

```

> restart :
  with(linalg) :
  with(plots) :

  print("Mesh vertices");
  R := 4;
  print(R, " points");
  print("3D-Coordinates of the ", R, " points in the rows");
  P := matrix(4, 3, [-2, 0, sqrt(2), 2, 0, sqrt(2), 0, -2, -sqrt(2), 0, 2, -sqrt(2)]);

  print("Mesh triangles");
  N := 4;
  print("Point Indices of the ", N, " triangles,");
  print("i_star(n,i) = index of point i in triangle n");
  print("Lexicographic order");
  i_star := matrix(N, 3, [1, 2, 3, 1, 2, 4, 1, 3, 4, 2, 3, 4]);

  #Calculation for nuber of edges"
  M := 0 :
  j_star0 := matrix(R*(R-1)*2^-1, 2) :
  for i1 from 1 to R-1 do:
    for i2 from i1+1 to R do:
      s := 0 :
      for n from 1 to N do:
        if i1 = i_star[n, 1] and i2 = i_star[n, 2] then s := 1 fi:
        if i1 = i_star[n, 1] and i2 = i_star[n, 3] then s := 1 fi:
        if i1 = i_star[n, 2] and i2 = i_star[n, 3] then s := 1 fi:
      od:#n
      if s = 1 then M := M + 1 : j_star0[M, 1] := i1 : j_star0[M, 2] := i2 : fi:
    od:od:#i1,i2

  print("M= ", M, "mesh edges");
  print("j_star(m,ell)= index of point ell in edge m");
  print("Lexicographic order");
  j_star := matrix(M, 2) :
  for m from 1 to M do: for ell from 1 to 2 do: j_star[m, ell] := j_star0[m, ell] : od:od:
  print(evalm(j_star));

  n_star := matrix(M, 2) :
  for m from 1 to M do:
    i1 := j_star[m, 1] : i2 := j_star[m, 2] :
    s := 0 :
    for n from 1 to N do:
      if i1 = i_star[n, 1] and i2 = i_star[n, 2] then s := s + 1 : n_star[m, s] := n : fi:
      if i1 = i_star[n, 1] and i2 = i_star[n, 3] then s := s + 1 : n_star[m, s] := n : fi:
      if i1 = i_star[n, 2] and i2 = i_star[n, 3] then s := s + 1 : n_star[m, s] := n : fi:
    od:#n
    if s = 1 then n_star[m, 2] := n_star[m, 1] : fi:
  od:#m
  print("n_star(m,ell)=index ell of neighboring triangle of edge m");
  print(evalm(n_star));

```

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k_star := matrix(M, 2) :
for m from 1 to M do:
i1 := j_star[m, 1] : i2 := j_star[m, 2] : n1 := n_star[m, 1] : n2 := n_star[m, 2] :
k_star[m, 1] := i_star[n1, 1] + i_star[n1, 2] + i_star[n1, 3] - i1 - i2 :
k_star[m, 2] := i_star[n2, 1] + i_star[n2, 2] + i_star[n2, 3] - i1 - i2 :
od:#m
print("k_star(m,1), k_star(m,2) indices of opposite vertices to edge m in neighboring
triangles");
print(evalm(k_star));

m_star := matrix(N, 3) :
for n from 1 to N do:
i1 := i_star[n, 1] : i2 := i_star[n, 2] : i3 := i_star[n, 3] :
for m from 1 to M do:
j1 := j_star[m, 1] : j2 := j_star[m, 2] :
if j1 = i1 and j2 = i2 then m_star[n, 3] := m : fi:
if j1 = i1 and j2 = i3 then m_star[n, 2] := m : fi:
if j1 = i2 and j2 = i3 then m_star[n, 1] := m : fi:
od:#m
od:#n
print("m_star(n,ell) = index of opposite edge of vertex ell of triangle n");
print(evalm(m_star));

print("Degrees of vertices, cycles of neighbors");

degp := vector(R) : startm := vector(R) : innerp := vector(R) :

for i from 1 to R do:
innerp[i] := 1 : degp[i] := 0 :
for m from 1 to M do:
if j_star[m, 1] = i or j_star[m, 2] = i then
if degp[i] = 0 then startm[i] := m : fi:
if innerp[i] = 1 and n_star[m, 1] = n_star[m, 2] then innerp[i] := 0 : startm[i] := m : fi:
degp[i] := degp[i] + 1 :
fi:#j_star
od:#m
od:#i
print("Degrees of vertices", evalm(degp));
print("Inner vertices", evalm(innerp));
print("Starting edges in cycles", evalm(startm));

for i from 1 to R do:
dp[i] := degp[i] + innerp[i] : cycm[i] := vector(dp[i]) : cycp[i] := vector(dp[i]) :
m := startm[i] : n := n_star[m, 1] : ip := j_star[m, 1] + j_star[m, 2] - i :
cycm[i][1] := m :
cycp[i][1] := ip :
for ell from 2 to dp[i] do:
n := n_star[m, 1] + n_star[m, 2] - n :
for k from 1 to 3 do:
if i_star[n, k] = ip then kk := k : fi:

```

```

od:#k
   $m := m\_star[n, kk] :$ 
   $ip := j\_star[m, 1] + j\_star[m, 2] - i :$ 
   $cycm[i][ell] := m :$ 
   $cycp[i][ell] := ip :$ 
od:#ell

  print("Cycle of edges from point", i, " ", evalm(cycm[i]), " Cycle of points",
    evalm(cycp[i])) :
od:#i

print("Guessing normal vectors");

nn := matrix(R, 3) :
nn2 := vector(R) :
u := vector(3) : v := vector(3) :

for i from 1 to R do:
  nn[i, 1] := 0 : nn[i, 2] := 0 : nn[i, 3] := 0 :
  nn2[i] := 0 :
  for ell from 2 to dp[i] do:
    for k from 1 to 3 do:
       $iii := cycp[i][ell] : ii := cycp[i][ell - 1] :$ 
       $u[k] := P[iii, k] - P[i, k] : v[k] := P[ii, k] - P[i, k] :$ 
    od:#k
     $nn[i, 1] := nn[i, 1] + u[2] \cdot v[3] - u[3] \cdot v[2] :$ 
     $nn[i, 2] := nn[i, 2] + u[3] \cdot v[1] - u[1] \cdot v[3] :$ 
     $nn[i, 3] := nn[i, 3] + u[1] \cdot v[2] - u[2] \cdot v[1] :$ 
  od:#ell
   $nn2[i] := nn2[i] + nn[i, 1]^2 + nn[i, 2]^2 + nn[i, 3]^2 :$ 
od:#i

print("Normal vectors", evalm(nn));
print("Norm squares", evalm(nn2));

print("Guessed Surface derivatives");
print("g_ij from point i toward j");
g := array(1..R, 1..R) :
for i from 1 to R do: for j from 1 to R do:
   $g[i, j] := vector(3, [0, 0, 0]) :$ 
od:od:#ij
  for i from 1 to R do:
    for ell from 1 to degp[i] do:
       $j := cycp[i][ell] :$ 
       $sij := nn[i, 1] \cdot (P[j, 1] - P[i, 1]) + nn[i, 2] \cdot (P[j, 2] - P[i, 2]) + nn[i, 3] \cdot (P[j, 3] - P[i, 3]) :$ 
       $sji := nn[j, 1] \cdot (P[i, 1] - P[j, 1]) + nn[j, 2] \cdot (P[i, 2] - P[j, 2]) + nn[j, 3] \cdot (P[i, 3] - P[j, 3]) :$ 
    od:#ell
    for k from 1 to 3 do:
       $g[i, j][k] := (P[j, k] - P[i, k]) - \frac{sij}{nn2[i]} \cdot nn[i, k] :$ 
    od:#k
  od:#i

```

