2 = i3 then m_star[n, 1] := m : fi:
od:#m
od:#n
print("m_star(n,ell) = index of opposite edge of vertex ell of triangle n");
print(evalm(m_star));


print("Degrees of vertices, cycles of neighbors");

degp := vector(R) : startm := vector(R) : innerp := vector(R) :


for i from 1 to R do:
innerp[i] := 1 : degp[i] := 0 :
for m from 1 to M do:
if j_star[m, 1] = i or j_star[m, 2] = i then
if degp[i] = 0 then startm[i] := m : fi:
if innerp[i] = 1 and n_star[m, 1] = n_star[m, 2] then innerp[i] := 0 : startm[i] := m : fi:
degp[i] := degp[i] + 1 :
fi:#j_star
od:#m
od:#i
print("Degrees of vertices", evalm(degp));
print("Inner vertices", evalm(innerp));
print("Starting edges in cycles", evalm(startm));

for i from 1 to R do:
dp[i] := degp[i] + innerp[i] : cycm[i] := vector(dp[i]) : cycp[i] := vector(dp[i]) :
m := startm[i] : n := n_star[m, 1] :  ip := j_star[m, 1] + j_star[m, 2] - i :
cycm[i][1] := m :
cycp[i][1] := ip :
for ell from 2 to dp[i] do:
n := n_star[m, 1] + n_star[m, 2] - n :
for k from 1 to 3 do:
if i_star[n, k] = ip then kk := k : fi:
```

```
od:#k
m := m_star[n, kk] :
ip := j_star[m, 1] + j_star[m, 2] - i :
cycm[i][ell] := m :
cycp[i][ell] := ip :
od:#ell

print("Cycle of edges from point", i, "    ", evalm(cycm[i]), "  Cycle of points",
    evalm(cycp[i])) :
od:#i


print("Guessing normal vectors");

nn := matrix(R, 3) :
nn2 := vector(R) :
u := vector(3) : v := vector(3) :

for i from 1 to R do:
nn[i, 1] := 0 : nn[i, 2] := 0 : nn[i, 3] := 0 :
nn2[i] := 0 :
for ell from 2 to dp[i] do:
for k  from 1 to 3 do:
iii := cycp[i][ell] : ii := cycp[i][ell - 1] :
u[k] := P[iii, k] - P[i, k] :   v[k] := P[ii, k] - P[i, k] :
od:#k
nn[i, 1] := nn[i, 1] + u[2]·v[3] - u[3]·v[2] :
nn[i, 2] := nn[i, 2] + u[3]·v[1] - u[1]·v[3] :
nn[i, 3] := nn[i, 3] + u[1]·v[2] - u[2]·v[1] :
od:#ell
nn2[i] := nn2[i] + nn[i, 1]^2 + nn[i, 2]^2 + nn[i, 3]^2 :
od:#i

print("Normal vectors", evalm(nn));
print("Norm squares", evalm(nn2));

print("Guessed Surface derivatives");
print("g_ij from point i toward j");
g := array(1 ..R, 1 ..R) :
for i from 1 to R do: for j from 1 to R do:
g[i, j] := vector(3, [0, 0, 0]) :
od:od:#ij
for i from 1 to R do:
for ell from 1 to degp[i] do:
j := cycp[i][ell] :
sij := nn[i, 1]·(P[j, 1] - P[i, 1]) + nn[i, 2]·(P[j, 2] - P[i, 2]) + nn[i, 3]·(P[j, 3] - P[i,
    3]) :
sji := nn[j, 1]·(P[i, 1] - P[j, 1]) + nn[j, 2]·(P[i, 2] - P[j, 2]) + nn[j, 3]·(P[i, 3] - P[j,
    3]) :
for k from 1 to 3 do:
g[i, j][k] := (P[j, k] - P[i, k]) - sij/nn2[i] ·nn[i, k] :
```

```
od:#k
print("g", i, j, "    ", evalm(g[i, j]), "    normalv", [nn[i, 1], nn[i, 2], nn[i, 3]]) :
od:#ell
od:#i


print(
  "_____________________________________________________________________________________\
  ___________");
print(
  "_____________________________________________________________________________________\
  ___________");

print("Checking if the g vectors are suitable in a polynomial construction");

print("g_,ell-g_ik) x g_ij not= 0 when   [p_i,p_j]  is a double edge between the mesh triangles
  [p_i,p_j,p_k],[pi,p_j,p_ell]");

suitable := 1 :
for m from 1 to M do:
i := j_star[m, 1] : j := j_star[m, 2] :
n1 := n_star[m, 1] : n2 := n_star[m, 2] :
if n1 ≠ n2 then
a1 := g[i, j][1] : a2 := g[i, j][2] : a3 := g[i, j][3] :
k1 := k_star[m, 1] : k2 := k_star[m, 2] :
b1 := g[i, k1][1] - g[i, k2][1] : b2 := g[i, k1][2] - g[i, k2][2] : b3 := g[i, k1][3] - g[i,
  k2][3] :
c1 := a2·b3 - b2·a3 :  c2 := a3·b1 - b3·a1 :  c3 := a1·b2 - b1·a2 :
c := c1^2 + c2^2 + c3^2 :  if c = 0  then suitable := 0 : fi:
if c = 0 then  print("g[", i, j, "]  ,  g[", i, k1, "]  ,  g[", i, k2, "]   FALSE") fi:
a1 := g[j, i][1] : a2 := g[j, i][2] : a3 := g[j, i][3] :
k1 := k_star[m, 1] : k2 := k_star[m, 2] :
b1 := g[j, k1][1] - g[j, k2][1] : b2 := g[j, k1][2] - g[j, k2][2] : b3 := g[j, k1][3] - g[j,
  k2][3] :
c1 := a2·b3 - b2·a3 :  c2 := a3·b1 - b3·a1 :  c3 := a1·b2 - b1·a2 :
c := c1^2 + c2^2 + c3^2 :  if c = 0  then suitable := 0 : fi:
if c = 0 then  print("g[", j, i, "]  ,  g[", j, k1, "]  ,  g[", j, k2, "]   FALSE") fi:
fi:
od:#m

if suitable = 1 then print("SUITABLE") : fi:
if suitable = 0 then print("NOT SUITABLE  - g data should be modified") : fi:

print(
  "_____________________________________________________________________________________\
  ___________");
print(
  "_____________________________________________________________________________________\
  ___________")
```

```
print("Basic RSD interpolation");


t := vector(3) : lambda := vector(R) :

print("Shape functions", "Pi_0=(", Phi, Theta, chi_0, chi_1,")");
Phi := t0^3·(10 − 15·t0 + 6·t0^2);
Theta := t0^3·(4 − 3·t0);
chi_0 := 30·t1^2·t2^2·t3;
chi_1 := 12·t1^2·t2^2·t3;

f := array( 1 .. N) : f_lambda := array(1 ..N) :
 for n from 1 to N do:
f[n] := vector(3, [0, 0, 0]) :  f_lambda[n] := vector(3, [0, 0, 0]) :
od:#n
p := array(1 ..R) :
for i from 1 to R do:
p[i] := vector(3, [P[i, 1], P[i, 2], P[i, 3]]) :
od:#i

for n from 1 to N do:

for i from 1 to 3 do:
ip := i_star[n, i] :
f[n] := evalm(f[n] + subs(t0 = t[i], Phi)·p[ip]) :
for j from 1 to 3 do:
jp := i_star[n, j] :
f[n] := evalm(f[n] + subs(t0 = t[i], Theta)·t[j]·g[ip, jp]) :
od:od:#ij
for i from 1 to 3 do: for j from 1 to 3 do:  for k from 1 to 3 do:
if i ≠ j and j ≠ k and k ≠ i then
 ip := i_star[n, i] : jp := i_star[n, j] :  kp := i_star[n, k] :
 f[n] := evalm( f[n] + subs(t1 = t[i], t2 = t[j], t3 = t[k], chi_0)·p[ip] + subs(t1 = t[i], t2
      = t[j], t3 = t[k], chi_1)·g[ip, jp]) :
fi:
od:od:od: #ijk

od:#n

for n from 1 to N do:
print("f^T,P,G_Pi0 over triangle ", n) :
 i1 := i_star[n, 1] :   i2 := i_star[n, 2] :   i3 := i_star[n, 3] :
for k  from 1 to 3 do:
f_lambda[n][k] := subs(t[1] = lambda[i1], t[2] = lambda[i2], t[3] = lambda[i3], f[n][k]) :
od:

print(evalm(f[n][1])) :  #print(evalm(f_lambda[n][1])) :
print(evalm(f[n][2])) :   #print(evalm(f_lambda[n][2])) :
print(evalm(f[n][3])) :   #print(evalm(f_lambda[n][3])) :

od:#n
```

```
print("f^T,P,G_Pi0   displayed");

f_su := matrix(N, 3) :
for n from 1 to N do:
for k from 1 to 3 do:
f_su[n, k] := subs(t[1] = sigma, t[2] = (1 − sigma)·tau, t[3] = (1 − sigma)·(1 − tau),
    f[n][k]) :

od:od: #kn

C1 := array(1..3) : C2 := array(1..3) : C3 := array(1..3) : C4 := array(1..3) :
for k from 1 to 3 do:
C1[k] := f_su[1, k] : C2[k] := f_su[2, k] : C3[k] := f_su[3, k] : C4[k] := f_su[4, k] :
od:

plot3d({C1, C2, C3, C4}, sigma = 0..1, tau = 0..1);

a := 0; b := 100^(−1); c := 49·100^(−1); d := 1 − 49·100^(−1);
plot3d({C3, C4}, sigma = a..b, tau = c..d);


print(
    "_____________________________________________________________________________\
    __________");
    print(
    "_____________________________________________________________________________\
    __________");



print("Edge corrections");

print("Row m: double edge m between points  i1,12 and opposite vertices j1,j2 in triangles n1,
    n2");
print("ordering:  i1<i2 ,  j1<j2  and n1<n2");

MM := 0 :
for m from 1 to M do:
if n_star[m, 1] ≠ n_star[m, 2] then MM := MM + 1 : fi:
od:#m
print("Number of double edges ", MM);

IJN := matrix(MM, 6) :
mm := 0 :
for m from 1 to M do:
n1 := n_star[m, 1] : n2 := n_star[m, 2] :
if n1 ≠ n2  then
mm := mm + 1 :
i1 := j_star[m, 1] : i2 := j_star[m, 2] :
j1 := k_star[m, 1] : j2 := k_star[m, 2] :
IJN[mm, 1] := i1 : IJN[mm, 2] := i2 :
```

```
 IJN[mm, 3] := j1 :  IJN[mm, 4] := j2 :
IJN[mm, 5] := n1 :  IJN[mm, 6] := n2 :
fi:
od:#m

 print("IJM  ", evalm(IJN));

 V := matrix(MM, 3) : VV := matrix(MM, 3) :  dV := matrix(MM, 3) :
Y := matrix(MM, 3) : W := matrix(MM, 3) :
 Bdet := vector(MM) :

 for m from 1 to MM do:
 i1 := IJN[m, 1] :  i2 := IJN[m, 2] :  j1 := IJN[m, 3] :  j2 := IJN[m, 4] :  n1 := IJN[m, 5] :  n2
       := IJN[m, 6] :
 print("_______________________________________________________") :
print("On (directed Edge ", m, "   points ", i1, " --> ", i2) :

for k from 1 to 3 do:
 F := f_lambda[n1][k] :

 print(k, F) :

 FL := subs(lambda[i1] = tau − 2^(−1)·sigma, lambda[i2] = 1 − tau − 2^(−1)·sigma, lambda[j1]
       = sigma, F) :
 V[m, k] := factor(subs(sigma = 0, diff(FL, sigma))) :
 F := f_lambda[n2][k] :
 FL := subs(lambda[i1] = tau − 2^(−1)·sigma, lambda[i2] = 1 − tau − 2^(−1)·sigma, lambda[j2]
       = sigma, F) :
 VV[m, k] := factor(subs(sigma = 0, diff(FL, sigma))) :
 dV[m, k] := factor(VV[m, k] − V[m, k]) :
 FL := subs(lambda[i1] = tau, lambda[i2] = 1 − tau, lambda[j2] = 0, F) :
 Y[m, k] := factor( diff(FL, tau)) :
 od:
 dv1 := dV[m, 1] : dv2 := dV[m, 2] : dv3 := dV[m, 3] :
 y1 := Y[m, 1] : y2 := Y[m, 2] : y3 := Y[m, 3] :
 W[m, 1] := factor(dv2·y3 − dv3·y2) : W[m, 2] := factor(dv3·y1 − dv1·y3) :  W[m, 3]
       := factor(dv1·y2 − dv2·y1) :
 v1 := V[m, 1] : v2 := V[m, 2] : v3 := V[m, 3] :
 w1 := W[m, 1] : w2 := W[m, 2] : w3 := W[m, 3] :
 bb := v1·w1 + v2·w2 + v3·w3 :
 Bdet[m] := factor(bb) :

 print("v", [V[m, 1], V[m, 2], V[m, 3]]) :
 print("vv", [VV[m, 1], VV[m, 2], VV[m, 3]]) :
 print("dv", [dV[m, 1], dV[m, 2], dV[m, 3]]) :
 print("y", [Y[m, 1], Y[m, 2], Y[m, 3]]) :
print("w", [W[m, 1], W[m, 2], W[m, 3]]) :
 print("b=det[v,dv,y]", Bdet[m]) :
 od:
```

```
print(
  "________________________________________________________________________________\
  __________");
   print(
  "________________________________________________________________________________\
  __________");


print("Cofactors of GCD(w_1,w_2,w_3)");


K := 3 :

 Q := vector(MM) :
 qw := matrix(MM, 3) :

a0 := matrix(1, K) :  a := matrix(1, K) :


for mmm from 1 to MM do:

i1 := IJN[mmm, 1] :     i2 := IJN[mmm, 2] :

print("W[", mmm, ",on points ", i1 , "-->", i2, "]");

 a0[1, 1] := W[mmm, 1] :
 a0[1, 2] := W[mmm, 2] :
 a0[1, 3] := W[mmm, 3] :
 print("Polynomials");
 print(a0[1, 1]) : print(a0[1, 2]) : print(a0[1, 3]) :

 for k from 1 to K do:
 a[1, k] := a0[1, k] :
 od:

 dd := vector(K) :  S := 0 :
 for k  from 1 to K do:
 if expand(a[1, k]) = 0 then dd[k] := -1 : fi:
 if expand(a[1, k] ) ≠ 0 then  dd[k] := degree(a[1, k]) : S := S + dd[k] : fi:
 od:#k
 #print("Degrees  ", evalm(dd));
  #print("Sum of positive degrees  ", S);


 mc := vector(K) :
 for k from 1 to K do:
 if dd[k] < 0 then mc[k] := 0 : fi:
 if dd[k] = 0 then mc[k] := a[1, k] : fi:
 if dd[k] > 0 then
b := a[1, k] : for j from 1 to dd[k] do: b := diff(b, tau)/j : od: mc[k] := b :
 fi:
```