```

od:#k
print("g", i, j, " ", evalm(g[i, j]), " normalv", [nn[i, 1], nn[i, 2], nn[i, 3]]) :
od:#ell
od:#i

print(
"
_____ \
_____");
print(
"
_____ \
_____");

print("Checking if the g vectors are suitable in a polynomial construction");

print("g_ell-g_ik x g_ij not= 0 when [p_i,p_j] is a double edge between the mesh triangles
[p_i,p_j,p_k],[p_i,p_j,p_ell]");

suitable := 1 :
for m from 1 to M do:
i := j_star[m, 1] : j := j_star[m, 2] :
n1 := n_star[m, 1] : n2 := n_star[m, 2] :
if n1 ≠ n2 then
a1 := g[i, j][1] : a2 := g[i, j][2] : a3 := g[i, j][3] :
k1 := k_star[m, 1] : k2 := k_star[m, 2] :
b1 := g[i, k1][1] - g[i, k2][1] : b2 := g[i, k1][2] - g[i, k2][2] : b3 := g[i, k1][3] - g[i,
k2][3] :
c1 := a2·b3 - b2·a3 : c2 := a3·b1 - b3·a1 : c3 := a1·b2 - b1·a2 :
c := c12 + c22 + c32 : if c=0 then suitable := 0 : fi:
if c=0 then print("g[, i, j, "] , g[, i, k1, "] , g[, i, k2, "] FALSE") fi:
a1 := g[j, i][1] : a2 := g[j, i][2] : a3 := g[j, i][3] :
k1 := k_star[m, 1] : k2 := k_star[m, 2] :
b1 := g[j, k1][1] - g[j, k2][1] : b2 := g[j, k1][2] - g[j, k2][2] : b3 := g[j, k1][3] - g[j,
k2][3] :
c1 := a2·b3 - b2·a3 : c2 := a3·b1 - b3·a1 : c3 := a1·b2 - b1·a2 :
c := c12 + c22 + c32 : if c=0 then suitable := 0 : fi:
if c=0 then print("g[, j, i, "] , g[, j, k1, "] , g[, j, k2, "] FALSE") fi:
fi:
od:#m

if suitable=1 then print("SUITABLE") : fi:
if suitable=0 then print("NOT SUITABLE - g data should be modified") : fi:

print(
"
_____ \
_____");
print(
"
_____ \
_____");

```

```
print("Basic RSD interpolation");
```

```
t := vector(3) : lambda := vector(R) :
```

```
print("Shape functions", "Pi_0=", Phi, Theta, chi_0, chi_1, " ");
```

```
Phi := t3 · (10 - 15 · t0 + 6 · t02);
```

```
Theta := t03 · (4 - 3 · t0);
```

```
chi_0 := 30 · t12 · t22 · t3;
```

```
chi_1 := 12 · t12 · t22 · t3;
```

```
f := array(1..N) : f_lambda := array(1..N) :
```

```
for n from 1 to N do:
```

```
f[n] := vector(3, [0, 0, 0]) : f_lambda[n] := vector(3, [0, 0, 0]) :
```

```
od:#n
```

```
p := array(1..R) :
```

```
for i from 1 to R do:
```

```
p[i] := vector(3, [P[i, 1], P[i, 2], P[i, 3]]) :
```

```
od:#i
```

```
for n from 1 to N do:
```

```
for i from 1 to 3 do:
```

```
ip := i_star[n, i] :
```

```
f[n] := evalm(f[n] + subs(t0 = t[i], Phi) · p[ip]) :
```

```
for j from 1 to 3 do:
```

```
jp := i_star[n, j] :
```

```
f[n] := evalm(f[n] + subs(t0 = t[i], Theta) · t[j] · g[ip, jp]) :
```

```
od:od:#ij
```

```
for i from 1 to 3 do: for j from 1 to 3 do: for k from 1 to 3 do:
```

```
if i ≠ j and j ≠ k and k ≠ i then
```

```
ip := i_star[n, i] : jp := i_star[n, j] : kp := i_star[n, k] :
```

```
f[n] := evalm(f[n] + subs(t1 = t[i], t2 = t[j], t3 = t[k], chi_0) · p[ip] + subs(t1 = t[i], t2 = t[j], t3 = t[k], chi_1) · g[ip, jp]) :
```

```
fi:
```

```
od:od:od: #ijk
```

```
od:#n
```

```
for n from 1 to N do:
```

```
print("f^T, P, G_Pi0 over triangle ", n) :
```

```
i1 := i_star[n, 1] : i2 := i_star[n, 2] : i3 := i_star[n, 3] :
```

```
for k from 1 to 3 do:
```

```
f_lambda[n][k] := subs(t[1] = lambda[i1], t[2] = lambda[i2], t[3] = lambda[i3], f[n][k]) :
```

```
od:
```

```
print(evalm(f[n][1])) : #print(evalm(f_lambda[n][1])) :
```

```
print(evalm(f[n][2])) : #print(evalm(f_lambda[n][2])) :
```

```
print(evalm(f[n][3])) : #print(evalm(f_lambda[n][3])) :
```

```
od:#n
```

```
print("f^T,P,G_Pi0 displayed");
```

```
f_su := matrix(N, 3) :
```

```
for n from 1 to N do:
```

```
for k from 1 to 3 do:
```

```
f_su[n, k] := subs(t[1] = sigma, t[2] = (1 - sigma) · tau, t[3] = (1 - sigma) · (1 - tau),  
f[n][k]) :
```

```
od:od: #kn
```

```
C1 := array(1..3) : C2 := array(1..3) : C3 := array(1..3) : C4 := array(1..3) :
```

```
for k from 1 to 3 do:
```

```
C1[k] := f_su[1, k] : C2[k] := f_su[2, k] : C3[k] := f_su[3, k] : C4[k] := f_su[4, k] :
```

```
od:
```

```
plot3d( {C1, C2, C3, C4}, sigma = 0..1, tau = 0..1);
```

```
a := 0; b := 100-1; c := 49·100-1; d := 1 - 49·100-1;
```

```
plot3d( {C3, C4}, sigma = a..b, tau = c..d);
```

```
print(
```

```
"  
_____"");
```

```
print(  
"  
_____"");
```

```
print("Edge corrections");
```

```
print("Row m: double edge m between points i1,i2 and opposite vertices j1,j2 in triangles n1,  
n2");
```

```
print("ordering: i1 < i2 , j1 < j2 and n1 < n2");
```

```
MM := 0 :
```

```
for m from 1 to M do:
```

```
if n_star[m, 1] ≠ n_star[m, 2] then MM := MM + 1 : fi:
```

```
od:#m
```

```
print("Number of double edges ", MM);
```

```
IJN := matrix(MM, 6) :
```

```
mm := 0 :
```

```
for m from 1 to M do:
```

```
n1 := n_star[m, 1] : n2 := n_star[m, 2] :
```

```
if n1 ≠ n2 then
```

```
mm := mm + 1 :
```

```
i1 := j_star[m, 1] : i2 := j_star[m, 2] :
```

```
j1 := k_star[m, 1] : j2 := k_star[m, 2] :
```

```
IJN[mm, 1] := i1 : IJN[mm, 2] := i2 :
```

```

IJN[mm, 3] := j1 : IJN[mm, 4] := j2 :
IJN[mm, 5] := n1 : IJN[mm, 6] := n2 :
fi:
od:#m

print("IJM ", evalm(IJN) );

V := matrix(MM, 3) : VV := matrix(MM, 3) : dV := matrix(MM, 3) :
Y := matrix(MM, 3) : W := matrix(MM, 3) :
Bdet := vector(MM) :

for m from 1 to MM do:
i1 := IJN[m, 1] : i2 := IJN[m, 2] : j1 := IJN[m, 3] : j2 := IJN[m, 4] : n1 := IJN[m, 5] : n2
:= IJN[m, 6] :
print("_____") :
print("On (directed Edge ", m, " points ", i1, " --> ", i2) :

for k from 1 to 3 do:
F := f_lambda[n1][k] :

print(k, F) :

FL := subs( lambda[i1] = tau - 2-1·sigma, lambda[i2] = 1 - tau - 2-1·sigma, lambda[j1]
= sigma, F) :
V[m, k] := factor(subs( sigma = 0, diff( FL, sigma) ) ) :
F := f_lambda[n2][k] :
FL := subs( lambda[i1] = tau - 2-1·sigma, lambda[i2] = 1 - tau - 2-1·sigma, lambda[j2]
= sigma, F) :
VV[m, k] := factor(subs( sigma = 0, diff( FL, sigma) ) ) :
dV[m, k] := factor( VV[m, k] - V[m, k] ) :
FL := subs( lambda[i1] = tau, lambda[i2] = 1 - tau, lambda[j2] = 0, F) :
Y[m, k] := factor( diff( FL, tau) ) :
od:
dv1 := dV[m, 1] : dv2 := dV[m, 2] : dv3 := dV[m, 3] :
y1 := Y[m, 1] : y2 := Y[m, 2] : y3 := Y[m, 3] :
W[m, 1] := factor( dv2·y3 - dv3·y2 ) : W[m, 2] := factor( dv3·y1 - dv1·y3 ) : W[m, 3]
:= factor( dv1·y2 - dv2·y1 ) :
v1 := V[m, 1] : v2 := V[m, 2] : v3 := V[m, 3] :
w1 := W[m, 1] : w2 := W[m, 2] : w3 := W[m, 3] :
bb := v1·w1 + v2·w2 + v3·w3 :
Bdet[m] := factor( bb ) :

print("v", [ V[m, 1], V[m, 2], V[m, 3] ] ) :
print("vv", [ VV[m, 1], VV[m, 2], VV[m, 3] ] ) :
print("dv", [ dV[m, 1], dV[m, 2], dV[m, 3] ] ) :
print("y", [ Y[m, 1], Y[m, 2], Y[m, 3] ] ) :
print("w", [ W[m, 1], W[m, 2], W[m, 3] ] ) :
print("b=det[v,dv,y]", Bdet[m] ) :
od:

```