```
od:#k
#print("maincoeff s  ", evalm(mc));

X := matrix(K, K) :
for i from 1 to K do: for j from 1 to K do: X[i, j] := 0 : od: X[i, i] := 1 : od:
#print("X initial  ", evalm(X) );

for s from 1 to S do:
#print("____________________________________________________________________") :
#print("STEP  ", s) :
#``
#print("polynomials", evalm(transpose(a) ) ) :

for k  from 1 to K do:
if expand(a[1, k]) =0 then dd[k] :=-1 : fi:
if expand(a[1, k] ) ≠ 0 then  dd[k] := degree(a[1, k]) : fi:
od:#k
for k from 1 to K do:
if dd[k] < 0 then mc[k] := 0 : fi:
if dd[k]=0 then mc[k] := a[1, k] : fi:
if dd[k] ≥ 1 then
b := a[1, k] : for j from 1 to dd[k] do: b := diff(b, tau)/j :od: mc[k] := b :
fi:
od:#k
#print("degrees  ", evalm(dd) ) :
#print("main coefficients  ", evalm(mc) ) :

#print("Normalization") :

for k from 1 to K do:
if dd[k] ≥ 0 then a[1, k] := expand(mc[k]^(-1)·a[1, k]) :fi:
od:#k
A := matrix(K, K) :
for i from 1 to K do: for j from 1 to K do:
A[i, j] := 0 :
if i =j and dd[j] ≥ 0 then A[i, j] := mc[j]^(-1) : fi:
if i =j and dd[j] < 0 then A[i, i] := 1 : fi:
od:od:#ij
X := evalm(X &*A) :
#print("New polynomials  ", evalm(transpose(a) ) ) :
# print("degrees  ", evalm(dd) ) :
#print("New X  with X=XA  ", evalm(X), "   A=", evalm(A) ) :
#``
#print("Reordering") :
P := matrix(K, K) :
for i from 1 to K do:for j from 1 to K do: P[i, j] := 0 : od: P[i, i] := 1 :od:
for n from 1 to K do:
for i from 1 to K − 1 do:
j := i + 1 :
if (0 ≤ dd[i] and  dd[i] < dd[j]) or dd[j] =-1 then
```

```
 ddd := dd[j] : dd[j] := dd[i] : dd[i] := ddd :
 aaa := a[1, j] : a[1, j] := a[1, i] : a[1, i] := aaa :
for k from 1 to K do:
 ppp := P[k, j] : P[k, j] := P[k, i] : P[k, i] := ppp :
 od:#k
 fi:
 od:#i
 od:#n
 X := evalm(X&*P) :
 #print("New polynomials ") :
 #print(evalm(transpose(a))) :
 #print("degrees  ", evalm(dd)) :
 #print("New X=XP  ", evalm(X), "  with   P=  ", evalm(P)) :

 #print("Degree decreasing") :
 L := 0 :
 for k from 1 to K do:
 if dd[k] =-1 then L := k  fi:
od:#k
 L := L + 1 :
 #print("First nonzero pol with index   L=", L) :


 #STOP condition
 if L < K then

 dddd := dd[L] - dd[L + 1] :
 #print("pol_L = ", a[1, L], "   pol_L+1 = ", a[1, L + 1], "    tau^d =", tau^dddd) :

 a[1, L] := expand(a[1, L] - tau^dddd·a[1, L + 1]) :
 #print("a_L-tau^dddd a_L+1 = ", a[1, L]) :
 B := matrix(K, K) :
 for i from 1 to K do: for j from 1 to K do:
 B[i, j] := 0 : od: B[i, i] := 1 :od:
 B[L + 1, L] := -tau^dddd :
 X := evalm(X&*B) :
 #print("New polynomials") :
 #print(evalm(transpose(a))) :
 #print("New X=XB  ", evalm(X), "  with   B=  ", evalm(B)) :

 fi: #STOPcond

 #STOP
 if L = K then s := S :fi:

 #print("CHEKING  a -a0 X=0") :
 DDD := evalm(a - a0&*X) :
 DD := evalm(DDD[1, 1]) :
 for k from 1 to K do: DD := expand(DD) :od:
#print(evalm(DD)) :
```

```
od:#s

GCD := evalm(a[1, K]) :
print(" GCD over double edge  ", mmm , " = ", GCD);
Q[mmm] := GCD :


#print("Cofactors in column 3 of X");

for i from 1 to K do: qw[mmm, i] := expand(X[i, L]) : od:


print("qw", mmm, 1, "= ", qw[mmm, 1]) :
print("qw", mmm, 2, "= ", qw[mmm, 2]) :
print("qw", mmm, 3, "= ", qw[mmm, 3]) :


# print("CHECKING GCD-a0_1 q_1-...-a0_K q_K =0");

# DD:=GCD:
#for k from 1 to K do: DD := expand(DD - a0[1, k]·qw[mmm, k]) : od:
#print(DD);

od:#mm


print(
    "________________________________________________________________________________\
    __________");
    print(
    "________________________________________________________________________________\
    __________");

print("Z polynomials along double edges");


Zq := matrix(MM, 3) :  ZqL := matrix(MM, 3) :


for mm from 1 to MM do:
 i1 := IJN[mm, 1] : i2 := IJN[mm, 2] :
print("Double edge ", mm, "   ", i1, "-->", i2) :
```

$$z1 := simplify\left(factor\left(\frac{Bdet[mm]}{Q[mm]\cdot \mathrm{tau}^2 \cdot (1-\mathrm{tau})^2} \right) \right) :$$

```
#print(z1) :

  for k from 1 to 3 do:
z2 := factor(qw[mm, k]) :

Zq[mm, k] := z1·z2 :
```

```
z0 := subs(tau = lambda[i1], z1) ·factor(subs(tau = 1 − lambda[i2], z2)) :
 z0 := z0 + factor(subs(tau = 1 −  lambda[i2], z1)) ·subs(tau = lambda[i1], z2) :
ZqL[mm, k] := 2^(−1)·z0 :
print("Zq[", mm, k, " resp. ZqL[", mm, k, "]") :
print(Zq[mm, k]) :
print(ZqL[mm, k]) :

od:#k
od:#mm

Fq := matrix(N, 3) :

for n from 1 to N do:
for k from 1 to 3 do: Fq[n, k] := f_lambda[n][k] : od:#k
od:#n

for mm from 1 to MM do:
i1 := IJN[mm, 1] :  i2 := IJN[mm, 2] :
j1 := IJN[mm, 3] :  j2 := IJN[mm, 4] :
n1 := IJN[mm, 5] :  n2 := IJN[mm, 6] :
 for k   from 1 to 3 do:
Fq[n1, k] := Fq[n1, k] −lambda[i1]^2·lambda[i2]^2·lambda[j1]· ZqL[mm, k] :
Fq[n2, k] := Fq[n2, k] −lambda[i1]^2·lambda[i2]^2·lambda[j2]· ZqL[mm, k] :
od:#k
od:#mm

for n from 1 to N do:
print("Triangle ", n, [i_star[n, 1], i_star[n, 2], i_star[n, 3]]) :
print("Spline over triangle ", n) :
for k from 1 to 3 do:
print(Fq[n, k]) :
od:#k
od:#n



print(
      "___________________________________________________________________________\
      __________");
       print(
      "___________________________________________________________________________\
      __________");

print("Final result displayed");