```

print(
  "
  _____ \
  _____");
print(
  "
  _____ \
  _____");

```

```

print("Cofactors of GCD(w_1,w_2,w_3)");

```

```

K := 3 :

```

```

Q := vector(MM) :
qw := matrix(MM, 3) :

```

```

a0 := matrix(1, K) : a := matrix(1, K) :

```

```

for mmm from 1 to MM do:

```

```

i1 := IJN[mmm, 1] : i2 := IJN[mmm, 2] :

```

```

print("W[" , mmm, ",on points " , i1 , "-->" , i2, "]");

```

```

a0[1, 1] := W[mmm, 1] :
a0[1, 2] := W[mmm, 2] :
a0[1, 3] := W[mmm, 3] :
print("Polynomials");
print(a0[1, 1]) : print(a0[1, 2]) : print(a0[1, 3]) :

```

```

for k from 1 to K do:

```

```

a[1, k] := a0[1, k] :

```

```

od:

```

```

dd := vector(K) : S := 0 :

```

```

for k from 1 to K do:

```

```

if expand(a[1, k]) = 0 then dd[k] := -1 : fi:

```

```

if expand(a[1, k]) ≠ 0 then dd[k] := degree(a[1, k]) : S := S + dd[k] : fi:

```

```

od:#k

```

```

#print("Degrees " , evalm(dd) );

```

```

#print("Sum of positive degrees " , S);

```

```

mc := vector(K) :

```

```

for k from 1 to K do:

```

```

if dd[k] < 0 then mc[k] := 0 : fi:

```

```

if dd[k] = 0 then mc[k] := a[1, k] : fi:

```

```

if dd[k] > 0 then

```

```

b := a[1, k] : for j from 1 to dd[k] do: b :=  $\frac{\text{diff}(b, \text{tau})}{j}$  : od: mc[k] := b :

```

```

fi:

```

```

od:#k
#print("maincoeff s ", evalm(mc));

X := matrix(K, K) :
for i from 1 to K do: for j from 1 to K do: X[i, j] := 0 : od: X[i, i] := 1 : od:
#print("X initial ", evalm(X) );

for s from 1 to S do:
#print("_____") :
#print("STEP ", s) :
#` `
#print("polynomials", evalm(transpose(a))) :

for k from 1 to K do:
if expand(a[1, k]) = 0 then dd[k] := -1 : fi:
if expand(a[1, k]) ≠ 0 then dd[k] := degree(a[1, k]) : fi:
od:#k
for k from 1 to K do:
if dd[k] < 0 then mc[k] := 0 : fi:
if dd[k] = 0 then mc[k] := a[1, k] : fi:
if dd[k] ≥ 1 then
b := a[1, k] : for j from 1 to dd[k] do: b :=  $\frac{\text{diff}(b, \text{tau})}{j}$  : od: mc[k] := b :
fi:
od:#k
#print("degrees ", evalm(dd)) :
#print("main coefficients ", evalm(mc)) :

#print("Normalization") :

for k from 1 to K do:
if dd[k] ≥ 0 then a[1, k] := expand(mc[k]-1·a[1, k]) : fi:
od:#k
A := matrix(K, K) :
for i from 1 to K do: for j from 1 to K do:
A[i, j] := 0 :
if i = j and dd[j] ≥ 0 then A[i, j] := mc[j]-1 : fi:
if i = j and dd[j] < 0 then A[i, i] := 1 : fi:
od:od:#ij
X := evalm(X & * A) :
#print("New polynomials ", evalm(transpose(a))) :
# print("degrees ", evalm(dd)) :
#print("New X with X=XA ", evalm(X), " A=", evalm(A)) :
#` `
#print("Reordering") :
P := matrix(K, K) :
for i from 1 to K do: for j from 1 to K do: P[i, j] := 0 : od: P[i, i] := 1 : od:
for n from 1 to K do:
for i from 1 to K - 1 do:
j := i + 1 :
if (0 ≤ dd[i] and dd[i] < dd[j]) or dd[j] = -1 then

```

```

ddd := dd[j] : dd[j] := dd[i] : dd[i] := ddd :
aaa := a[1,j] : a[1,j] := a[1,i] : a[1,i] := aaa :
for k from 1 to K do:
ppp := P[k,j] : P[k,j] := P[k,i] : P[k,i] := ppp :
od:#k
fi:
od:#i
od:#n
X := evalm(X&*P) :
#print("New polynomials ") :
#print(evalm(transpose(a))) :
#print("degrees ", evalm(dd)) :
#print("New X=XP ", evalm(X), " with P= ", evalm(P)) :

#print("Degree decreasing") :
L := 0 :
for k from 1 to K do:
if dd[k]=-1 then L := k fi:
od:#k
L := L + 1 :
#print("First nonzero pol with index L=", L) :

#STOP condition
if L < K then

dddd := dd[L] - dd[L + 1] :
#print("pol_L = ", a[1, L], " pol_L+1 = ", a[1, L + 1], " tau^d = ", taudddd) :

a[1, L] := expand(a[1, L] - taudddd.a[1, L + 1]) :
#print("a_L-taudddda_L+1 = ", a[1, L]) :
B := matrix(K, K) :
for i from 1 to K do: for j from 1 to K do:
B[i, j] := 0 : od: B[i, i] := 1 : od:
B[L + 1, L] := -taudddd :
X := evalm(X&*B) :
#print("New polynomials") :
#print(evalm(transpose(a))) :
#print("New X=XB ", evalm(X), " with B= ", evalm(B)) :

fi: #STOPcond

#STOP
if L = K then s := S : fi:

#print("CHEKING a -a0 X=0") :
DDD := evalm(a - a0&*X) :
DD := evalm(DDD[1, 1]) :
for k from 1 to K do: DD := expand(DD) : od:
#print(evalm(DD)) :

```

od:#s

```
GCD := evalm(a[1, K]) :  
print(" GCD over double edge ", mmm, " = ", GCD);  
Q[mmm] := GCD :
```

```
#print("Cofactors in column 3 of X");
```

```
for i from 1 to K do: qw[mmm, i] := expand(X[i, L]) : od:
```

```
print("qw", mmm, 1, " = ", qw[mmm, 1]) :  
print("qw", mmm, 2, " = ", qw[mmm, 2]) :  
print("qw", mmm, 3, " = ", qw[mmm, 3]) :
```

```
# print("CHECKING GCD-a0_1 q_1-...-a0_K q_K =0");
```

```
# DD:=GCD:
```

```
#for k from 1 to K do: DD := expand(DD - a0[1, k]·qw[mmm, k]) : od:  
#print(DD);
```

od:#mm

```
print(  
"  
_____" );  
print(  
"  
_____" );
```

```
print("Z polynomials along double edges");
```

```
Zq := matrix(MM, 3) : ZqL := matrix(MM, 3) :
```

```
for mm from 1 to MM do:
```

```
  i1 := IJN[mm, 1] : i2 := IJN[mm, 2] :  
  print("Double edge ", mm, " ", i1, "-->", i2) :
```

```
  z1 := simplify( factor(  $\frac{Bdet[mm]}{Q[mm] \cdot \tau^2 \cdot (1 - \tau)^2}$  ) ) ) :
```

```
  print(z1) :
```

```
    for k from 1 to 3 do:
```

```
      z2 := factor(qw[mm, k]) :
```

```
      Zq[mm, k] := z1·z2 :
```

```

z0 := subs(tau = lambda[i1], z1) * factor(subs(tau = 1 - lambda[i2], z2)) :
z0 := z0 + factor(subs(tau = 1 - lambda[i2], z1)) * subs(tau = lambda[i1], z2) :
ZqL[mm, k] := 2-1 · z0 :
print("Zq[" , mm, k, " resp. ZqL[" , mm, k, "]) :
print(Zq[mm, k]) :
print(ZqL[mm, k]) :

```

```

od:#k
od:#mm

```

```

Fq := matrix(N, 3) :

```

```

for n from 1 to N do:
for k from 1 to 3 do: Fq[n, k] := f_lambda[n][k] : od:#k
od:#n

```

```

for mm from 1 to MM do:
i1 := IJN[mm, 1] : i2 := IJN[mm, 2] :
j1 := IJN[mm, 3] : j2 := IJN[mm, 4] :
n1 := IJN[mm, 5] : n2 := IJN[mm, 6] :
for k from 1 to 3 do:
Fq[n1, k] := Fq[n1, k] - lambda[i1]2 · lambda[i2]2 · lambda[j1] · ZqL[mm, k] :
Fq[n2, k] := Fq[n2, k] - lambda[i1]2 · lambda[i2]2 · lambda[j2] · ZqL[mm, k] :
od:#k
od:#mm

```

```

for n from 1 to N do:
print("Triangle ", n, [i_star[n, 1], i_star[n, 2], i_star[n, 3]]) :
print("Spline over triangle ", n) :
for k from 1 to 3 do:
print(Fq[n, k]) :
od:#k
od:#n

```

```

print(
"_____ \
_____");
print(
"_____ \
_____");

```

```

print("Final result displayed");

```

```

Fqts := matrix(N, 3) :
for n from 1 to N do:
i1 := i_star[n, 1] : i2 := i_star[n, 2] : i3 := i_star[n, 3] :
for k from 1 to 3 do:
Fqts[n, k] := subs(lambda[i1] = tau, lambda[i2] = (1 - tau) · sigma, lambda[i3] = (1 - tau) · (1

```