Fqts := matrix(N, 3) :
for n from 1 to N do:
i1 := i_star[n, 1] : i2 := i_star[n, 2] : i3 := i_star[n, 3] :
for k from 1 to 3 do:
Fqts[n, k] := subs(lambda[i1] = tau, lambda[i2] = (1 − tau) ·sigma, lambda[i3] = (1 − tau) · (1
```

$- \text{sigma}), Fq[n, k]) :$
$\#print(Fqts[n, k]) :$
od:$\#k$
od:$\#n$

$Cq1 := [Fqts[1, 1], Fqts[1, 2], Fqts[1, 3]] : Cq2 := [Fqts[2, 1], Fqts[2, 2], Fqts[2, 3]] :$
$Cq3 := [Fqts[3, 1], Fqts[3, 2], Fqts[3, 3]] : Cq4 := [Fqts[4, 1], Fqts[4, 2], Fqts[4, 3]] :$

$plot3d(\{Cq1, Cq2, Cq3, Cq4\}, \text{tau} = 0..1, \text{sigma} = 0..1);$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 99 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 90 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 80 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 70 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 60 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 50 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 40 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 30 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 20 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 10 \cdot 10^{-2}..100 \cdot 10^{-2});$

$plot3d(\{Cq1, Cq2\}, \text{tau} = 0 \cdot 10^{-2}..100 \cdot 10^{-2}, \text{sigma} = 0 \cdot 10^{-2}..100 \cdot 10^{-2});$

"Mesh vertices"

$R := 4$

4, " points"

"3D-Coordinates of the ", 4, " points in the rows"

$$P := \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"Mesh triangles"

$N := 4$

"Point Indices of the ", 4, " triangles,"
"i_star(n,i) = index of point i in triangle n"
"Lexicographic order"

$$i\_star := \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix}$$

"M= ", 6, "mesh edges"
"j_star(m,ell)= index of point ell in edge m"
"Lexicographic order"

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 2 & 3 \\ 2 & 4 \\ 3 & 4 \end{bmatrix}$$

"n_star(m,ell)=index ell of neighboring triangle of edge m"

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 3 \\ 1 & 4 \\ 2 & 4 \\ 3 & 4 \end{bmatrix}$$

"k_star(m,1), k_star(m,2) indices of opposite vertices to edge m in neighboring triangles"

$$\begin{bmatrix} 3 & 4 \\ 2 & 4 \\ 2 & 3 \\ 1 & 4 \\ 1 & 3 \\ 1 & 2 \end{bmatrix}$$

"m_star(n,ell) = index of opposite edge of vertex ell of triangle n"

$$\begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 1 \\ 6 & 3 & 2 \\ 6 & 5 & 4 \end{bmatrix}$$

"Degrees of vertices, cycles of neighbors"

"Degrees of vertices", $\begin{bmatrix} 3 & 3 & 3 & 3 \end{bmatrix}$

"Inner vertices", $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$

"Starting edges in cycles", $\begin{bmatrix} 1 & 1 & 2 & 3 \end{bmatrix}$

"Cycle of edges from point", 1, " ", $\begin{bmatrix} 1 & 3 & 2 & 1 \end{bmatrix}$, " Cycle of points", $\begin{bmatrix} 2 & 4 & 3 & 2 \end{bmatrix}$

"Cycle of edges from point", 2, " ", $\begin{bmatrix} 1 & 5 & 4 & 1 \end{bmatrix}$, " Cycle of points", $\begin{bmatrix} 1 & 4 & 3 & 1 \end{bmatrix}$

"Cycle of edges from point", 3, " ", $\begin{bmatrix} 2 & 6 & 4 & 2 \end{bmatrix}$, " Cycle of points", $\begin{bmatrix} 1 & 4 & 2 & 1 \end{bmatrix}$

"Cycle of edges from point", 4, " ", $\begin{bmatrix} 3 & 6 & 5 & 3 \end{bmatrix}$, " Cycle of points", $\begin{bmatrix} 1 & 3 & 2 & 1 \end{bmatrix}$

"Guessing normal vectors"

"Normal vectors", $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

"Norm squares", $\begin{bmatrix} 3 & 1 & 1 & 1 \end{bmatrix}$

"Guessed Surface derivatives"

"g_ij from point i toward j"

"g", 1, 2, " ", $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \end{bmatrix}$, " normalv", [1, 1, 1]

"g", 1, 4, " ", $\begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$, " normalv", [1, 1, 1]

"g", 1, 3, " ", $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$, " normalv", [1, 1, 1]

"g", 2, 1, " ", $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, " normalv", [1, 0, 0]

"g", 2, 4, " ", $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$, " normalv", [1, 0, 0]

"g", 2, 3, " ", $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$, " normalv", [1, 0, 0]

"g", 3, 1, " ", $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, " normalv", [0, −1, 0]

"g", 3, 4, " ", $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$, " normalv", [0, −1, 0]

"g", 3, 2, " ", $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, " normalv", [0, −1, 0]

"g", 4, 1, " ", $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, " normalv", [0, 0, 1]

"g", 4, 3, " ", $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$, " normalv", [0, 0, 1]

"g", 4, 2, " ", $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, " normalv", [0, 0, 1]

"____\
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"____\
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"Checking if the g vectors are suitable in a polynomial construction"
"g_,ell-g_ik) x g_ij not= 0 when [p_i,p_j] is a double edge between the mesh triangles [p_i,p_j, p_k],[pi,p_j,p_ell]"
"g[", 2, 1, "] , g[", 2, 3, "] , g[", 2, 4, "] FALSE"
"g[", 3, 1, "] , g[", 3, 2, "] , g[", 3, 4, "] FALSE"
"g[", 4, 1, "] , g[", 4, 2, "] , g[", 4, 3, "] FALSE"
"NOT SUITABLE - g data should be modified"
"____\
________"
"____\
________"
"Basic RSD interpolation"

$$(\ )^2$$

"Shape functions", "Pi_0=(", Φ, Θ, *chi_0*, *chi_1*, ")"

$$\Phi := t0^3\left(6\, t0^2 - 15\, t0 + 10\right)$$

$$\Theta := t0^3\,(4 - 3\, t0)$$

$$chi\_0 := 30\, t1^2\, t2^2\, t3$$

$$chi\_1 := 12\, t1^2\, t2^2\, t3$$

"f^T,P,G_Pi0 over triangle ", 1

$$42\, t_2^2 t_3^2 t_1 + 38\, t_1^2 t_2^2 t_3 - 4\, t_1^2 t_3^2 t_2 + \frac{2}{3}\, t_1^3\,(4 - 3\, t_1)\, t_2 - \frac{1}{3}\, t_1^3\,(4 - 3\, t_1)\, t_3 + t_2^3\left(6\, t_2^2 - 15\, t_2 + 10\right) + t_3^3\,(4 - 3\, t_3)\, t_2$$

$$42\, t_2^2 t_3^2 t_1 + 38\, t_1^2 t_3^2 t_2 - 4\, t_1^2 t_2^2 t_3 - \frac{1}{3}\, t_1^3\,(4 - 3\, t_1)\, t_2 + \frac{2}{3}\, t_1^3\,(4 - 3\, t_1)\, t_3 + t_2^3\,(4 - 3\, t_2)\, t_3 + t_3^3\left(6\, t_3^2 - 15\, t_3 + 10\right)$$

$$-4\, t_1^2 t_3^2 t_2 - 4\, t_1^2 t_2^2 t_3 - \frac{1}{3}\, t_1^3\,(4 - 3\, t_1)\, t_2 - \frac{1}{3}\, t_1^3\,(4 - 3\, t_1)\, t_3$$

"f^T,P,G_Pi0 over triangle ", 2

$$42\, t_2^2 t_3^2 t_1 + 38\, t_1^2 t_2^2 t_3 - 4\, t_1^2 t_3^2 t_2 + \frac{2}{3}\, t_1^3\,(4 - 3\, t_1)\, t_2 - \frac{1}{3}\, t_1^3\,(4 - 3\, t_1)\, t_3 + t_2^3\left(6\, t_2^2 - 15\, t_2 + 10\right) + t_3^3\,(4 - 3\, t_3)\, t_2$$

$$-4\, t_1^2 t_3^2 t_2 - 4\, t_1^2 t_2^2 t_3 - \frac{1}{3}\, t_1^3\,(4 - 3\, t_1)\, t_2 - \frac{1}{3}\, t_1^3\,(4 - 3\, t_1)\, t_3$$

$$42\, t_2^2 t_3^2 t_1 + 38\, t_1^2 t_3^2 t_2 - 4\, t_1^2 t_2^2 t_3 - \frac{1}{3}\, t_1^3\,(4 - 3\, t_1)\, t_2 + \frac{2}{3}\, t_1^3\,(4 - 3\, t_1)\, t_3 + t_2^3\,(4 - 3\, t_2)\, t_3 + t_3^3\left(6\, t_3^2 - 15\, t_3 + 10\right)$$

"f^T,P,G_Pi0 over triangle ", 3

$$-4\, t_1^2 t_3^2 t_2 - 4\, t_1^2 t_2^2 t_3 - \frac{1}{3}\, t_1^3 \left(4 - 3\, t_1\right) t_2 - \frac{1}{3}\, t_1^3 \left(4 - 3\, t_1\right) t_3$$

$$42\, t_2^2 t_3^2 t_1 + 38\, t_1^2 t_2^2 t_3 - 4\, t_1^2 t_3^2 t_2 + \frac{2}{3}\, t_1^3 \left(4 - 3\, t_1\right) t_2 - \frac{1}{3}\, t_1^3 \left(4 - 3\, t_1\right) t_3 + t_2^3 \left(6\, t_2^2 - 15\, t_2 + 10\right) + t_3^3 \left(4 - 3\, t_3\right) t_2$$

$$42\, t_2^2 t_3^2 t_1 + 38\, t_1^2 t_3^2 t_2 - 4\, t_1^2 t_2^2 t_3 - \frac{1}{3}\, t_1^3 \left(4 - 3\, t_1\right) t_2 + \frac{2}{3}\, t_1^3 \left(4 - 3\, t_1\right) t_3 + t_2^3 \left(4 - 3\, t_2\right) t_3 + t_3^3 \left(6\, t_3^2 - 15\, t_3 + 10\right)$$

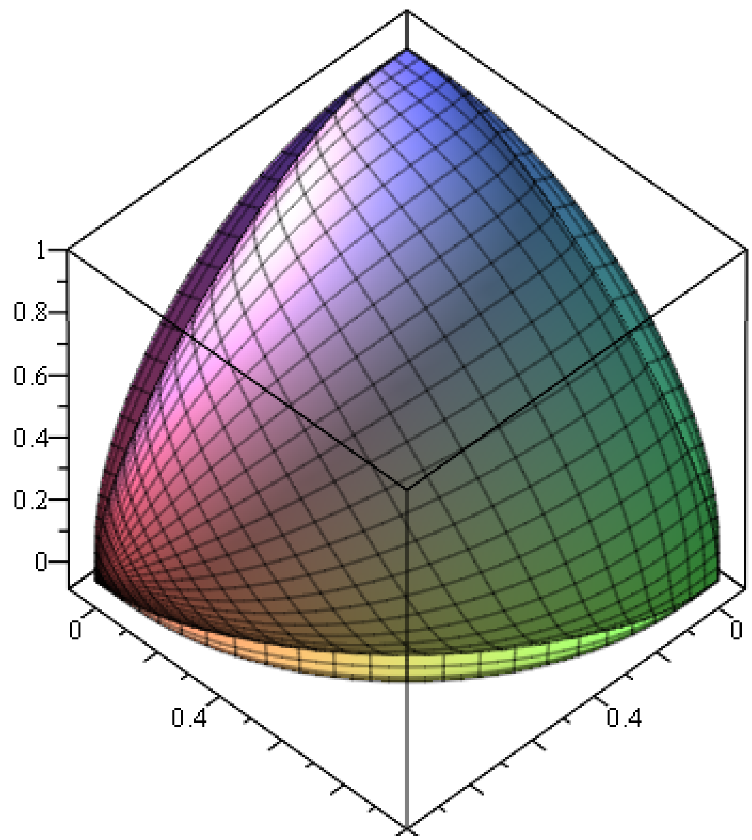
"f^T,P,G_Pi0 over triangle ", 4

$$42\, t_1^2 t_3^2 t_2 + 42\, t_1^2 t_2^2 t_3 + t_1^3 \left(6\, t_1^2 - 15\, t_1 + 10\right) + t_2^3 \left(4 - 3\, t_2\right) t_1 + t_3^3 \left(4 - 3\, t_3\right) t_1$$