```

    - sigma), Fq[n, k]) :
#print(Fqts[n, k]) :
od:#k
od:#n

```

```

Cq1 := [Fqts[1, 1], Fqts[1, 2], Fqts[1, 3]] : Cq2 := [Fqts[2, 1], Fqts[2, 2], Fqts[2, 3]] :
Cq3 := [Fqts[3, 1], Fqts[3, 2], Fqts[3, 3]] : Cq4 := [Fqts[4, 1], Fqts[4, 2], Fqts[4, 3]] :

```

```

plot3d( {Cq1, Cq2, Cq3, Cq4}, tau = 0..1, sigma = 0..1);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 99·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 90·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 80·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 70·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 60·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 50·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 40·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 30·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 20·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 10·10-2..100·10-2);

```

```

plot3d( {Cq1, Cq2}, tau = 0·10-2..100·10-2, sigma = 0·10-2..100·10-2);

```

"Mesh vertices"

$R := 4$

4, " points"

"3D-Coordinates of the ", 4, " points in the rows"

$$P := \begin{bmatrix} -2 & 0 & \sqrt{2} \\ 2 & 0 & \sqrt{2} \\ 0 & -2 & -\sqrt{2} \\ 0 & 2 & -\sqrt{2} \end{bmatrix}$$

"Mesh triangles"

$N := 4$

"Point Indices of the ", 4, " triangles,"

"i_star(n,i) = index of point i in triangle n"

"Lexicographic order"

$$i_star := \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix}$$

"M= ", 6, "mesh edges"

"j_star(m,ell)= index of point ell in edge m"

"Lexicographic order"

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 2 & 3 \\ 2 & 4 \\ 3 & 4 \end{bmatrix}$$

"n_star(m,ell)=index ell of neighboring triangle of edge m"

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 3 \\ 1 & 4 \\ 2 & 4 \\ 3 & 4 \end{bmatrix}$$

"k_star(m,1), k_star(m,2) indices of opposite vertices to edge m in neighboring triangles"

$$\begin{bmatrix} 3 & 4 \\ 2 & 4 \\ 2 & 3 \\ 1 & 4 \\ 1 & 3 \\ 1 & 2 \end{bmatrix}$$

"m_star(n,ell) = index of opposite edge of vertex ell of triangle n"

$$\begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 1 \\ 6 & 3 & 2 \\ 6 & 5 & 4 \end{bmatrix}$$

"Degrees of vertices, cycles of neighbors"

"Degrees of vertices", $\begin{bmatrix} 3 & 3 & 3 & 3 \end{bmatrix}$

"Inner vertices", $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$

"Starting edges in cycles", $\begin{bmatrix} 1 & 1 & 2 & 3 \end{bmatrix}$

"Cycle of edges from point", 1, " ", $\begin{bmatrix} 1 & 3 & 2 & 1 \end{bmatrix}$, " Cycle of points", $\begin{bmatrix} 2 & 4 & 3 & 2 \end{bmatrix}$

"Cycle of edges from point", 2, " ", $\begin{bmatrix} 1 & 5 & 4 & 1 \end{bmatrix}$, " Cycle of points", $\begin{bmatrix} 1 & 4 & 3 & 1 \end{bmatrix}$

"Cycle of edges from point", 3, " ", $\begin{bmatrix} 2 & 6 & 4 & 2 \end{bmatrix}$, " Cycle of points", $\begin{bmatrix} 1 & 4 & 2 & 1 \end{bmatrix}$

"Cycle of edges from point", 4, " ", $\begin{bmatrix} 3 & 6 & 5 & 3 \end{bmatrix}$, " Cycle of points", $\begin{bmatrix} 1 & 3 & 2 & 1 \end{bmatrix}$

"Guessing normal vectors"

"Normal vectors", $\begin{bmatrix} 8\sqrt{2} & 0 & -8 \\ 8\sqrt{2} & 0 & 8 \\ 0 & 8\sqrt{2} & 8 \\ 0 & 8\sqrt{2} & -8 \end{bmatrix}$

"Norm squares", $\begin{bmatrix} 192 & 192 & 192 & 192 \end{bmatrix}$

"Guessed Surface derivatives"

"g_ij from point i toward j"

"g", 1, 2, " ", $\begin{bmatrix} \frac{4}{3} & 0 & \frac{4}{3}\sqrt{2} \end{bmatrix}$, " normalv", $\begin{bmatrix} 8\sqrt{2} & 0 & -8 \end{bmatrix}$

"g", 1, 4, " ", $\begin{bmatrix} -\frac{2}{3} & 2 & -\frac{2}{3}\sqrt{2} \end{bmatrix}$, " normalv", $\begin{bmatrix} 8\sqrt{2} & 0 & -8 \end{bmatrix}$

$$\text{"g", 1, 3, " "}, \left[-\frac{2}{3} \quad -2 \quad -\frac{2}{3} \sqrt{2} \right], \text{" normalv", } [8\sqrt{2}, 0, -8]$$

$$\text{"g", 2, 1, " "}, \left[-\frac{4}{3} \quad 0 \quad \frac{4}{3} \sqrt{2} \right], \text{" normalv", } [8\sqrt{2}, 0, 8]$$

$$\text{"g", 2, 4, " "}, \left[\frac{2}{3} \quad 2 \quad -\frac{2}{3} \sqrt{2} \right], \text{" normalv", } [8\sqrt{2}, 0, 8]$$

$$\text{"g", 2, 3, " "}, \left[\frac{2}{3} \quad -2 \quad -\frac{2}{3} \sqrt{2} \right], \text{" normalv", } [8\sqrt{2}, 0, 8]$$

$$\text{"g", 3, 1, " "}, \left[-2 \quad -\frac{2}{3} \quad \frac{2}{3} \sqrt{2} \right], \text{" normalv", } [0, 8\sqrt{2}, 8]$$

$$\text{"g", 3, 4, " "}, \left[0 \quad \frac{4}{3} \quad -\frac{4}{3} \sqrt{2} \right], \text{" normalv", } [0, 8\sqrt{2}, 8]$$

$$\text{"g", 3, 2, " "}, \left[2 \quad -\frac{2}{3} \quad \frac{2}{3} \sqrt{2} \right], \text{" normalv", } [0, 8\sqrt{2}, 8]$$

$$\text{"g", 4, 1, " "}, \left[-2 \quad \frac{2}{3} \quad \frac{2}{3} \sqrt{2} \right], \text{" normalv", } [0, 8\sqrt{2}, -8]$$

$$\text{"g", 4, 3, " "}, \left[0 \quad -\frac{4}{3} \quad -\frac{4}{3} \sqrt{2} \right], \text{" normalv", } [0, 8\sqrt{2}, -8]$$

$$\text{"g", 4, 2, " "}, \left[2 \quad \frac{2}{3} \quad \frac{2}{3} \sqrt{2} \right], \text{" normalv", } [0, 8\sqrt{2}, -8]$$

" _____ \

_____ "

" _____ \

_____ "

"Checking if the g vectors are suitable in a polynomial construction"

"g_ell-g_ik) x g_ij not= 0 when [p_i,p_j] is a double edge between the mesh triangles [p_i,p_j, p_k],[pi,p_j,p_ell]"

"SUITABLE"

" _____ \

_____ "

" _____ \

_____ "

"Basic RSD interpolation"

"Shape functions", "Pi_0=(", Φ , Θ , χ_0 , χ_1 , ")"

$$\Phi := t\theta^3 (6 t\theta^2 - 15 t\theta + 10)$$

$$\Theta := t\theta^3 (4 - 3 t\theta)$$

$$\chi_0 := 30 t\theta^2 t^2 t^3$$

$$chi_I := 12 t l^2 t^2 t_3$$

"f^T,P,G_Pi0 over triangle ", 1

$$\begin{aligned} & 92 t_2^2 t_3^2 t_1 - 92 t_1^2 t_3^2 t_2 - 2 t_1^3 (6 t_1^2 - 15 t_1 + 10) + \frac{4}{3} t_1^3 (4 - 3 t_1) t_2 - \frac{2}{3} t_1^3 (4 - 3 t_1) t_3 + 2 \\ & t_2^3 (6 t_2^2 - 15 t_2 + 10) - \frac{4}{3} t_2^3 (4 - 3 t_2) t_1 + \frac{2}{3} t_2^3 (4 - 3 t_2) t_3 - 2 t_3^3 (4 - 3 t_3) t_1 + 2 \\ & t_3^3 (4 - 3 t_3) t_2 \\ & - 92 t_2^2 t_3^2 t_1 - 92 t_1^2 t_3^2 t_2 - 2 t_1^3 (4 - 3 t_1) t_3 - 2 t_2^3 (4 - 3 t_2) t_3 - 2 t_3^3 (6 t_3^2 - 15 t_3 + 10) - \frac{2}{3} \\ & t_3^3 (4 - 3 t_3) t_1 - \frac{2}{3} t_3^3 (4 - 3 t_3) t_2 \end{aligned}$$

$$\begin{aligned} & 92 t_1^2 t_2^2 t_3 \sqrt{2} + t_1^3 (6 t_1^2 - 15 t_1 + 10) \sqrt{2} + \frac{4}{3} t_1^3 (4 - 3 t_1) t_2 \sqrt{2} - \frac{2}{3} t_1^3 (4 - 3 t_1) t_3 \sqrt{2} \\ & + t_2^3 (6 t_2^2 - 15 t_2 + 10) \sqrt{2} + \frac{4}{3} t_2^3 (4 - 3 t_2) t_1 \sqrt{2} - \frac{2}{3} t_2^3 (4 - 3 t_2) t_3 \sqrt{2} - t_3^3 (6 t_3^2 \\ & - 15 t_3 + 10) \sqrt{2} + \frac{2}{3} t_3^3 (4 - 3 t_3) t_1 \sqrt{2} + \frac{2}{3} t_3^3 (4 - 3 t_3) t_2 \sqrt{2} \end{aligned}$$

"f^T,P,G_Pi0 over triangle ", 2

$$\begin{aligned} & 92 t_2^2 t_3^2 t_1 - 92 t_1^2 t_3^2 t_2 - 2 t_1^3 (6 t_1^2 - 15 t_1 + 10) + \frac{4}{3} t_1^3 (4 - 3 t_1) t_2 - \frac{2}{3} t_1^3 (4 - 3 t_1) t_3 + 2 \\ & t_2^3 (6 t_2^2 - 15 t_2 + 10) - \frac{4}{3} t_2^3 (4 - 3 t_2) t_1 + \frac{2}{3} t_2^3 (4 - 3 t_2) t_3 - 2 t_3^3 (4 - 3 t_3) t_1 + 2 \\ & t_3^3 (4 - 3 t_3) t_2 \\ & 92 t_2^2 t_3^2 t_1 + 92 t_1^2 t_3^2 t_2 + 2 t_1^3 (4 - 3 t_1) t_3 + 2 t_2^3 (4 - 3 t_2) t_3 + 2 t_3^3 (6 t_3^2 - 15 t_3 + 10) + \frac{2}{3} \\ & t_3^3 (4 - 3 t_3) t_1 + \frac{2}{3} t_3^3 (4 - 3 t_3) t_2 \end{aligned}$$

$$\begin{aligned} & 92 t_1^2 t_2^2 t_3 \sqrt{2} + t_1^3 (6 t_1^2 - 15 t_1 + 10) \sqrt{2} + \frac{4}{3} t_1^3 (4 - 3 t_1) t_2 \sqrt{2} - \frac{2}{3} t_1^3 (4 - 3 t_1) t_3 \sqrt{2} \\ & + t_2^3 (6 t_2^2 - 15 t_2 + 10) \sqrt{2} + \frac{4}{3} t_2^3 (4 - 3 t_2) t_1 \sqrt{2} - \frac{2}{3} t_2^3 (4 - 3 t_2) t_3 \sqrt{2} - t_3^3 (6 t_3^2 \\ & - 15 t_3 + 10) \sqrt{2} + \frac{2}{3} t_3^3 (4 - 3 t_3) t_1 \sqrt{2} + \frac{2}{3} t_3^3 (4 - 3 t_3) t_2 \sqrt{2} \end{aligned}$$

"f^T,P,G_Pi0 over triangle ", 3

$$\begin{aligned} & - 92 t_1^2 t_3^2 t_2 - 92 t_1^2 t_2^2 t_3 - 2 t_1^3 (6 t_1^2 - 15 t_1 + 10) - \frac{2}{3} t_1^3 (4 - 3 t_1) t_2 - \frac{2}{3} t_1^3 (4 - 3 t_1) t_3 - 2 \\ & t_2^3 (4 - 3 t_2) t_1 - 2 t_3^3 (4 - 3 t_3) t_1 \\ & 92 t_1^2 t_3^2 t_2 - 92 t_1^2 t_2^2 t_3 - 2 t_1^3 (4 - 3 t_1) t_2 + 2 t_1^3 (4 - 3 t_1) t_3 - 2 t_2^3 (6 t_2^2 - 15 t_2 + 10) - \frac{2}{3} \\ & t_2^3 (4 - 3 t_2) t_1 + \frac{4}{3} t_2^3 (4 - 3 t_2) t_3 + 2 t_3^3 (6 t_3^2 - 15 t_3 + 10) + \frac{2}{3} t_3^3 (4 - 3 t_3) t_1 - \frac{4}{3} \\ & t_3^3 (4 - 3 t_3) t_2 \end{aligned}$$

$$\begin{aligned}
& -92 \ell_2^2 \ell_3^2 t_1 \sqrt{2} + \ell_1^3 (6 \ell_1^2 - 15 t_1 + 10) \sqrt{2} - \frac{2}{3} \ell_1^3 (4 - 3 t_1) t_2 \sqrt{2} - \frac{2}{3} \ell_1^3 (4 - 3 t_1) t_3 \sqrt{2} \\
& - \ell_2^3 (6 \ell_2^2 - 15 t_2 + 10) \sqrt{2} + \frac{2}{3} \ell_2^3 (4 - 3 t_2) t_1 \sqrt{2} - \frac{4}{3} \ell_2^3 (4 - 3 t_2) t_3 \sqrt{2} - \ell_3^3 (6 \ell_3^2 \\
& - 15 t_3 + 10) \sqrt{2} + \frac{2}{3} \ell_3^3 (4 - 3 t_3) t_1 \sqrt{2} - \frac{4}{3} \ell_3^3 (4 - 3 t_3) t_2 \sqrt{2}
\end{aligned}$$

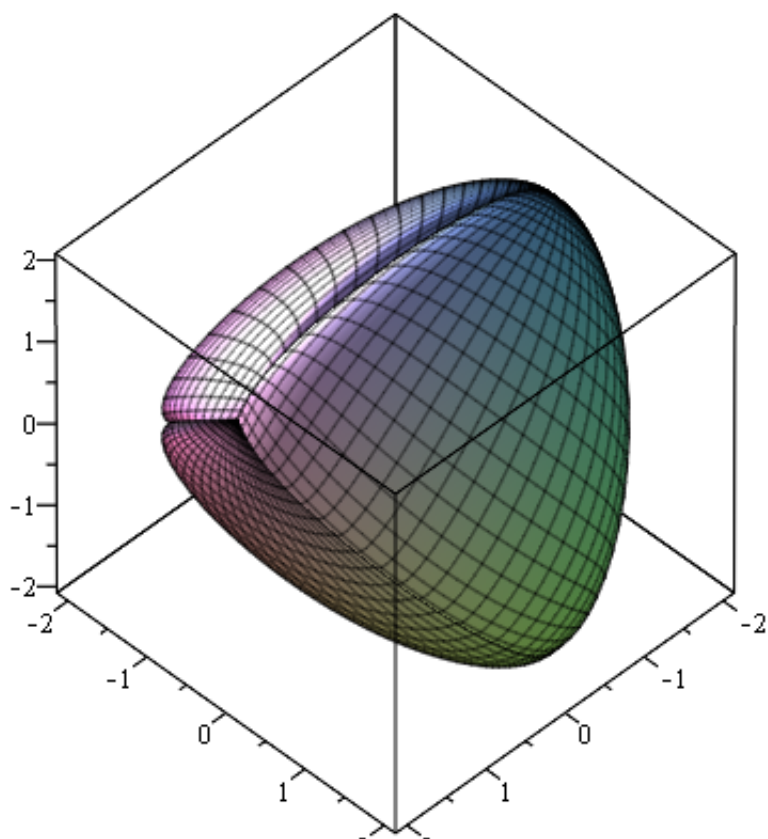
"f^T,P,G_Pi0 over triangle ", 4

$$\begin{aligned}
& 92 \ell_1^2 \ell_3^2 t_2 + 92 \ell_1^2 \ell_2^2 t_3 + 2 \ell_1^3 (6 \ell_1^2 - 15 t_1 + 10) + \frac{2}{3} \ell_1^3 (4 - 3 t_1) t_2 + \frac{2}{3} \ell_1^3 (4 - 3 t_1) t_3 + 2 \\
& \ell_2^3 (4 - 3 t_2) t_1 + 2 \ell_3^3 (4 - 3 t_3) t_1
\end{aligned}$$

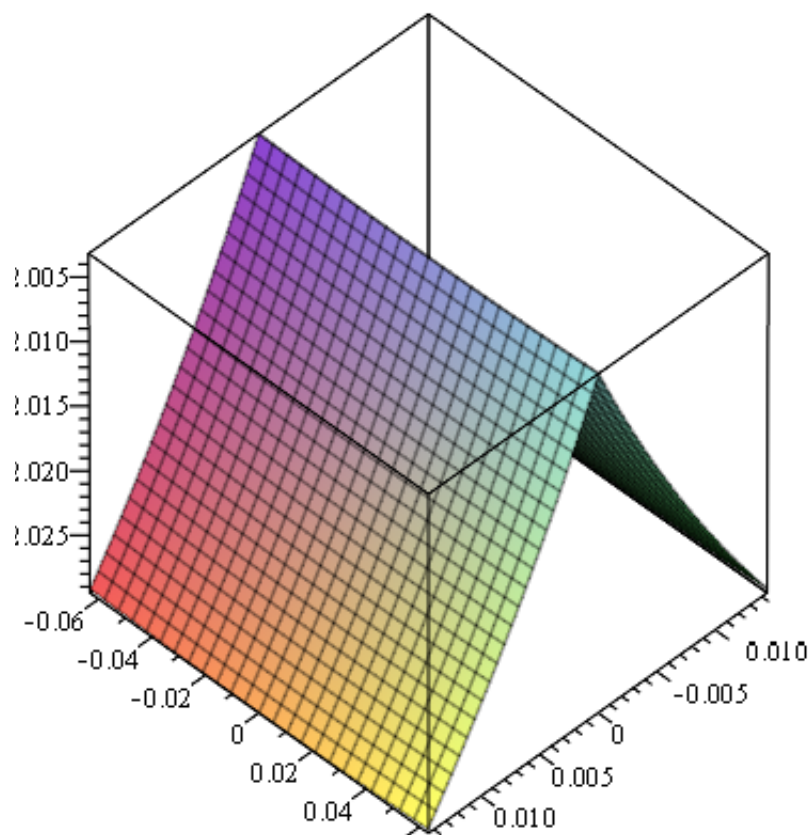
$$\begin{aligned}
& 92 \ell_1^2 \ell_3^2 t_2 - 92 \ell_1^2 \ell_2^2 t_3 - 2 \ell_1^3 (4 - 3 t_1) t_2 + 2 \ell_1^3 (4 - 3 t_1) t_3 - 2 \ell_2^3 (6 \ell_2^2 - 15 t_2 + 10) - \frac{2}{3} \\
& \ell_2^3 (4 - 3 t_2) t_1 + \frac{4}{3} \ell_2^3 (4 - 3 t_2) t_3 + 2 \ell_3^3 (6 \ell_3^2 - 15 t_3 + 10) + \frac{2}{3} \ell_3^3 (4 - 3 t_3) t_1 - \frac{4}{3} \\
& \ell_3^3 (4 - 3 t_3) t_2
\end{aligned}$$

$$\begin{aligned}
& -92 \ell_2^2 \ell_3^2 t_1 \sqrt{2} + \ell_1^3 (6 \ell_1^2 - 15 t_1 + 10) \sqrt{2} - \frac{2}{3} \ell_1^3 (4 - 3 t_1) t_2 \sqrt{2} - \frac{2}{3} \ell_1^3 (4 - 3 t_1) t_3 \sqrt{2} \\
& - \ell_2^3 (6 \ell_2^2 - 15 t_2 + 10) \sqrt{2} + \frac{2}{3} \ell_2^3 (4 - 3 t_2) t_1 \sqrt{2} - \frac{4}{3} \ell_2^3 (4 - 3 t_2) t_3 \sqrt{2} - \ell_3^3 (6 \ell_3^2 \\
& - 15 t_3 + 10) \sqrt{2} + \frac{2}{3} \ell_3^3 (4 - 3 t_3) t_1 \sqrt{2} - \frac{4}{3} \ell_3^3 (4 - 3 t_3) t_2 \sqrt{2}
\end{aligned}$$

"f^T,P,G_Pi0 displayed"



$$\begin{aligned}a &:= 0 \\b &:= \frac{1}{100} \\c &:= \frac{49}{100} \\d &:= \frac{51}{100}\end{aligned}$$



"

"

"

"Edge corrections"

"Row m: double edge m between points i1,i2 and opposite vertices j1,j2 in triangles n1,n2"

"ordering: i1 < i2 , j1 < j2 and n1 < n2"

"Number of double edges ", 6

"IJM ",

1	2	3	4	1	2
1	3	2	4	1	3
1	4	2	3	2	3
2	3	1	4	1	4
2	4	1	3	2	4
3	4	1	2	3	4

"

"

"On (directed Edge ", 1, " points ", 1, " --> ", 2

$$1, 92 \lambda_2^2 \lambda_3^2 \lambda_1 - 92 \lambda_1^2 \lambda_3^2 \lambda_2 - 2 \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 + 2 \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) - \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 + \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 - 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 + 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_2$$

$$2, -92 \lambda_2^2 \lambda_3^2 \lambda_1 - 92 \lambda_1^2 \lambda_3^2 \lambda_2 - 2 \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 - 2 \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 - 2 \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 + 10) - \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 - \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_2$$

$$3, 92 \lambda_1^2 \lambda_2^2 \lambda_3 \sqrt{2} + \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) \sqrt{2} + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 \sqrt{2} - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 \sqrt{2} + \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) \sqrt{2} + \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 \sqrt{2} - \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 \sqrt{2} - \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 + 10) \sqrt{2} + \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 \sqrt{2} + \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_2 \sqrt{2}$$

$$\text{"v"}, \left[\frac{4}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1), 12 \tau^4 - 24 \tau^3 + 12 \tau^2 - 2, \frac{2}{3} \sqrt{2} (3 \tau - 2 + \sqrt{2}) (3 \tau - 1 + \sqrt{2}) (-3 \tau + 2 + \sqrt{2}) (-3 \tau + 1 + \sqrt{2}) \right]$$