$$42\, t_2^2 t_3^2 t_1 + 42\, t_1^2 t_2^2 t_3 + t_1^3 \left(4 - 3\, t_1\right) t_2 + t_2^3 \left(6\, t_2^2 - 15\, t_2 + 10\right) + t_3^3 \left(4 - 3\, t_3\right) t_2$$

$$42\, t_2^2 t_3^2 t_1 + 42\, t_1^2 t_3^2 t_2 + t_1^3 \left(4 - 3\, t_1\right) t_3 + t_2^3 \left(4 - 3\, t_2\right) t_3 + t_3^3 \left(6\, t_3^2 - 15\, t_3 + 10\right)$$

"f^T,P,G_Pi0   displayed"

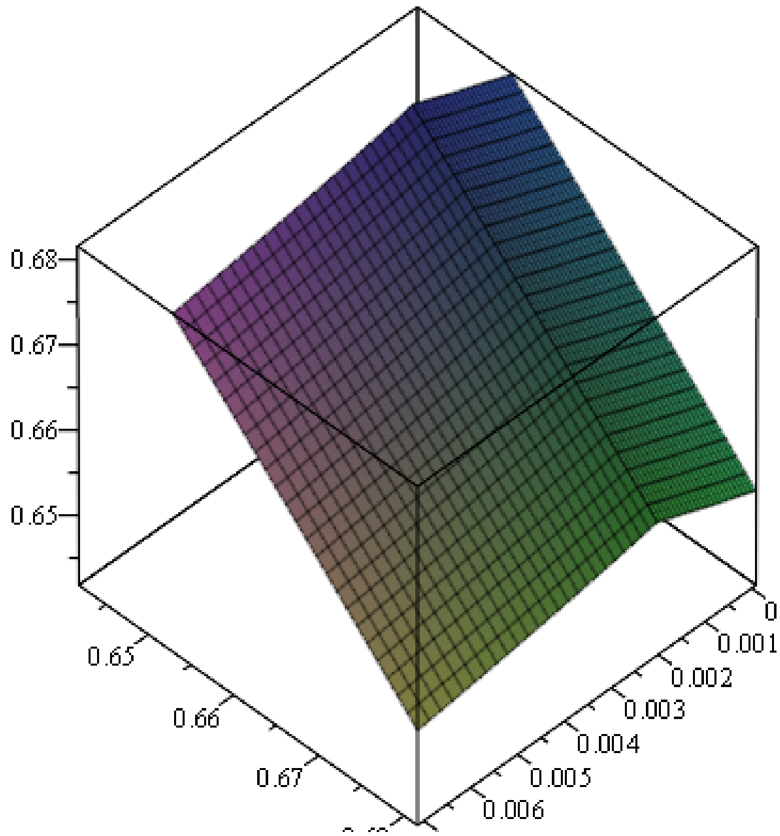


$a := 0$

$$b := \frac{1}{100}$$

$$c := \frac{49}{100}$$

$$d := \frac{51}{100}$$



"____\
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"____\
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"Edge corrections"

"Row m: double edge m between points i1,12 and opposite vertices j1,j2 in triangles n1,n2"

"ordering: i1<i2 , j1<j2 and n1<n2"

"Number of double edges ", 6

$$"IJM ", \left[\begin{array}{cccccc} 1 & 2 & 3 & 4 & 1 & 2 \\ 1 & 3 & 2 & 4 & 1 & 3 \\ 1 & 4 & 2 & 3 & 2 & 3 \\ 2 & 3 & 1 & 4 & 1 & 4 \\ 2 & 4 & 1 & 3 & 2 & 4 \\ 3 & 4 & 1 & 2 & 3 & 4 \end{array}\right]$$

"____"

"On (directed Edge ", 1, "   points ", 1, " --> ", 2

$$1, 42\,\lambda_2^2\lambda_3^2\lambda_1 + 38\,\lambda_1^2\lambda_2^2\lambda_3 - 4\,\lambda_1^2\lambda_3^2\lambda_2 + \frac{2}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_2 - \frac{1}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_3 + \lambda_2^3\left(6\,\lambda_2^2 - 15\,\lambda_2 + 10\right) + \lambda_3^3\left(4 - 3\,\lambda_3\right)\lambda_2$$

$$2, 42\,\lambda_2^2\lambda_3^2\lambda_1 + 38\,\lambda_1^2\lambda_3^2\lambda_2 - 4\,\lambda_1^2\lambda_2^2\lambda_3 - \frac{1}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_2 + \frac{2}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_3 + \lambda_2^3\left(4 - 3\,\lambda_2\right)\lambda_3 + \lambda_3^3\left(6\,\lambda_3^2 - 15\,\lambda_3 + 10\right)$$

$$3, -4\,\lambda_1^2\lambda_3^2\lambda_2 - 4\,\lambda_1^2\lambda_2^2\lambda_3 - \frac{1}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_2 - \frac{1}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_3$$

$$"v", \left[\frac{1}{3}\,\tau^2\left(63\,\tau^2 - 122\,\tau + 57\right), -\frac{15}{2}\,\tau^4 + \frac{46}{3}\,\tau^3 - 8\,\tau^2 + 1, -\frac{1}{6}\,\tau^2\left(9\,\tau^2 - 20\,\tau + 12\right)\right]$$

$$"vv", \left[\frac{1}{3}\,\tau^2\left(63\,\tau^2 - 122\,\tau + 57\right), -\frac{1}{6}\,\tau^2\left(9\,\tau^2 - 20\,\tau + 12\right), -\frac{15}{2}\,\tau^4 + \frac{46}{3}\,\tau^3 - 8\,\tau^2 + 1\right]$$

$$"dv", \left[0, 6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1, -6\,\tau^4 + 12\,\tau^3 - 6\,\tau^2 + 1\right]$$

$$"y", \left[-\frac{2}{3}\,\tau^2\left(30\,\tau^2 - 62\,\tau + 33\right), -\frac{1}{3}\,\tau^2\left(3\,\tau - 2\right)\left(5\,\tau - 6\right), -\frac{1}{3}\,\tau^2\left(3\,\tau - 2\right)\left(5\,\tau - 6\right)\right]$$

$$"w", \left[-\frac{2}{3}\left(6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1\right)\tau^2\left(3\,\tau - 2\right)\left(5\,\tau - 6\right), \frac{2}{3}\left(6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1\right)\tau^2\left(30\,\tau^2 - 62\,\tau + 33\right), \frac{2}{3}\left(6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1\right)\tau^2\left(30\,\tau^2 - 62\,\tau + 33\right)\right]$$

$$"b=det[v,dv,y]", -\frac{2}{3}\,\tau^2\left(6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1\right)\left(585\,\tau^4 - 1146\,\tau^3 + 553\,\tau^2 - 4\,\tau - 33\right)\left(-1 + \tau\right)^2$$

"____"

"On (directed Edge ", 2, "   points ", 1, " --> ", 3

$$1, 42\,\lambda_2^2\lambda_3^2\lambda_1 + 38\,\lambda_1^2\lambda_2^2\lambda_3 - 4\,\lambda_1^2\lambda_3^2\lambda_2 + \frac{2}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_2 - \frac{1}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_3 + \lambda_2^3\left(6\,\lambda_2^2 - 15\,\lambda_2 + 10\right) + \lambda_3^3\left(4 - 3\,\lambda_3\right)\lambda_2$$

$$2, 42\,\lambda_2^2\lambda_3^2\lambda_1 + 38\,\lambda_1^2\lambda_3^2\lambda_2 - 4\,\lambda_1^2\lambda_2^2\lambda_3 - \frac{1}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_2 + \frac{2}{3}\,\lambda_1^3\left(4 - 3\,\lambda_1\right)\lambda_3 + \lambda_2^3\left(4 - 3\,\lambda_2\right)\lambda_3 + \lambda_3^3\left(6\,\lambda_3^2 - 15\,\lambda_3 + 10\right)$$

$$3, -4\lambda_1^2\lambda_3^2\lambda_2 - 4\lambda_1^2\lambda_2^2\lambda_3 - \frac{1}{3}\lambda_1^3(4-3\lambda_1)\lambda_2 - \frac{1}{3}\lambda_1^3(4-3\lambda_1)\lambda_3$$

"v", $\left[-\frac{15}{2}\tau^4 + \frac{46}{3}\tau^3 - 8\tau^2 + 1, \frac{1}{3}\tau^2(63\tau^2 - 122\tau + 57), -\frac{1}{6}\tau^2(9\tau^2 - 20\tau + 12)\right]$

"vv", $\left[-\frac{1}{6}\tau^2(9\tau^2 - 20\tau + 12), \frac{1}{3}\tau^2(63\tau^2 - 122\tau + 57), -\frac{15}{2}\tau^4 + \frac{46}{3}\tau^3 - 8\tau^2 + 1\right]$

"dv", $[6\tau^4 - 12\tau^3 + 6\tau^2 - 1, 0, -6\tau^4 + 12\tau^3 - 6\tau^2 + 1]$

"y", $\left[-\frac{1}{3}\tau^2(3\tau-2)(5\tau-6), -\frac{2}{3}\tau^2(30\tau^2 - 62\tau + 33), -\frac{1}{3}\tau^2(3\tau-2)(5\tau-6)\right]$

"w", $\left[-\frac{2}{3}(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)\tau^2(30\tau^2 - 62\tau + 33), \frac{2}{3}(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)\tau^2(3\tau-2)(5\tau-6), -\frac{2}{3}(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)\tau^2(30\tau^2 - 62\tau + 33)\right]$

"b=det[v,dv,y]", $\frac{2}{3}\tau^2(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)(585\tau^4 - 1146\tau^3 + 553\tau^2 - 4\tau - 33)(-1 + \tau)^2$

"____"

"On (directed Edge ", 3, "   points ", 1, " --> ", 4

$$1, 42\lambda_2^2\lambda_4^2\lambda_1 + 38\lambda_1^2\lambda_2^2\lambda_4 - 4\lambda_1^2\lambda_4^2\lambda_2 + \frac{2}{3}\lambda_1^3(4-3\lambda_1)\lambda_2 - \frac{1}{3}\lambda_1^3(4-3\lambda_1)\lambda_4 + \lambda_2^3(6\lambda_2^2 - 15\lambda_2 + 10) + \lambda_4^3(4-3\lambda_4)\lambda_2$$

$$2, -4\lambda_1^2\lambda_4^2\lambda_2 - 4\lambda_1^2\lambda_2^2\lambda_4 - \frac{1}{3}\lambda_1^3(4-3\lambda_1)\lambda_2 - \frac{1}{3}\lambda_1^3(4-3\lambda_1)\lambda_4$$

$$3, 42\lambda_2^2\lambda_4^2\lambda_1 + 38\lambda_1^2\lambda_4^2\lambda_2 - 4\lambda_1^2\lambda_2^2\lambda_4 - \frac{1}{3}\lambda_1^3(4-3\lambda_1)\lambda_2 + \frac{2}{3}\lambda_1^3(4-3\lambda_1)\lambda_4 + \lambda_2^3(4-3\lambda_2)\lambda_4 + \lambda_4^3(6\lambda_4^2 - 15\lambda_4 + 10)$$

"v", $\left[-\frac{15}{2}\tau^4 + \frac{46}{3}\tau^3 - 8\tau^2 + 1, -\frac{1}{6}\tau^2(9\tau^2 - 20\tau + 12), \frac{1}{3}\tau^2(63\tau^2 - 122\tau + 57)\right]$

"vv", $\left[-\frac{1}{6}\tau^2(9\tau^2 - 20\tau + 12), -\frac{15}{2}\tau^4 + \frac{46}{3}\tau^3 - 8\tau^2 + 1, \frac{1}{3}\tau^2(63\tau^2 - 122\tau + 57)\right]$

"dv", $[6\tau^4 - 12\tau^3 + 6\tau^2 - 1, -6\tau^4 + 12\tau^3 - 6\tau^2 + 1, 0]$

"y", $\left[-\frac{1}{3}\tau^2(3\tau-2)(5\tau-6), -\frac{1}{3}\tau^2(3\tau-2)(5\tau-6), -\frac{2}{3}\tau^2(30\tau^2 - 62\tau + 33)\right]$

"w", $\left[\frac{2}{3}(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)\tau^2(30\tau^2 - 62\tau + 33), \frac{2}{3}(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)\tau^2(30\tau^2 - 62\tau + 33), -\frac{2}{3}(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)\tau^2(3\tau-2)(5\tau-6)\right]$

"b=det[v,dv,y]", $-\frac{2}{3}\tau^2(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)(585\tau^4 - 1146\tau^3 + 553\tau^2 - 4\tau - 33)(-1 + \tau)^2$

"____"

"On (directed Edge ", 4, "   points ", 2, " --> ", 3

$1, 42\,\lambda_2^2\,\lambda_3^2\,\lambda_1 + 38\,\lambda_1^2\,\lambda_2^2\,\lambda_3 - 4\,\lambda_1^2\,\lambda_3^2\,\lambda_2 + \frac{2}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_2 - \frac{1}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_3 + \lambda_2^3\,(6\,\lambda_2^2 - 15\,\lambda_2 + 10) + \lambda_3^3\,(4-3\,\lambda_3)\,\lambda_2$

$2, 42\,\lambda_2^2\,\lambda_3^2\,\lambda_1 + 38\,\lambda_1^2\,\lambda_3^2\,\lambda_2 - 4\,\lambda_1^2\,\lambda_2^2\,\lambda_3 - \frac{1}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_2 + \frac{2}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_3 + \lambda_2^3\,(4-3\,\lambda_2)\,\lambda_3 + \lambda_3^3\,(6\,\lambda_3^2 - 15\,\lambda_3 + 10)$

$3, -4\,\lambda_1^2\,\lambda_3^2\,\lambda_2 - 4\,\lambda_1^2\,\lambda_2^2\,\lambda_3 - \frac{1}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_2 - \frac{1}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_3$

"v", $\left[\frac{1}{2}\,(45\,\tau^2 - 2\,\tau - 1)\,(-1+\tau)^2, \frac{1}{2}\,\tau^2\,(45\,\tau^2 - 88\,\tau + 42), 0\right]$

"vv", $\left[\frac{1}{2}\,(45\,\tau^2 - 2\,\tau - 1)\,(-1+\tau)^2, \frac{1}{2}\,\tau^2\,(45\,\tau^2 - 88\,\tau + 42), -6\,\tau^4 + 12\,\tau^3 - 6\,\tau^2 + 1\right]$

"dv", $[0, 0, -6\,\tau^4 + 12\,\tau^3 - 6\,\tau^2 + 1]$

"y", $[(15\,\tau^2 + 2\,\tau + 1)\,(-1+\tau)^2, -\tau^2\,(15\,\tau^2 - 32\,\tau + 18), 0]$

"w", $[-(6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1)\,\tau^2\,(15\,\tau^2 - 32\,\tau + 18), -(6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1)\,(15\,\tau^2 + 2\,\tau + 1)\,(-1+\tau)^2, 0]$

"b=det[v,dv,y]", $-\tau^2\,(6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1)\,(675\,\tau^4 - 1350\,\tau^3 + 679\,\tau^2 - 4\,\tau + 12)\,(-1+\tau)^2$

"____"

"On (directed Edge ", 5, "   points ", 2, " --> ", 4

$1, 42\,\lambda_2^2\,\lambda_4^2\,\lambda_1 + 38\,\lambda_1^2\,\lambda_2^2\,\lambda_4 - 4\,\lambda_1^2\,\lambda_4^2\,\lambda_2 + \frac{2}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_2 - \frac{1}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_4 + \lambda_2^3\,(6\,\lambda_2^2 - 15\,\lambda_2 + 10) + \lambda_4^3\,(4-3\,\lambda_4)\,\lambda_2$

$2, -4\,\lambda_1^2\,\lambda_4^2\,\lambda_2 - 4\,\lambda_1^2\,\lambda_2^2\,\lambda_4 - \frac{1}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_2 - \frac{1}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_4$

$3, 42\,\lambda_2^2\,\lambda_4^2\,\lambda_1 + 38\,\lambda_1^2\,\lambda_4^2\,\lambda_2 - 4\,\lambda_1^2\,\lambda_2^2\,\lambda_4 - \frac{1}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_2 + \frac{2}{3}\,\lambda_1^3\,(4-3\,\lambda_1)\,\lambda_4 + \lambda_2^3\,(4-3\,\lambda_2)\,\lambda_4 + \lambda_4^3\,(6\,\lambda_4^2 - 15\,\lambda_4 + 10)$

"v", $\left[\frac{1}{2}\,(45\,\tau^2 - 2\,\tau - 1)\,(-1+\tau)^2, 0, \frac{1}{2}\,\tau^2\,(45\,\tau^2 - 88\,\tau + 42)\right]$

"vv", $\left[\frac{1}{2}\,(45\,\tau^2 - 2\,\tau - 1)\,(-1+\tau)^2, -6\,\tau^4 + 12\,\tau^3 - 6\,\tau^2 + 1, \frac{1}{2}\,\tau^2\,(45\,\tau^2 - 88\,\tau + 42)\right]$

"dv", $[0, -6\,\tau^4 + 12\,\tau^3 - 6\,\tau^2 + 1, 0]$

"y", $[(15\,\tau^2 + 2\,\tau + 1)\,(-1+\tau)^2, 0, -\tau^2\,(15\,\tau^2 - 32\,\tau + 18)]$

"w", $[(6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1)\,\tau^2\,(15\,\tau^2 - 32\,\tau + 18), 0, (6\,\tau^4 - 12\,\tau^3 + 6\,\tau^2 - 1)\,(15\,\tau^2 + 2\,\tau + 1)\,(-1+\tau)^2]$

$$\text{"b=det[v,dv,y]"},\ \tau^2\left(6\tau^4-12\tau^3+6\tau^2-1\right)\left(675\tau^4-1350\tau^3+679\tau^2-4\tau+12\right)(-1+\tau)^2$$

"____"

"On (directed Edge ", 6, "   points ", 3, " --> ", 4

$$1,\ -4\lambda_1^2\lambda_4^2\lambda_3-4\lambda_1^2\lambda_3^2\lambda_4-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_3-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_4$$

$$2,\ 42\lambda_3^2\lambda_4^2\lambda_1+38\lambda_1^2\lambda_3^2\lambda_4-4\lambda_1^2\lambda_4^2\lambda_3+\frac{2}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_3-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_4+\lambda_3^3\left(6\lambda_3^2-15\lambda_3+10\right)+\lambda_4^3\left(4-3\lambda_4\right)\lambda_3$$

$$3,\ 42\lambda_3^2\lambda_4^2\lambda_1+38\lambda_1^2\lambda_4^2\lambda_3-4\lambda_1^2\lambda_3^2\lambda_4-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_3+\frac{2}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_4+\lambda_3^3\left(4-3\lambda_3\right)\lambda_4+\lambda_4^3\left(6\lambda_4^2-15\lambda_4+10\right)$$

$$\text{"v"},\ \left[0,\ \frac{1}{2}\left(45\tau^2-2\tau-1\right)(-1+\tau)^2,\ \frac{1}{2}\tau^2\left(45\tau^2-88\tau+42\right)\right]$$

$$\text{"vv"},\ \left[-6\tau^4+12\tau^3-6\tau^2+1,\ \frac{1}{2}\left(45\tau^2-2\tau-1\right)(-1+\tau)^2,\ \frac{1}{2}\tau^2\left(45\tau^2-88\tau+42\right)\right]$$

$$\text{"dv"},\ \left[-6\tau^4+12\tau^3-6\tau^2+1,\ 0,\ 0\right]$$

$$\text{"y"},\ \left[0,\ \left(15\tau^2+2\tau+1\right)(-1+\tau)^2,\ -\tau^2\left(15\tau^2-32\tau+18\right)\right]$$

$$\text{"w"},\ \left[0,\ -\left(6\tau^4-12\tau^3+6\tau^2-1\right)\tau^2\left(15\tau^2-32\tau+18\right),\ -\left(6\tau^4-12\tau^3+6\tau^2-1\right)\left(15\tau^2+2\tau+1\right)(-1+\tau)^2\right]$$

$$\text{"b=det[v,dv,y]"},\ -\tau^2\left(6\tau^4-12\tau^3+6\tau^2-1\right)\left(675\tau^4-1350\tau^3+679\tau^2-4\tau+12\right)(-1+\tau)^2$$

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"Cofactors of GCD(w_1,w_2,w_3)"

"W[", 1, ",on points ", 1, "-->", 2, "]"

"Polynomials"

$$-\frac{2}{3}\left(6\tau^4-12\tau^3+6\tau^2-1\right)\tau^2\left(3\tau-2\right)\left(5\tau-6\right)$$

$$\frac{2}{3}\left(6\tau^4-12\tau^3+6\tau^2-1\right)\tau^2\left(30\tau^2-62\tau+33\right)$$

$$\frac{2}{3}\left(6\tau^4-12\tau^3+6\tau^2-1\right)\tau^2\left(30\tau^2-62\tau+33\right)$$

$$\text{" GCD over double edge "},\ 1,\ \text{" = "},\ \tau^6-2\tau^5+\tau^4-\frac{1}{6}\tau^2$$

$$\text{"qw"},\ 1,\ 1,\ \text{"= "},\ -\frac{17}{90}+\frac{1}{3}\tau$$

$$"qw", 1, 2, "= ", -\frac{17}{180} + \frac{1}{6}\tau$$

$$"qw", 1, 3, "= ", \frac{1}{30}$$

"W[", 2, ",on points ", 1, "-->", 3, "]"

"Polynomials"

$$-\frac{2}{3}\left(6\tau^4 - 12\tau^3 + 6\tau^2 - 1\right)\tau^2\left(30\tau^2 - 62\tau + 33\right)$$

$$\frac{2}{3}\left(6\tau^4 - 12\tau^3 + 6\tau^2 - 1\right)\tau^2\left(3\tau - 2\right)\left(5\tau - 6\right)$$

$$-\frac{2}{3}\left(6\tau^4 - 12\tau^3 + 6\tau^2 - 1\right)\tau^2\left(30\tau^2 - 62\tau + 33\right)$$

$$" \text{ GCD over double edge } ", 2, " = ", \tau^6 - 2\tau^5 + \tau^4 - \frac{1}{6}\tau^2$$

$$"qw", 2, 1, "= ", 0$$

$$"qw", 2, 2, "= ", \frac{17}{90} - \frac{1}{3}\tau$$

$$"qw", 2, 3, "= ", \frac{11}{180} - \frac{1}{6}\tau$$

"W[", 3, ",on points ", 1, "-->", 4, "]"

"Polynomials"

$$\frac{2}{3}\left(6\tau^4 - 12\tau^3 + 6\tau^2 - 1\right)\tau^2\left(30\tau^2 - 62\tau + 33\right)$$

$$\frac{2}{3}\left(6\tau^4 - 12\tau^3 + 6\tau^2 - 1\right)\tau^2\left(30\tau^2 - 62\tau + 33\right)$$

$$-\frac{2}{3}\left(6\tau^4 - 12\tau^3 + 6\tau^2 - 1\right)\tau^2\left(3\tau - 2\right)\left(5\tau - 6\right)$$

$$" \text{ GCD over double edge } ", 3, " = ", \tau^6 - 2\tau^5 + \tau^4 - \frac{1}{6}\tau^2$$

$$"qw", 3, 1, "= ", 0$$

$$"qw", 3, 2, "= ", -\frac{11}{180} + \frac{1}{6}\tau$$

$$"qw", 3, 3, "= ", -\frac{17}{90} + \frac{1}{3}\tau$$

"W[", 4, ",on points ", 2, "-->", 3, "]"

"Polynomials"

$$-\left(6\tau^4 - 12\tau^3 + 6\tau^2 - 1\right)\tau^2\left(15\tau^2 - 32\tau + 18\right)$$

$$-\left(6\tau^4 - 12\tau^3 + 6\tau^2 - 1\right)\left(15\tau^2 + 2\tau + 1\right)\left(-1 + \tau\right)^2$$

$$0$$

$$" \text{ GCD over double edge } ", 4, " = ", \tau^4 - 2\tau^3 + \tau^2 - \frac{1}{6}$$

$$"qw", 4, 1, "= ", \frac{179}{2346} - \frac{40}{391}\tau - \frac{205}{391}\tau^2 + \frac{150}{391}\tau^3$$

$$"qw", 4, 2, "= ", -\frac{1}{6} + \frac{245}{391}\tau^2 - \frac{150}{391}\tau^3$$

$$"qw", 4, 3, "= ", 0$$

"W[", 5, ",on points ", 2, "-->", 4, "]"

"Polynomials"

$$(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)\tau^2(15\tau^2 - 32\tau + 18)$$

$$0$$

$$(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)(15\tau^2 + 2\tau + 1)(-1 + \tau)^2$$