$$\text{"vv"}, \left[\frac{4}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1), -12 \tau^4 + 24 \tau^3 - 12 \tau^2 + 2, \frac{2}{3} \sqrt{2} (3 \tau - 2 + \sqrt{2}) (3 \tau - 1 + \sqrt{2}) (-3 \tau + 2 + \sqrt{2}) (-3 \tau + 1 + \sqrt{2}) \right]$$

$$\text{"dv"}, [0, -24 \tau^4 + 48 \tau^3 - 24 \tau^2 + 4, 0]$$

$$\text{"y"}, \left[-\frac{4}{3} - 80 \tau^2 + 160 \tau^3 - 80 \tau^4, 0, \frac{4}{3} \sqrt{2} (2 \tau^2 - 2 \tau - 1) (2 \tau - 1) \right]$$

$$\text{"w"}, \left[-\frac{16}{3} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) \sqrt{2} (2 \tau^2 - 2 \tau - 1) (2 \tau - 1), 0, -\frac{16}{3} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (60 \tau^4 - 120 \tau^3 + 60 \tau^2 + 1) \right]$$

$$\text{"b=det[v,dv,y]}, -\frac{32}{9} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) \tau^2 (\tau - 1)^2$$

"_____"

"On (directed Edge ", 2, " points ", 1, " --> ", 3

$$1, 92 \lambda_2^2 \lambda_3^2 \lambda_1 - 92 \lambda_1^2 \lambda_3^2 \lambda_2 - 2 \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 - 3 \lambda_1) \lambda_3 + 2 \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) - \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 + \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 - 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 + 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_2$$

$$2, -92 \lambda_2^2 \lambda_3^2 \lambda_1 - 92 \lambda_1^2 \lambda_3^2 \lambda_2 - 2 \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 - 2 \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 - 2 \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 + 10) - \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 - \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_2$$

$$3, 92 \lambda_1^2 \lambda_2^2 \lambda_3 \sqrt{2} + \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) \sqrt{2} + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 \sqrt{2} - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 \sqrt{2} + \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) \sqrt{2} + \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 \sqrt{2} - \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 \sqrt{2} - \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 + 10) \sqrt{2} + \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 \sqrt{2} + \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_2 \sqrt{2}$$

$$"v", \left[3 - 64 \tau^2 + \frac{368}{3} \tau^3 - 60 \tau^4, -44 \tau^2 + \frac{280}{3} \tau^3 - 48 \tau^4 - \frac{1}{3}, -\frac{1}{3} \sqrt{2} (18 \tau^4 - 28 \tau^3 + 6 \tau^2 - 1) \right]$$

$$"vv", \left[1 - 52 \tau^2 + \frac{296}{3} \tau^3 - 48 \tau^4, \frac{5}{3} - 56 \tau^2 + \frac{352}{3} \tau^3 - 60 \tau^4, \frac{1}{3} \sqrt{2} (18 \tau^4 - 44 \tau^3 + 30 \tau^2 - 5) \right]$$

$$"dv", \left[12 \tau^4 - 24 \tau^3 + 12 \tau^2 - 2, -12 \tau^4 + 24 \tau^3 - 12 \tau^2 + 2, 2 \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) \right]$$

$$"y", \left[-40 \tau^4 + \frac{224}{3} \tau^3 - 32 \tau^2 - 2, -\frac{2}{3} + 48 \tau^2 - \frac{256}{3} \tau^3 + 40 \tau^4, \frac{2}{3} \sqrt{2} (60 \tau^4 - 120 \tau^3 + 60 \tau^2 + 1) \right]$$

$$"w", \left[-\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 - 62 \tau + 33) \tau^2, -\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 + 2 \tau + 1) (\tau - 1)^2, -\frac{16}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1) (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) \right]$$

$$"b=\det[v,dv,y]", \frac{32}{9} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) \tau^2 (\tau - 1)^2$$

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 "On (directed Edge ", 3, " points ", 1, " --> ", 4

$$1, 92 \lambda_2^2 \lambda_4^2 \lambda_1 - 92 \lambda_1^2 \lambda_4^2 \lambda_2 - 2 \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_4 + 2 \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) - \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 + \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_4 - 2 \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 + 2 \lambda_4^3 (4 - 3 \lambda_4) \lambda_2$$

$$2, 92 \lambda_2^2 \lambda_4^2 \lambda_1 + 92 \lambda_1^2 \lambda_4^2 \lambda_2 + 2 \lambda_1^3 (4 - 3 \lambda_1) \lambda_4 + 2 \lambda_2^3 (4 - 3 \lambda_2) \lambda_4 + 2 \lambda_4^3 (6 \lambda_4^2 - 15 \lambda_4$$

$$\begin{aligned}
& + 10) + \frac{2}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 + \frac{2}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_2 \\
& 3, 92 \lambda_1^2 \lambda_2^2 \lambda_4 \sqrt{2} + \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) \sqrt{2} + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 \sqrt{2} - \frac{2}{3} \lambda_1^3 (4 \\
& - 3 \lambda_1) \lambda_4 \sqrt{2} + \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) \sqrt{2} + \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 \sqrt{2} - \frac{2}{3} \lambda_2^3 (4 \\
& - 3 \lambda_2) \lambda_4 \sqrt{2} - \lambda_4^3 (6 \lambda_4^2 - 15 \lambda_4 + 10) \sqrt{2} + \frac{2}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 \sqrt{2} + \frac{2}{3} \lambda_4^3 (4 \\
& - 3 \lambda_4) \lambda_2 \sqrt{2} \\
& "v", \left[3 - 64 \tau^2 + \frac{368}{3} \tau^3 - 60 \tau^4, 44 \tau^2 - \frac{280}{3} \tau^3 + 48 \tau^4 + \frac{1}{3}, -\frac{1}{3} \sqrt{2} (18 \tau^4 - 28 \tau^3 \right. \\
& \left. + 6 \tau^2 - 1) \right] \\
& "vv", \left[1 - 52 \tau^2 + \frac{296}{3} \tau^3 - 48 \tau^4, 56 \tau^2 - \frac{352}{3} \tau^3 + 60 \tau^4 - \frac{5}{3}, \frac{1}{3} \sqrt{2} (18 \tau^4 - 44 \tau^3 \right. \\
& \left. + 30 \tau^2 - 5) \right] \\
& "dv", \left[12 \tau^4 - 24 \tau^3 + 12 \tau^2 - 2, 12 \tau^4 - 24 \tau^3 + 12 \tau^2 - 2, 2 \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) \right] \\
& "y", \left[-40 \tau^4 + \frac{224}{3} \tau^3 - 32 \tau^2 - 2, \frac{2}{3} - 48 \tau^2 + \frac{256}{3} \tau^3 - 40 \tau^4, \frac{2}{3} \sqrt{2} (60 \tau^4 - 120 \tau^3 \right. \\
& \left. + 60 \tau^2 + 1) \right] \\
& "w", \left[\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 - 62 \tau + 33) \tau^2, -\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 \right. \\