$$" GCD over double edge ", 5, " = ", \tau^4 - 2\tau^3 + \tau^2 - \frac{1}{6}$$

$$"qw", 5, 1, "= ", -\frac{179}{2346} + \frac{40}{391}\tau + \frac{205}{391}\tau^2 - \frac{150}{391}\tau^3$$

$$"qw", 5, 2, "= ", 0$$

$$"qw", 5, 3, "= ", \frac{1}{6} - \frac{245}{391}\tau^2 + \frac{150}{391}\tau^3$$

"W[", 6, ",on points ", 3, "-->", 4, "]"

"Polynomials"

$$0$$

$$-(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)\tau^2(15\tau^2 - 32\tau + 18)$$

$$-(6\tau^4 - 12\tau^3 + 6\tau^2 - 1)(15\tau^2 + 2\tau + 1)(-1 + \tau)^2$$

$$" GCD over double edge ", 6, " = ", \tau^4 - 2\tau^3 + \tau^2 - \frac{1}{6}$$

$$"qw", 6, 1, "= ", 0$$

$$"qw", 6, 2, "= ", \frac{179}{2346} - \frac{40}{391}\tau - \frac{205}{391}\tau^2 + \frac{150}{391}\tau^3$$

$$"qw", 6, 3, "= ", -\frac{1}{6} + \frac{245}{391}\tau^2 - \frac{150}{391}\tau^3$$

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"Z polynomials along double edges"

"Double edge ", 1, "   ", 1, "-->", 2

"Zq[", 1, 1, " resp. ZqL[", 1, 1, "]"

$$-\frac{4(585\tau^4 - 1146\tau^3 + 553\tau^2 - 4\tau - 33)\left(-\frac{17}{90} + \frac{1}{3}\tau\right)}{\tau^2}$$

$$-\frac{2(585\lambda_1^4 - 1146\lambda_1^3 + 553\lambda_1^2 - 4\lambda_1 - 33)\left(\frac{13}{90} - \frac{1}{3}\lambda_2\right)}{\lambda_1^2}$$

$$-\frac{2\left(585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45\right)\left(-\frac{17}{90}+\frac{1}{3}\lambda_1\right)}{\left(-1+\lambda_2\right)^2}$$

"Zq[", 1, 2, " resp. ZqL[", 1, 2, "]"

$$-\frac{4\left(585\tau^4-1146\tau^3+553\tau^2-4\tau-33\right)\left(-\frac{17}{180}+\frac{1}{6}\tau\right)}{\tau^2}$$

$$-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{13}{180}-\frac{1}{6}\lambda_2\right)}{\lambda_1^2}$$

$$-\frac{2\left(585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45\right)\left(-\frac{17}{180}+\frac{1}{6}\lambda_1\right)}{\left(-1+\lambda_2\right)^2}$$

"Zq[", 1, 3, " resp. ZqL[", 1, 3, "]"

$$-\frac{2}{15}\,\frac{585\tau^4-1146\tau^3+553\tau^2-4\tau-33}{\tau^2}$$

$$-\frac{1}{15}\,\frac{585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33}{\lambda_1^2}-\frac{1}{15}\,\frac{585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45}{\left(-1+\lambda_2\right)^2}$$

"Double edge ", 2, "   ", 1, "-->", 3

"Zq[", 2, 1, " resp. ZqL[", 2, 1, "]"

0

0

"Zq[", 2, 2, " resp. ZqL[", 2, 2, "]"

$$\frac{4\left(585\tau^4-1146\tau^3+553\tau^2-4\tau-33\right)\left(\frac{17}{90}-\frac{1}{3}\tau\right)}{\tau^2}$$

$$\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(-\frac{13}{90}+\frac{1}{3}\lambda_3\right)}{\lambda_1^2}$$

$$+\frac{2\left(585\lambda_3^4-1194\lambda_3^3+625\lambda_3^2-4\lambda_3-45\right)\left(\frac{17}{90}-\frac{1}{3}\lambda_1\right)}{\left(-1+\lambda_3\right)^2}$$

"Zq[", 2, 3, " resp. ZqL[", 2, 3, "]"

$$\frac{4\left(585\tau^4-1146\tau^3+553\tau^2-4\tau-33\right)\left(\frac{11}{180}-\frac{1}{6}\tau\right)}{\tau^2}$$

$$\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(-\frac{19}{180}+\frac{1}{6}\lambda_3\right)}{\lambda_1^2}$$

$$+\frac{2\left(585\lambda_3^4-1194\lambda_3^3+625\lambda_3^2-4\lambda_3-45\right)\left(\frac{11}{180}-\frac{1}{6}\lambda_1\right)}{\left(-1+\lambda_3\right)^2}$$

"Double edge ", 3, "   ", 1, "-->", 4

"Zq[", 3, 1, " resp. ZqL[", 3, 1, "]"

0

0

"Zq[", 3, 2, " resp. ZqL[", 3, 2, "]"

$$-\frac{4\left(585\tau^4-1146\tau^3+553\tau^2-4\tau-33\right)\left(-\frac{11}{180}+\frac{1}{6}\tau\right)}{\tau^2}$$

$$-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{19}{180}-\frac{1}{6}\lambda_4\right)}{\lambda_1^2}$$

$$-\frac{2\left(585\lambda_4^4-1194\lambda_4^3+625\lambda_4^2-4\lambda_4-45\right)\left(-\frac{11}{180}+\frac{1}{6}\lambda_1\right)}{\left(-1+\lambda_4\right)^2}$$

"Zq[", 3, 3, " resp. ZqL[", 3, 3, "]"

$$-\frac{4\left(585\tau^4-1146\tau^3+553\tau^2-4\tau-33\right)\left(-\frac{17}{90}+\frac{1}{3}\tau\right)}{\tau^2}$$

$$-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{13}{90}-\frac{1}{3}\lambda_4\right)}{\lambda_1^2}$$

$$-\frac{2\left(585\lambda_4^4-1194\lambda_4^3+625\lambda_4^2-4\lambda_4-45\right)\left(-\frac{17}{90}+\frac{1}{3}\lambda_1\right)}{\left(-1+\lambda_4\right)^2}$$

"Double edge ", 4, "   ", 2, "-->", 3

"Zq[", 4, 1, " resp. ZqL[", 4, 1, "]"

$$\left(-4050\tau^4+8100\tau^3-4074\tau^2+24\tau-72\right)\left(\frac{179}{2346}-\frac{40}{391}\tau-\frac{205}{391}\tau^2+\frac{150}{391}\tau^3\right)$$

$$\frac{1}{2}\left(-4050\lambda_2^4+8100\lambda_2^3-4074\lambda_2^2+24\lambda_2-72\right)\left(-\frac{1}{6}+\frac{245}{391}\lambda_3^2-\frac{150}{391}\lambda_3^3\right)+\frac{1}{2}\Big($$
$$-4050\lambda_3^4+8100\lambda_3^3-4074\lambda_3^2+24\lambda_3-72\Big)\left(\frac{179}{2346}-\frac{40}{391}\lambda_2-\frac{205}{391}\lambda_2^2+\frac{150}{391}\lambda_2^3\right)$$

"Zq[", 4, 2, " resp. ZqL[", 4, 2, "]"

$$\left(-4050\,\tau^4+8100\,\tau^3-4074\,\tau^2+24\,\tau-72\right)\left(-\frac{1}{6}+\frac{245}{391}\,\tau^2-\frac{150}{391}\,\tau^3\right)$$

$$\frac{1}{2}\left(-4050\,\lambda_2^4+8100\,\lambda_2^3-4074\,\lambda_2^2+24\,\lambda_2-72\right)\left(\frac{179}{2346}-\frac{205}{391}\,\lambda_3^2-\frac{40}{391}\,\lambda_3+\frac{150}{391}\,\lambda_3^3\right)$$
$$+\frac{1}{2}\left(-4050\,\lambda_3^4+8100\,\lambda_3^3-4074\,\lambda_3^2+24\,\lambda_3-72\right)\left(-\frac{1}{6}+\frac{245}{391}\,\lambda_2^2-\frac{150}{391}\,\lambda_2^3\right)$$

"Zq[", 4, 3, " resp. ZqL[", 4, 3, "]"

0

0

"Double edge ", 5, "   ", 2, "-->", 4

"Zq[", 5, 1, " resp. ZqL[", 5, 1, "]"

$$\left(4050\,\tau^4-8100\,\tau^3+4074\,\tau^2-24\,\tau+72\right)\left(-\frac{179}{2346}+\frac{40}{391}\,\tau+\frac{205}{391}\,\tau^2-\frac{150}{391}\,\tau^3\right)$$

$$\frac{1}{2}\left(4050\,\lambda_2^4-8100\,\lambda_2^3+4074\,\lambda_2^2-24\,\lambda_2+72\right)\left(\frac{1}{6}-\frac{245}{391}\,\lambda_4^2+\frac{150}{391}\,\lambda_4^3\right)+\frac{1}{2}\left(4050\,\lambda_4^4\right.$$
$$\left.-8100\,\lambda_4^3+4074\,\lambda_4^2-24\,\lambda_4+72\right)\left(-\frac{179}{2346}+\frac{40}{391}\,\lambda_2+\frac{205}{391}\,\lambda_2^2-\frac{150}{391}\,\lambda_2^3\right)$$

"Zq[", 5, 2, " resp. ZqL[", 5, 2, "]"

0

0

"Zq[", 5, 3, " resp. ZqL[", 5, 3, "]"

$$\left(4050\,\tau^4-8100\,\tau^3+4074\,\tau^2-24\,\tau+72\right)\left(\frac{1}{6}-\frac{245}{391}\,\tau^2+\frac{150}{391}\,\tau^3\right)$$

$$\frac{1}{2}\left(4050\,\lambda_2^4-8100\,\lambda_2^3+4074\,\lambda_2^2-24\,\lambda_2+72\right)\left(-\frac{179}{2346}+\frac{205}{391}\,\lambda_4^2+\frac{40}{391}\,\lambda_4-\frac{150}{391}\,\lambda_4^3\right)$$
$$+\frac{1}{2}\left(4050\,\lambda_4^4-8100\,\lambda_4^3+4074\,\lambda_4^2-24\,\lambda_4+72\right)\left(\frac{1}{6}-\frac{245}{391}\,\lambda_2^2+\frac{150}{391}\,\lambda_2^3\right)$$

"Double edge ", 6, "   ", 3, "-->", 4

"Zq[", 6, 1, " resp. ZqL[", 6, 1, "]"

0

0

"Zq[", 6, 2, " resp. ZqL[", 6, 2, "]"

$$\left(-4050\,\tau^4+8100\,\tau^3-4074\,\tau^2+24\,\tau-72\right)\left(\frac{179}{2346}-\frac{40}{391}\,\tau-\frac{205}{391}\,\tau^2+\frac{150}{391}\,\tau^3\right)$$

$$\frac{1}{2}\left(-4050\,\lambda_3^4+8100\,\lambda_3^3-4074\,\lambda_3^2+24\,\lambda_3-72\right)\left(-\frac{1}{6}+\frac{245}{391}\,\lambda_4^2-\frac{150}{391}\,\lambda_4^3\right)+\frac{1}{2}\left(\right.$$
$$\left.-4050\,\lambda_4^4+8100\,\lambda_4^3-4074\,\lambda_4^2+24\,\lambda_4-72\right)\left(\frac{179}{2346}-\frac{205}{391}\,\lambda_3^2-\frac{40}{391}\,\lambda_3+\frac{150}{391}\,\lambda_3^3\right)$$

"Zq[", 6, 3, " resp. ZqL[", 6, 3, "]"

$$\left(-4050\,\tau^4+8100\,\tau^3-4074\,\tau^2+24\,\tau-72\right)\left(-\frac{1}{6}+\frac{245}{391}\,\tau^2-\frac{150}{391}\,\tau^3\right)$$

$$\frac{1}{2}\left(-4050\,\lambda_3^4+8100\,\lambda_3^3-4074\,\lambda_3^2+24\,\lambda_3-72\right)\left(\frac{179}{2346}-\frac{205}{391}\,\lambda_4^2-\frac{40}{391}\,\lambda_4+\frac{150}{391}\,\lambda_4^3\right)$$

$$+\frac{1}{2}\left(-4050\lambda_4^4+8100\lambda_4^3-4074\lambda_4^2+24\lambda_4-72\right)\left(-\frac{1}{6}+\frac{245}{391}\lambda_3^2-\frac{150}{391}\lambda_3^3\right)$$

"Triangle ", 1, [1, 2, 3]

"Spline over triangle ", 1

$$-\left(\frac{1}{2}\left(-4050\lambda_2^4+8100\lambda_2^3-4074\lambda_2^2+24\lambda_2-72\right)\left(-\frac{1}{6}+\frac{245}{391}\lambda_3^2-\frac{150}{391}\lambda_3^3\right)+\frac{1}{2}\left(-4050\lambda_3^4+8100\lambda_3^3-4074\lambda_3^2+24\lambda_3-72\right)\left(\frac{179}{2346}-\frac{40}{391}\lambda_2-\frac{205}{391}\lambda_2^2+\frac{150}{391}\lambda_2^3\right)\right)$$
$$\lambda_2^2\lambda_3^2\lambda_1-\left(-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{13}{90}-\frac{1}{3}\lambda_2\right)}{\lambda_1^2}-\frac{2\left(585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45\right)\left(-\frac{17}{90}+\frac{1}{3}\lambda_1\right)}{\left(-1+\lambda_2\right)^2}\right)\lambda_1^2\lambda_2^2\lambda_3+42\lambda_2^2\lambda_3^2\lambda_1$$
$$+38\lambda_1^2\lambda_2^2\lambda_3-4\lambda_1^2\lambda_3^2\lambda_2+\frac{2}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_2-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_3+\lambda_2^3\left(6\lambda_2^2-15\lambda_2+10\right)+\lambda_3^3\left(4-3\lambda_3\right)\lambda_2$$

$$-\left(\frac{1}{2}\left(-4050\lambda_2^4+8100\lambda_2^3-4074\lambda_2^2+24\lambda_2-72\right)\left(\frac{179}{2346}-\frac{205}{391}\lambda_3^2-\frac{40}{391}\lambda_3+\frac{150}{391}\lambda_3^3\right)+\frac{1}{2}\left(-4050\lambda_3^4+8100\lambda_3^3-4074\lambda_3^2+24\lambda_3-72\right)\left(-\frac{1}{6}+\frac{245}{391}\lambda_2^2-\frac{150}{391}\lambda_2^3\right)\right)$$
$$\lambda_2^2\lambda_3^2\lambda_1-\left(\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(-\frac{13}{90}+\frac{1}{3}\lambda_3\right)}{\lambda_1^2}+\frac{2\left(585\lambda_3^4-1194\lambda_3^3+625\lambda_3^2-4\lambda_3-45\right)\left(\frac{17}{90}-\frac{1}{3}\lambda_1\right)}{\left(-1+\lambda_3\right)^2}\right)\lambda_1^2\lambda_3^2\lambda_2-\Bigg($$
$$-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{13}{180}-\frac{1}{6}\lambda_2\right)}{\lambda_1^2}-\frac{2\left(585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45\right)\left(-\frac{17}{180}+\frac{1}{6}\lambda_1\right)}{\left(-1+\lambda_2\right)^2}\Bigg)\lambda_1^2\lambda_2^2\lambda_3+42\lambda_2^2\lambda_3^2\lambda_1$$
$$+38\lambda_1^2\lambda_3^2\lambda_2-4\lambda_1^2\lambda_2^2\lambda_3-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_2+\frac{2}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_3+\lambda_2^3\left(4-3\lambda_2\right)\lambda_3+\lambda_3^3\left(6\lambda_3^2-15\lambda_3+10\right)$$

$$-\left(\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(-\frac{19}{180}+\frac{1}{6}\lambda_3\right)}{\lambda_1^2}\right.$$

$$\left.+\frac{2\left(585\lambda_3^4-1194\lambda_3^3+625\lambda_3^2-4\lambda_3-45\right)\left(\frac{11}{180}-\frac{1}{6}\lambda_1\right)}{\left(-1+\lambda_3\right)^2}\right)\lambda_1^2\lambda_3^2\lambda_2-\left(\right.$$

$$-\frac{1}{15}\frac{585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33}{\lambda_1^2}$$

$$\left.-\frac{1}{15}\frac{585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45}{\left(-1+\lambda_2\right)^2}\right)\lambda_1^2\lambda_2^2\lambda_3-4\lambda_1^2\lambda_3^2\lambda_2-4\lambda_1^2\lambda_2^2\lambda_3-\frac{1}{3}$$

$$\lambda_1^3\left(4-3\lambda_1\right)\lambda_2-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_3$$

"Triangle ", 2, [1, 2, 4]

"Spline over triangle ", 2

$$-\left(\frac{1}{2}\left(4050\lambda_2^4-8100\lambda_2^3+4074\lambda_2^2-24\lambda_2+72\right)\left(\frac{1}{6}-\frac{245}{391}\lambda_4^2+\frac{150}{391}\lambda_4^3\right)+\frac{1}{2}\left(4050\right.\right.$$

$$\left.\left.\lambda_4^4-8100\lambda_4^3+4074\lambda_4^2-24\lambda_4+72\right)\left(-\frac{179}{2346}+\frac{40}{391}\lambda_2+\frac{205}{391}\lambda_2^2-\frac{150}{391}\lambda_2^3\right)\right)\lambda_2^2$$

$$\lambda_4^2\lambda_1-\left(-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{13}{90}-\frac{1}{3}\lambda_2\right)}{\lambda_1^2}\right.$$

$$\left.-\frac{2\left(585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45\right)\left(-\frac{17}{90}+\frac{1}{3}\lambda_1\right)}{\left(-1+\lambda_2\right)^2}\right)\lambda_1^2\lambda_2^2\lambda_4+42\lambda_2^2\lambda_4^2\lambda_1$$

$$+38\lambda_1^2\lambda_2^2\lambda_4-4\lambda_1^2\lambda_4^2\lambda_2+\frac{2}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_2-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_4+\lambda_2^3\left(6\lambda_2^2-15\lambda_2\right.$$

$$\left.+10\right)+\lambda_4^3\left(4-3\lambda_4\right)\lambda_2$$

$$-\left(-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{19}{180}-\frac{1}{6}\lambda_4\right)}{\lambda_1^2}\right.$$

$$\left.-\frac{2\left(585\lambda_4^4-1194\lambda_4^3+625\lambda_4^2-4\lambda_4-45\right)\left(-\frac{11}{180}+\frac{1}{6}\lambda_1\right)}{\left(-1+\lambda_4\right)^2}\right)\lambda_1^2\lambda_4^2\lambda_2-\left(\right.$$

$$-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{13}{180}-\frac{1}{6}\lambda_2\right)}{\lambda_1^2}$$

$$-\frac{2\left(585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45\right)\left(-\frac{17}{180}+\frac{1}{6}\lambda_1\right)}{\left(-1+\lambda_2\right)^2}\Bigg)\lambda_1^2\lambda_2^2\lambda_4-4\lambda_1^2\lambda_4^2\lambda_2$$

$$-4\lambda_1^2\lambda_2^2\lambda_4-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_2-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_4$$

$$-\Bigg(\frac{1}{2}\left(4050\lambda_2^4-8100\lambda_2^3+4074\lambda_2^2-24\lambda_2+72\right)\left(-\frac{179}{2346}+\frac{205}{391}\lambda_4^2+\frac{40}{391}\lambda_4-\frac{150}{391}\lambda_4^3\right)+\frac{1}{2}\left(4050\lambda_4^4-8100\lambda_4^3+4074\lambda_4^2-24\lambda_4+72\right)\left(\frac{1}{6}-\frac{245}{391}\lambda_2^2+\frac{150}{391}\lambda_2^3\right)\Bigg)\lambda_2^2$$

$$\lambda_4^2\lambda_1-\Bigg(-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{13}{90}-\frac{1}{3}\lambda_4\right)}{\lambda_1^2}$$

$$-\frac{2\left(585\lambda_4^4-1194\lambda_4^3+625\lambda_4^2-4\lambda_4-45\right)\left(-\frac{17}{90}+\frac{1}{3}\lambda_1\right)}{\left(-1+\lambda_4\right)^2}\Bigg)\lambda_1^2\lambda_4^2\lambda_2-\Bigg($$

$$-\frac{1}{15}\frac{585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33}{\lambda_1^2}$$

$$-\frac{1}{15}\frac{585\lambda_2^4-1194\lambda_2^3+625\lambda_2^2-4\lambda_2-45}{\left(-1+\lambda_2\right)^2}\Bigg)\lambda_1^2\lambda_2^2\lambda_4+42\lambda_2^2\lambda_4^2\lambda_1+38\lambda_1^2\lambda_4^2\lambda_2-4$$

$$\lambda_1^2\lambda_2^2\lambda_4-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_2+\frac{2}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_4+\lambda_2^3\left(4-3\lambda_2\right)\lambda_4+\lambda_4^3\left(6\lambda_4^2\right.$$

$$\left.-15\lambda_4+10\right)$$

"Triangle ", 3, [1, 3, 4]

"Spline over triangle ", 3

$$-4\lambda_1^2\lambda_4^2\lambda_3-4\lambda_1^2\lambda_3^2\lambda_4-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_3-\frac{1}{3}\lambda_1^3\left(4-3\lambda_1\right)\lambda_4$$

$$-\Bigg(\frac{1}{2}\left(-4050\lambda_3^4+8100\lambda_3^3-4074\lambda_3^2+24\lambda_3-72\right)\left(-\frac{1}{6}+\frac{245}{391}\lambda_4^2-\frac{150}{391}\lambda_4^3\right)+\frac{1}{2}\left(-4050\lambda_4^4+8100\lambda_4^3-4074\lambda_4^2+24\lambda_4-72\right)\left(\frac{179}{2346}-\frac{205}{391}\lambda_3^2-\frac{40}{391}\lambda_3+\frac{150}{391}\lambda_3^3\right)\Bigg)$$

$$\lambda_3^2\lambda_4^2\lambda_1-\Bigg(-\frac{2\left(585\lambda_1^4-1146\lambda_1^3+553\lambda_1^2-4\lambda_1-33\right)\left(\frac{19}{180}-\frac{1}{6}\lambda_4\right)}{\lambda_1^2}$$