& \left. + 6 \tau^2 - 1) (30 \tau^2 + 2 \tau + 1) (\tau - 1)^2, \frac{16}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1) (6 \tau^4 - 12 \tau^3 \right. \\
& \left. + 6 \tau^2 - 1) \right] \\
& "b=\det[v,dv,y]", -\frac{32}{9} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau \\
& - 63) \tau^2 (\tau - 1)^2
\end{aligned}$$

"On (directed Edge ", 4, " points ", 2, " --> ", 3

$$\begin{aligned}
& 1, 92 \lambda_2^2 \lambda_3^2 \lambda_1 - 92 \lambda_1^2 \lambda_3^2 \lambda_2 - 2 \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 - \frac{2}{3} \lambda_1^3 (4 \\
& - 3 \lambda_1) \lambda_3 + 2 \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) - \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 + \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 - 2 \\
& \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 + 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_2 \\
& 2, -92 \lambda_2^2 \lambda_3^2 \lambda_1 - 92 \lambda_1^2 \lambda_3^2 \lambda_2 - 2 \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 - 2 \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 - 2 \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 \\
& + 10) - \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 - \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_2
\end{aligned}$$

$$3, 92 \lambda_1^2 \lambda_2^2 \lambda_3 \sqrt{2} + \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) \sqrt{2} + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 \sqrt{2} - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 \sqrt{2} + \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) \sqrt{2} + \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 \sqrt{2} - \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 \sqrt{2} - \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 + 10) \sqrt{2} + \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 \sqrt{2} + \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_2 \sqrt{2}$$

$$"v", \left[-3 + 64 \tau^2 - \frac{368}{3} \tau^3 + 60 \tau^4, -44 \tau^2 + \frac{280}{3} \tau^3 - 48 \tau^4 - \frac{1}{3}, -\frac{1}{3} \sqrt{2} (18 \tau^4 - 28 \tau^3 + 6 \tau^2 - 1) \right]$$

$$"vv", \left[-1 + 52 \tau^2 - \frac{296}{3} \tau^3 + 48 \tau^4, \frac{5}{3} - 56 \tau^2 + \frac{352}{3} \tau^3 - 60 \tau^4, \frac{1}{3} \sqrt{2} (18 \tau^4 - 44 \tau^3 + 30 \tau^2 - 5) \right]$$

$$"dv", \left[-12 \tau^4 + 24 \tau^3 - 12 \tau^2 + 2, -12 \tau^4 + 24 \tau^3 - 12 \tau^2 + 2, 2 \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) \right]$$

$$"y", \left[40 \tau^4 - \frac{224}{3} \tau^3 + 32 \tau^2 + 2, -\frac{2}{3} + 48 \tau^2 - \frac{256}{3} \tau^3 + 40 \tau^4, \frac{2}{3} \sqrt{2} (60 \tau^4 - 120 \tau^3 + 60 \tau^2 + 1) \right]$$

$$"w", \left[-\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 - 62 \tau + 33) \tau^2, \frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 + 2 \tau + 1) (\tau - 1)^2, \frac{16}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1) (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) \right]$$

$$"b=\det[v,dv,y]", -\frac{32}{9} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) \tau^2 (\tau - 1)^2$$

"On (directed Edge ", 5, " points ", 2, " --> ", 4

$$1, 92 \lambda_2^2 \lambda_4^2 \lambda_1 - 92 \lambda_1^2 \lambda_4^2 \lambda_2 - 2 \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_4 + 2 \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) - \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 + \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_4 - 2 \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 + 2 \lambda_4^3 (4 - 3 \lambda_4) \lambda_2$$

$$2, 92 \lambda_2^2 \lambda_4^2 \lambda_1 + 92 \lambda_1^2 \lambda_4^2 \lambda_2 + 2 \lambda_1^3 (4 - 3 \lambda_1) \lambda_4 + 2 \lambda_2^3 (4 - 3 \lambda_2) \lambda_4 + 2 \lambda_4^3 (6 \lambda_4^2 - 15 \lambda_4 + 10) + \frac{2}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 + \frac{2}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_2$$

$$\begin{aligned}
& 3, 92 \lambda_1^2 \lambda_2^2 \lambda_4 \sqrt{2} + \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) \sqrt{2} + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 \sqrt{2} - \frac{2}{3} \lambda_1^3 (4 \\
& \quad - 3 \lambda_1) \lambda_4 \sqrt{2} + \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) \sqrt{2} + \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 \sqrt{2} - \frac{2}{3} \lambda_2^3 (4 \\
& \quad - 3 \lambda_2) \lambda_4 \sqrt{2} - \lambda_4^3 (6 \lambda_4^2 - 15 \lambda_4 + 10) \sqrt{2} + \frac{2}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 \sqrt{2} + \frac{2}{3} \lambda_4^3 (4 \\
& \quad - 3 \lambda_4) \lambda_2 \sqrt{2} \\
\text{"v"}, & \left[-3 + 64 \tau^2 - \frac{368}{3} \tau^3 + 60 \tau^4, 44 \tau^2 - \frac{280}{3} \tau^3 + 48 \tau^4 + \frac{1}{3}, -\frac{1}{3} \sqrt{2} (18 \tau^4 - 28 \tau^3 \right. \\
& \quad \left. + 6 \tau^2 - 1) \right] \\
\text{"vv"}, & \left[-1 + 52 \tau^2 - \frac{296}{3} \tau^3 + 48 \tau^4, 56 \tau^2 - \frac{352}{3} \tau^3 + 60 \tau^4 - \frac{5}{3}, \frac{1}{3} \sqrt{2} (18 \tau^4 - 44 \tau^3 \right. \\
& \quad \left. + 30 \tau^2 - 5) \right] \\
\text{"dv"}, & \left[-12 \tau^4 + 24 \tau^3 - 12 \tau^2 + 2, 12 \tau^4 - 24 \tau^3 + 12 \tau^2 - 2, 2 \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) \right] \\
\text{"y"}, & \left[40 \tau^4 - \frac{224}{3} \tau^3 + 32 \tau^2 + 2, \frac{2}{3} - 48 \tau^2 + \frac{256}{3} \tau^3 - 40 \tau^4, \frac{2}{3} \sqrt{2} (60 \tau^4 - 120 \tau^3 \right. \\
& \quad \left. + 60 \tau^2 + 1) \right] \\
\text{"w"}, & \left[\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 - 62 \tau + 33) \tau^2, \frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 \right. \\
& \quad \left. - 1) (30 \tau^2 + 2 \tau + 1) (\tau - 1)^2, -\frac{16}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1) (6 \tau^4 - 12 \tau^3 + 6 \tau^2 \right. \\
& \quad \left. - 1) \right] \\
\text{"b=det[v,dv,y"]}, & \frac{32}{9} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau \\
& \quad - 63) \tau^2 (\tau - 1)^2 \\