$$-\frac{2\left(585\lambda_4^4-1194\lambda_4^3+625\lambda_4^2-4\lambda_4-45\right)\left(-\frac{11}{180}+\frac{1}{6}\lambda_1\right)}{\left(-1+\lambda_4\right)^2}\Bigg)\lambda_1^2\lambda_4^2\lambda_3$$

$$- \left(\frac{2 \left(585 \lambda_1^4 - 1146 \lambda_1^3 + 553 \lambda_1^2 - 4 \lambda_1 - 33\right) \left(-\frac{13}{90} + \frac{1}{3} \lambda_3\right)}{\lambda_1^2} + \frac{2 \left(585 \lambda_3^4 - 1194 \lambda_3^3 + 625 \lambda_3^2 - 4 \lambda_3 - 45\right) \left(\frac{17}{90} - \frac{1}{3} \lambda_1\right)}{\left(-1 + \lambda_3\right)^2} \right) \lambda_1^2 \lambda_3^2 \lambda_4 + 42 \lambda_3^2 \lambda_4^2 \lambda_1 + 38 \lambda_1^2 \lambda_3^2 \lambda_4 - 4 \lambda_1^2 \lambda_4^2 \lambda_3 + \frac{2}{3} \lambda_1^3 \left(4 - 3 \lambda_1\right) \lambda_3 - \frac{1}{3} \lambda_1^3 \left(4 - 3 \lambda_1\right) \lambda_4 + \lambda_3^3 \left(6 \lambda_3^2 - 15 \lambda_3 + 10\right) + \lambda_4^3 \left(4 - 3 \lambda_4\right) \lambda_3$$

$$- \left(\frac{1}{2} \left(-4050 \lambda_3^4 + 8100 \lambda_3^3 - 4074 \lambda_3^2 + 24 \lambda_3 - 72\right) \left(\frac{179}{2346} - \frac{205}{391} \lambda_4^2 - \frac{40}{391} \lambda_4 + \frac{150}{391} \lambda_4^3\right) + \frac{1}{2} \left(-4050 \lambda_4^4 + 8100 \lambda_4^3 - 4074 \lambda_4^2 + 24 \lambda_4 - 72\right) \left(-\frac{1}{6} + \frac{245}{391} \lambda_3^2 - \frac{150}{391} \lambda_3^3\right) \right) \lambda_3^2 \lambda_4^2 \lambda_1 - \left(-\frac{2 \left(585 \lambda_1^4 - 1146 \lambda_1^3 + 553 \lambda_1^2 - 4 \lambda_1 - 33\right) \left(\frac{13}{90} - \frac{1}{3} \lambda_4\right)}{\lambda_1^2} - \frac{2 \left(585 \lambda_4^4 - 1194 \lambda_4^3 + 625 \lambda_4^2 - 4 \lambda_4 - 45\right) \left(-\frac{17}{90} + \frac{1}{3} \lambda_1\right)}{\left(-1 + \lambda_4\right)^2} \right) \lambda_1^2 \lambda_4^2 \lambda_3$$

$$- \left(\frac{2 \left(585 \lambda_1^4 - 1146 \lambda_1^3 + 553 \lambda_1^2 - 4 \lambda_1 - 33\right) \left(-\frac{19}{180} + \frac{1}{6} \lambda_3\right)}{\lambda_1^2} + \frac{2 \left(585 \lambda_3^4 - 1194 \lambda_3^3 + 625 \lambda_3^2 - 4 \lambda_3 - 45\right) \left(\frac{11}{180} - \frac{1}{6} \lambda_1\right)}{\left(-1 + \lambda_3\right)^2} \right) \lambda_1^2 \lambda_3^2 \lambda_4 + 42 \lambda_3^2 \lambda_4^2 \lambda_1 + 38 \lambda_1^2 \lambda_4^2 \lambda_3 - 4 \lambda_1^2 \lambda_3^2 \lambda_4 - \frac{1}{3} \lambda_1^3 \left(4 - 3 \lambda_1\right) \lambda_3 + \frac{2}{3} \lambda_1^3 \left(4 - 3 \lambda_1\right) \lambda_4 + \lambda_3^3 \left(4 - 3 \lambda_3\right) \lambda_4 + \lambda_4^3 \left(6 \lambda_4^2 - 15 \lambda_4 + 10\right)$$

"Triangle ", 4, [2, 3, 4]

"Spline over triangle ", 4

$$- \left(\frac{1}{2} \left(4050 \lambda_2^4 - 8100 \lambda_2^3 + 4074 \lambda_2^2 - 24 \lambda_2 + 72\right) \left(\frac{1}{6} - \frac{245}{391} \lambda_4^2 + \frac{150}{391} \lambda_4^3\right) + \frac{1}{2} \left(4050 \lambda_4^4 - 8100 \lambda_4^3 + 4074 \lambda_4^2 - 24 \lambda_4 + 72\right) \left(-\frac{179}{2346} + \frac{40}{391} \lambda_2 + \frac{205}{391} \lambda_2^2 - \frac{150}{391} \lambda_2^3\right) \right) \lambda_2^2 \lambda_4^2 \lambda_3 - \left(\frac{1}{2} \left(-4050 \lambda_2^4 + 8100 \lambda_2^3 - 4074 \lambda_2^2 + 24 \lambda_2 - 72\right) \left(-\frac{1}{6} + \frac{245}{391} \lambda_3^2 - \frac{150}{391} \lambda_3^3\right) + \frac{1}{2} \left(-4050 \lambda_3^4 + 8100 \lambda_3^3 - 4074 \lambda_3^2 + 24 \lambda_3 - 72\right) \left(\frac{179}{2346} - \frac{40}{391} \lambda_2 - \frac{205}{391} \lambda_2^2\right.$$

$$+\frac{150}{391}\lambda_2^3\Big)\Big)\lambda_2^2\lambda_3^2\lambda_4+42\lambda_2^2\lambda_4^2\lambda_3+42\lambda_2^2\lambda_3^2\lambda_4+\lambda_2^3\left(6\lambda_2^2-15\lambda_2+10\right)+\lambda_3^3\left(4-3\lambda_3\right)\lambda_2+\lambda_4^3\left(4-3\lambda_4\right)\lambda_2$$

$$-\Big(\frac{1}{2}\left(-4050\lambda_3^4+8100\lambda_3^3-4074\lambda_3^2+24\lambda_3-72\right)\left(-\frac{1}{6}+\frac{245}{391}\lambda_4^2-\frac{150}{391}\lambda_4^3\right)+\frac{1}{2}\left(-4050\lambda_4^4+8100\lambda_4^3-4074\lambda_4^2+24\lambda_4-72\right)\left(\frac{179}{2346}-\frac{205}{391}\lambda_3^2-\frac{40}{391}\lambda_3+\frac{150}{391}\lambda_3^3\right)\Big)\lambda_3^2\lambda_4^2\lambda_2-\Big(\frac{1}{2}\left(-4050\lambda_2^4+8100\lambda_2^3-4074\lambda_2^2+24\lambda_2-72\right)\left(\frac{179}{2346}-\frac{205}{391}\lambda_3^2-\frac{40}{391}\lambda_3+\frac{150}{391}\lambda_3^3\right)+\frac{1}{2}\left(-4050\lambda_3^4+8100\lambda_3^3-4074\lambda_3^2+24\lambda_3-72\right)\left(-\frac{1}{6}+\frac{245}{391}\lambda_2^2-\frac{150}{391}\lambda_2^3\right)\Big)\lambda_2^2\lambda_3^2\lambda_4+42\lambda_3^2\lambda_4^2\lambda_2+42\lambda_2^2\lambda_3^2\lambda_4+\lambda_2^3\left(4-3\lambda_2\right)\lambda_3+\lambda_3^3\left(6\lambda_3^2-15\lambda_3+10\right)+\lambda_4^3\left(4-3\lambda_4\right)\lambda_3$$

$$-\Big(\frac{1}{2}\left(-4050\lambda_3^4+8100\lambda_3^3-4074\lambda_3^2+24\lambda_3-72\right)\left(\frac{179}{2346}-\frac{205}{391}\lambda_4^2-\frac{40}{391}\lambda_4+\frac{150}{391}\lambda_4^3\right)+\frac{1}{2}\left(-4050\lambda_4^4+8100\lambda_4^3-4074\lambda_4^2+24\lambda_4-72\right)\left(-\frac{1}{6}+\frac{245}{391}\lambda_3^2-\frac{150}{391}\lambda_3^3\right)\Big)\lambda_3^2\lambda_4^2\lambda_2-\Big(\frac{1}{2}\left(4050\lambda_2^4-8100\lambda_2^3+4074\lambda_2^2-24\lambda_2+72\right)\left(-\frac{179}{2346}+\frac{205}{391}\lambda_4^2+\frac{40}{391}\lambda_4-\frac{150}{391}\lambda_4^3\right)+\frac{1}{2}\left(4050\lambda_4^4-8100\lambda_4^3+4074\lambda_4^2-24\lambda_4+72\right)\left(\frac{1}{6}-\frac{245}{391}\lambda_2^2+\frac{150}{391}\lambda_2^3\right)\Big)\lambda_2^2\lambda_4^2\lambda_3+42\lambda_3^2\lambda_4^2\lambda_2+42\lambda_2^2\lambda_4^2\lambda_3+\lambda_2^3\left(4-3\lambda_2\right)\lambda_4+\lambda_3^3\left(4-3\lambda_3\right)\lambda_4+\lambda_4^3\left(6\lambda_4^2-15\lambda_4+10\right)$$

"____\
______"

"____\
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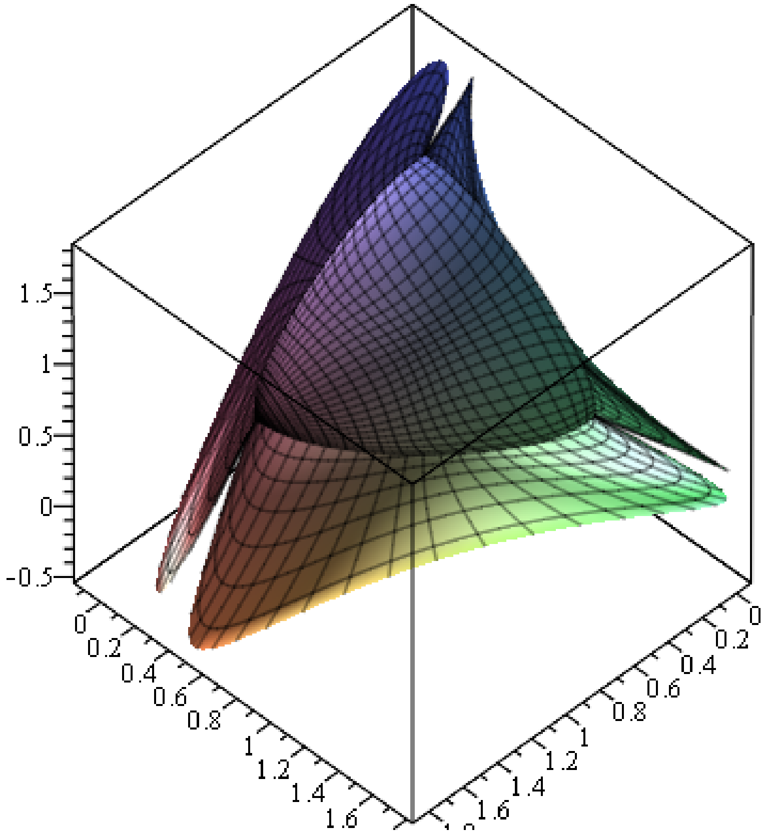
"Final result displayed"



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0
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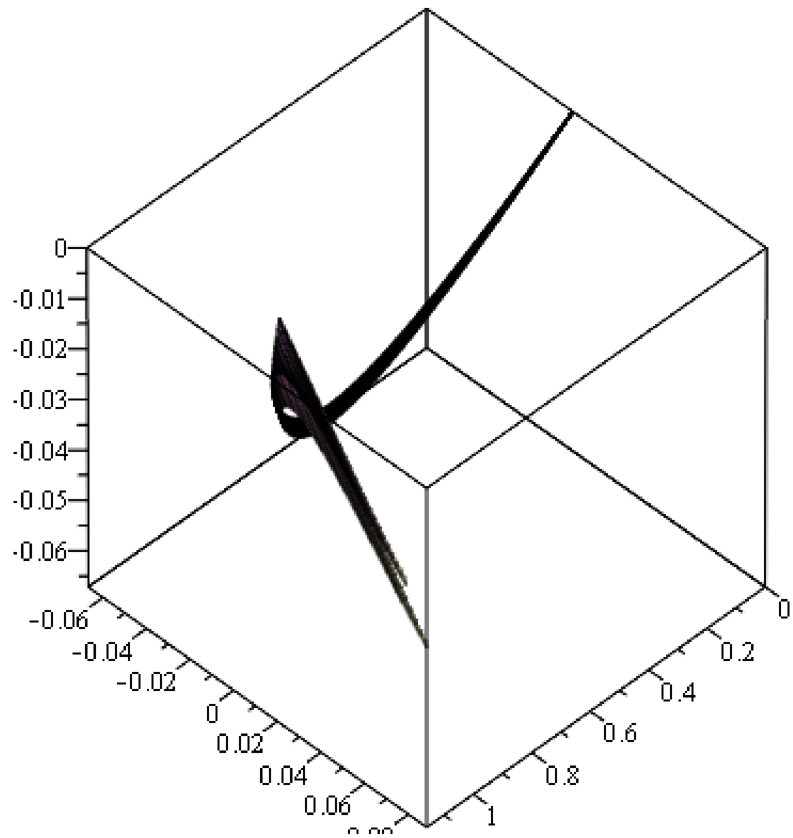
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6



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-0.04
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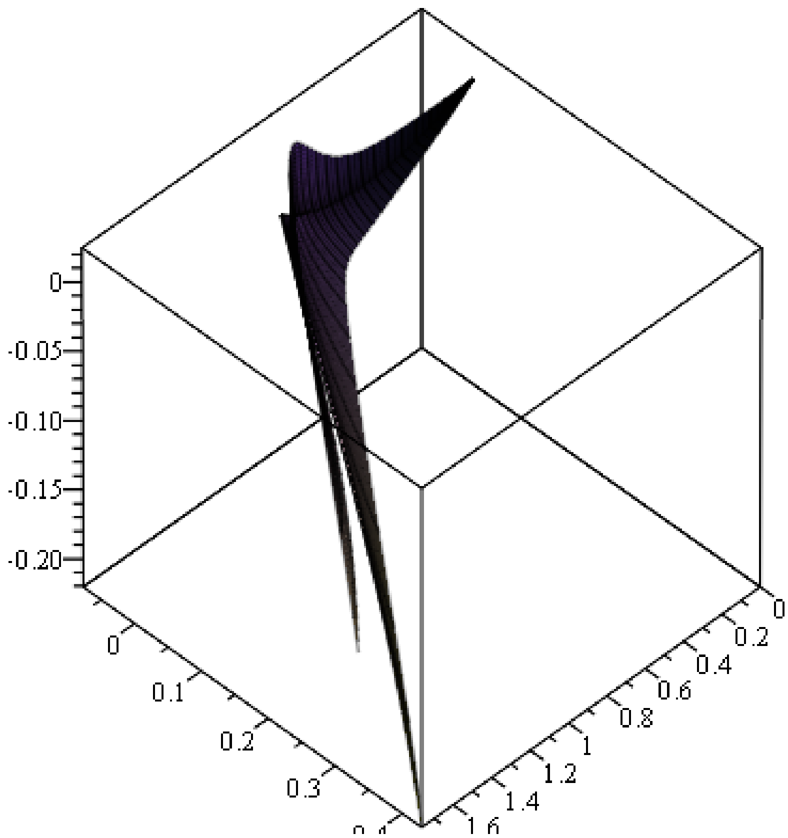
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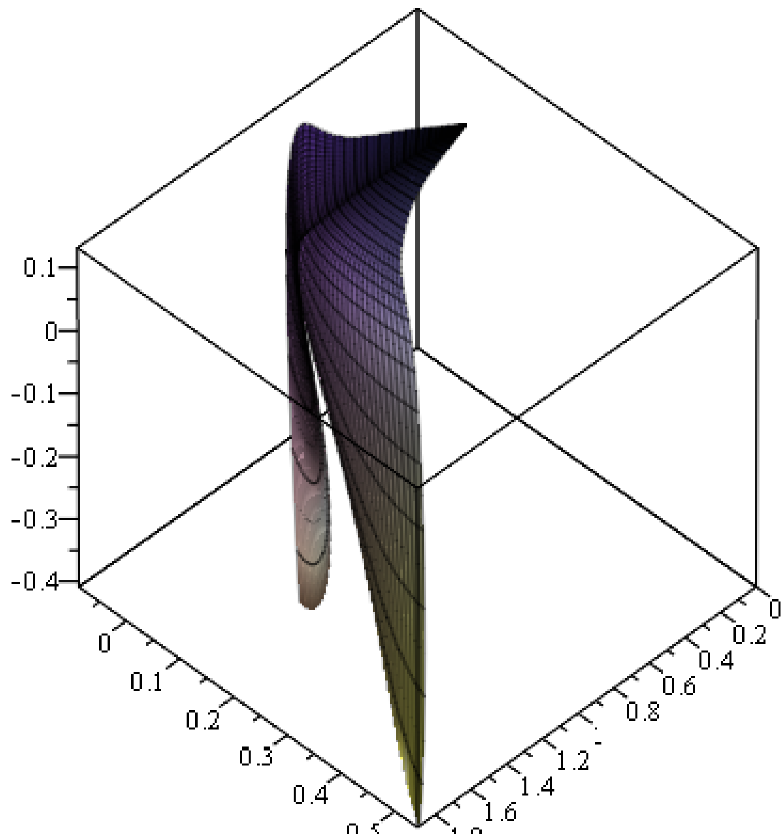
1.4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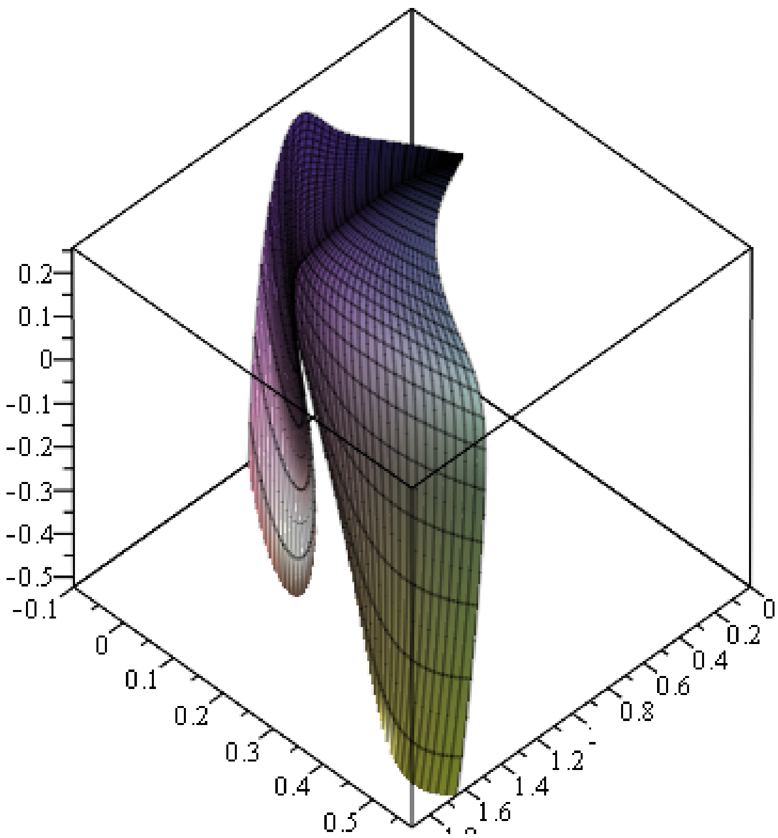
1.6



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0.5
0
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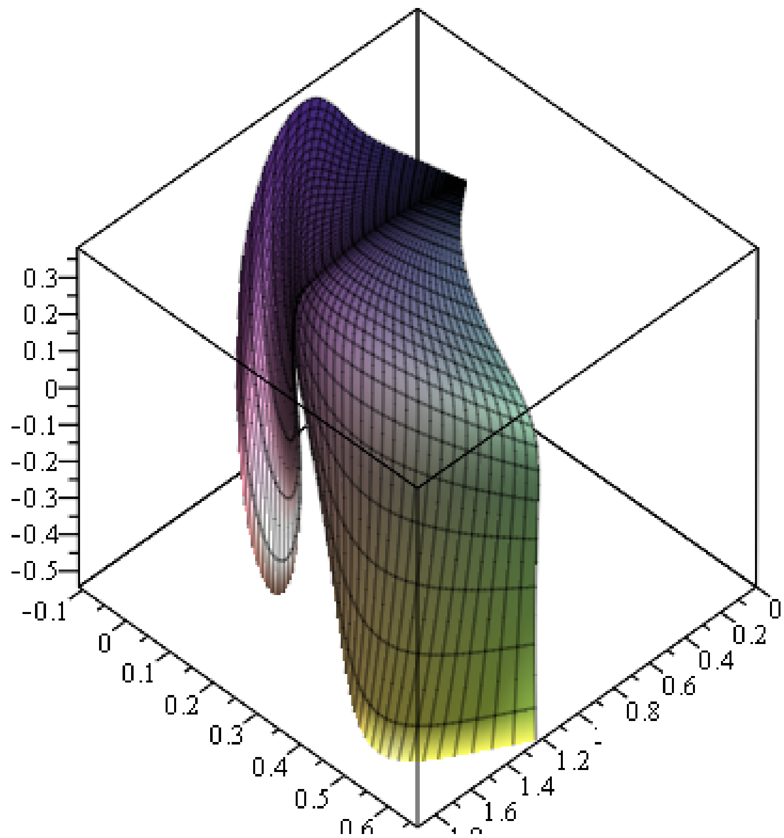
0.4
0.6
0.8
1.2
1.4
1.6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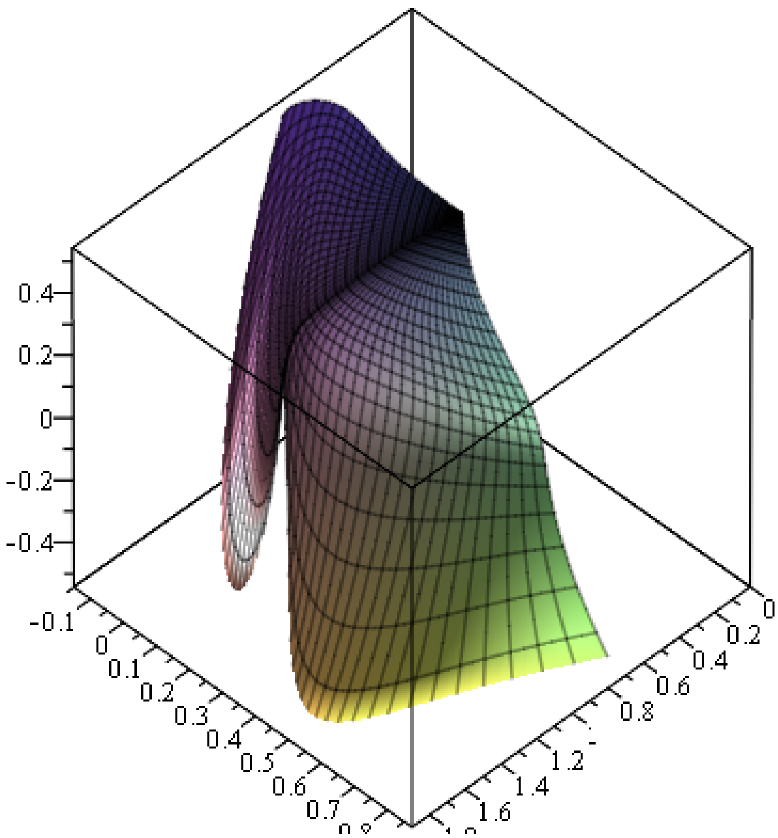
0.6
0
0.2
0.4
0.6
0.8
1.2
1.4
1.6



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0.1
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0.6
0.8
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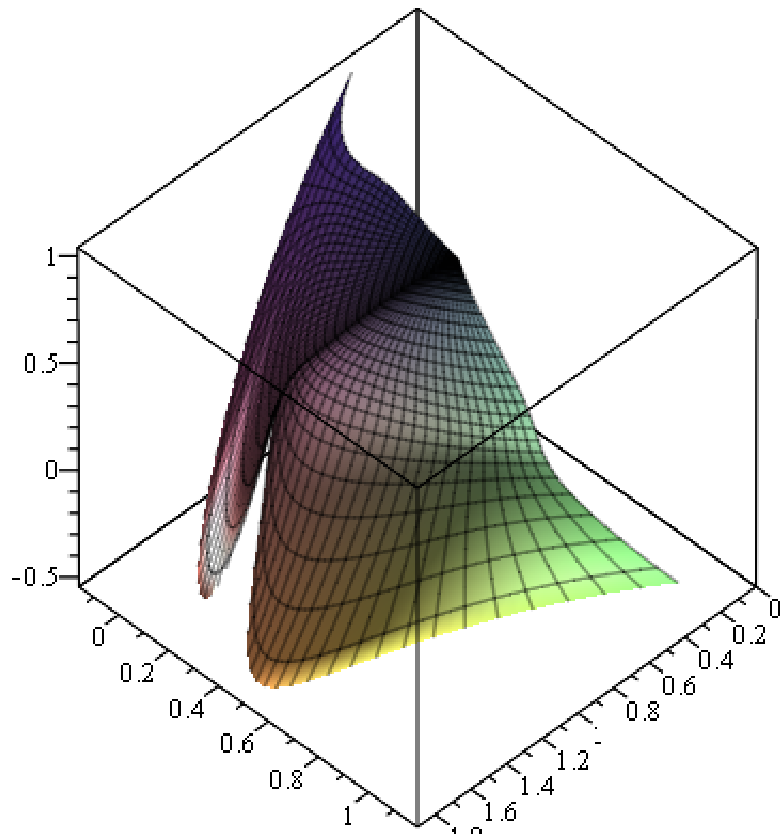
1.4
1.6



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0
0.2
0.4
0.6
0.8
1
0
0.2
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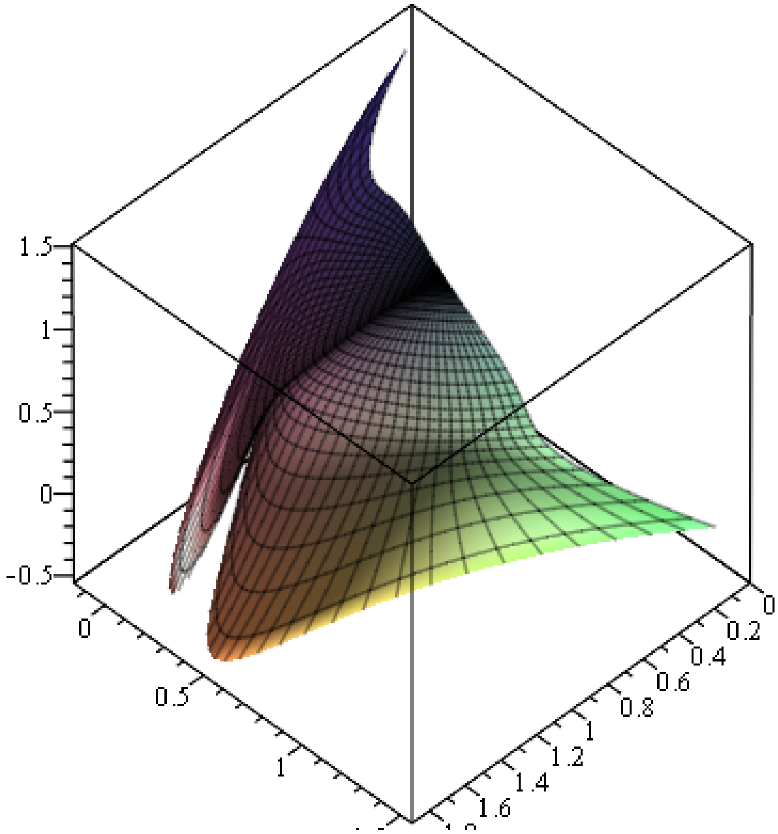
0.6
0.8
1.2
1.4
1.6



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0
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0
0.5
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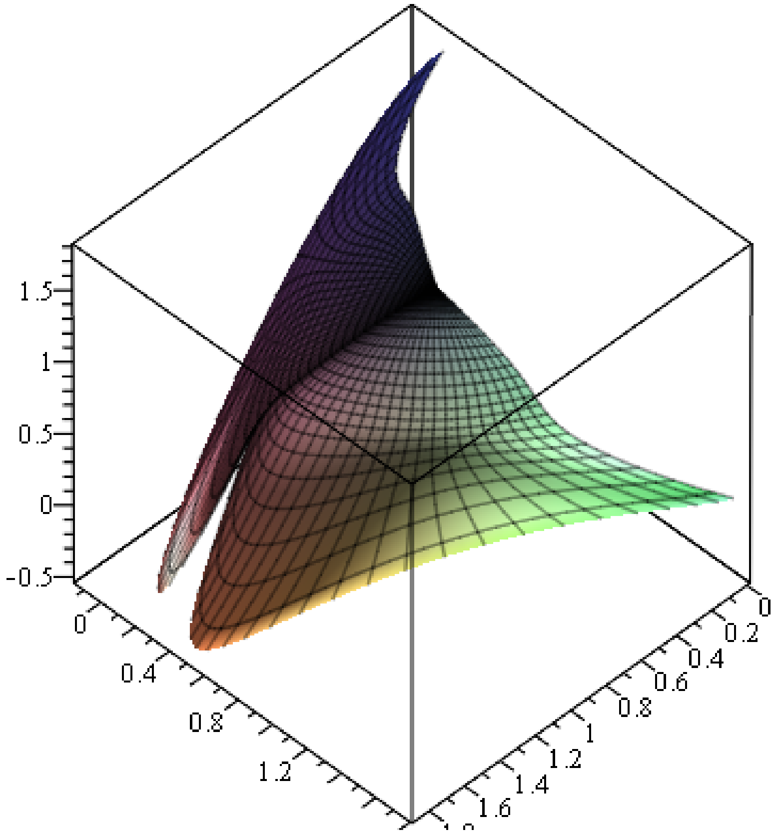
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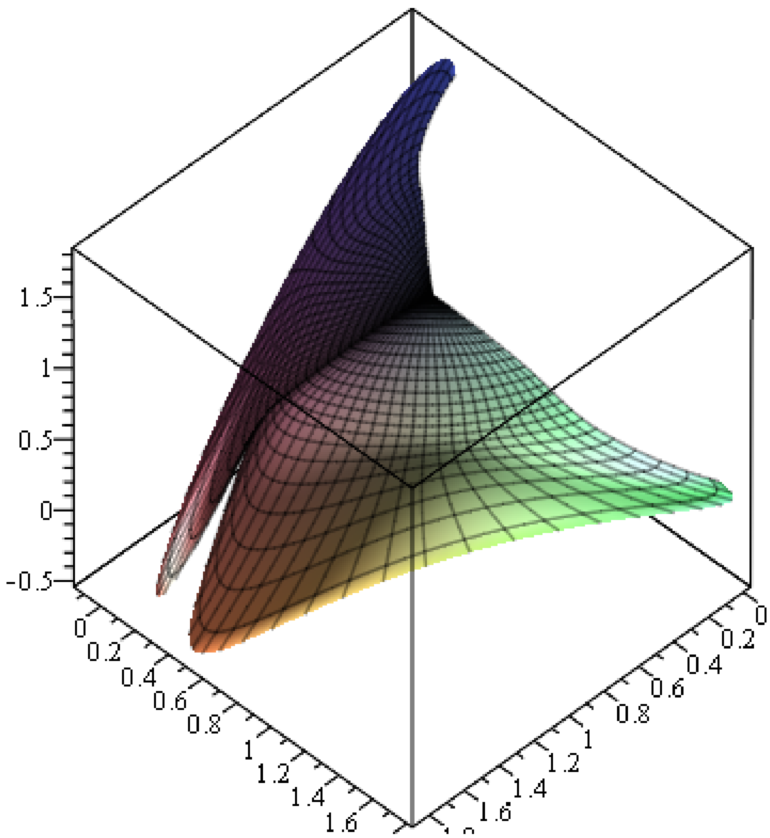
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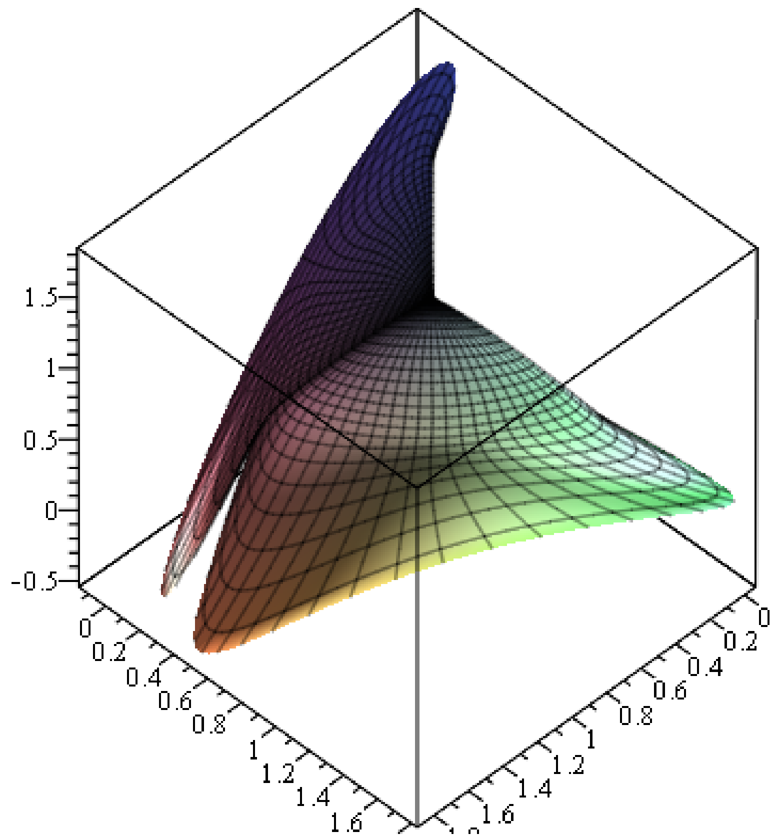
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