& \text{"} \frac{\text{"On (directed Edge ", 6, " points ", 3, " --> ", 4}}{
\end{aligned}$$

$$\begin{aligned}
& 1, -92 \lambda_1^2 \lambda_4^2 \lambda_3 - 92 \lambda_1^2 \lambda_3^2 \lambda_4 - 2 \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 - \frac{2}{3} \lambda_1^3 (4 \\
& \quad - 3 \lambda_1) \lambda_4 - 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 - 2 \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 \\
& 2, 92 \lambda_1^2 \lambda_4^2 \lambda_3 - 92 \lambda_1^2 \lambda_3^2 \lambda_4 - 2 \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 + 2 \lambda_1^3 (4 - 3 \lambda_1) \lambda_4 - 2 \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 \\
& \quad + 10) - \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 + \frac{4}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_4 + 2 \lambda_4^3 (6 \lambda_4^2 - 15 \lambda_4 + 10) + \frac{2}{3} \\
& \quad \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 - \frac{4}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_3 \\
& 3, -92 \lambda_3^2 \lambda_4^2 \lambda_1 \sqrt{2} + \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) \sqrt{2} - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 \sqrt{2} - \frac{2}{3} \lambda_1^3 (4
\end{aligned}$$

$$\begin{aligned}
& -3\lambda_1)\lambda_4\sqrt{2}-\lambda_3^3\left(6\lambda_3^2-15\lambda_3+10\right)\sqrt{2}+\frac{2}{3}\lambda_3^3\left(4-3\lambda_3\right)\lambda_1\sqrt{2}-\frac{4}{3}\lambda_3^3\left(4\right. \\
& \left.-3\lambda_3\right)\lambda_4\sqrt{2}-\lambda_4^3\left(6\lambda_4^2-15\lambda_4+10\right)\sqrt{2}+\frac{2}{3}\lambda_4^3\left(4-3\lambda_4\right)\lambda_1\sqrt{2}-\frac{4}{3}\lambda_4^3\left(4\right. \\
& \left.-3\lambda_4\right)\lambda_3\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
\text{"v"}, & \left[12\tau^4-24\tau^3+12\tau^2-2,\frac{4}{3}\left(2\tau-1\right)\left(2\tau^2-2\tau-1\right),-\frac{2}{3}\sqrt{2}\left(3\tau-2\right.\right. \\
& \left.\left.+\sqrt{2}\right)\left(3\tau-1+\sqrt{2}\right)\left(-3\tau+2+\sqrt{2}\right)\left(-3\tau+1+\sqrt{2}\right)\right]
\end{aligned}$$

$$\begin{aligned}
\text{"vv"}, & \left[-12\tau^4+24\tau^3-12\tau^2+2,\frac{4}{3}\left(2\tau-1\right)\left(2\tau^2-2\tau-1\right),-\frac{2}{3}\sqrt{2}\left(3\tau-2\right.\right. \\
& \left.\left.+\sqrt{2}\right)\left(3\tau-1+\sqrt{2}\right)\left(-3\tau+2+\sqrt{2}\right)\left(-3\tau+1+\sqrt{2}\right)\right]
\end{aligned}$$

$$\text{"dv"}, \left[-24\tau^4+48\tau^3-24\tau^2+4,0,0\right]$$

$$\text{"y"}, \left[0,-\frac{4}{3}-80\tau^2+160\tau^3-80\tau^4,-\frac{4}{3}\sqrt{2}\left(2\tau^2-2\tau-1\right)\left(2\tau-1\right)\right]$$

$$\begin{aligned}
\text{"w"}, & \left[0,-\frac{16}{3}\left(6\tau^4-12\tau^3+6\tau^2-1\right)\sqrt{2}\left(2\tau^2-2\tau-1\right)\left(2\tau-1\right),\frac{16}{3}\left(6\tau^4-12\tau^3\right.\right. \\
& \left.\left.+6\tau^2-1\right)\left(60\tau^4-120\tau^3+60\tau^2+1\right)\right]
\end{aligned}$$

$$\begin{aligned}
\text{"b=det[v,dv,y]}, & -\frac{32}{9}\sqrt{2}\left(6\tau^4-12\tau^3+6\tau^2-1\right)\left(4860\tau^4-9720\tau^3+4892\tau^2-32\tau\right. \\
& \left.-63\right)\tau^2\left(\tau-1\right)^2
\end{aligned}$$

"_____\"

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"_____\"

"_____"

"Cofactors of GCD(w_1,w_2,w_3)"

"W[" , 1, ",on points " , 1, "-->" , 2, "]"

"Polynomials"

$$-\frac{16}{3}\left(6\tau^4-12\tau^3+6\tau^2-1\right)\sqrt{2}\left(2\tau^2-2\tau-1\right)\left(2\tau-1\right)$$

0

$$-\frac{16}{3}\left(6\tau^4-12\tau^3+6\tau^2-1\right)\left(60\tau^4-120\tau^3+60\tau^2+1\right)$$

$$\text{" GCD over double edge " , 1, "=" , }\tau^4-2\tau^3+\tau^2-\frac{1}{6}$$

$$\text{"qw", 1, 1, "=" , }\frac{15}{4864}\tau\sqrt{2}-\frac{255}{19456}\sqrt{2}-\frac{225}{4864}\sqrt{2}\tau^3+\frac{675}{9728}\tau^2\sqrt{2}$$

$$\text{"qw", 1, 2, "=" , }0$$

$$\text{"qw", 1, 3, "=" , }-\frac{49}{9728}-\frac{15}{2432}\tau+\frac{15}{2432}\tau^2$$

"W[", 2, ",on points ", 1, "-->", 3, "]"

"Polynomials"

$$\begin{aligned}
& -\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 - 62 \tau + 33) \tau^2 \\
& -\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 + 2 \tau + 1) (\tau - 1)^2 \\
& -\frac{16}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1) (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1)
\end{aligned}$$

$$\text{" GCD over double edge "}, 2, "=" , \tau^4 - 2 \tau^3 + \tau^2 - \frac{1}{6}$$

$$\text{"qw", 2, 1, "=" , } -\frac{345}{19456} \tau \sqrt{2} + \frac{637}{38912} \sqrt{2} + \frac{225}{4864} \sqrt{2} \tau^3 - \frac{645}{9728} \tau^2 \sqrt{2}$$

$$\text{"qw", 2, 2, "=" , } -\frac{833}{38912} \sqrt{2} + \frac{225}{19456} \tau \sqrt{2} + \frac{705}{9728} \tau^2 \sqrt{2} - \frac{225}{4864} \sqrt{2} \tau^3$$

$$\text{"qw", 2, 3, "=" , } \frac{225}{19456} - \frac{225}{9728} \tau$$

"W[", 3, ",on points ", 1, "-->", 4, "]"

"Polynomials"

$$\begin{aligned}
& \frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 - 62 \tau + 33) \tau^2 \\
& -\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 + 2 \tau + 1) (\tau - 1)^2 \\
& \frac{16}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1) (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1)
\end{aligned}$$

$$\text{" GCD over double edge "}, 3, "=" , \tau^4 - 2 \tau^3 + \tau^2 - \frac{1}{6}$$

$$\text{"qw", 3, 1, "=" , } \frac{345}{19456} \tau \sqrt{2} - \frac{637}{38912} \sqrt{2} - \frac{225}{4864} \sqrt{2} \tau^3 + \frac{645}{9728} \tau^2 \sqrt{2}$$

$$\text{"qw", 3, 2, "=" , } -\frac{833}{38912} \sqrt{2} + \frac{225}{19456} \tau \sqrt{2} + \frac{705}{9728} \tau^2 \sqrt{2} - \frac{225}{4864} \sqrt{2} \tau^3$$

$$\text{"qw", 3, 3, "=" , } -\frac{225}{19456} + \frac{225}{9728} \tau$$

"W[", 4, ",on points ", 2, "-->", 3, "]"

"Polynomials"

$$\begin{aligned}
& -\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 - 62 \tau + 33) \tau^2 \\
& \frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 + 2 \tau + 1) (\tau - 1)^2 \\
& \frac{16}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1) (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1)
\end{aligned}$$

$$\text{" GCD over double edge "}, 4, "=" , \tau^4 - 2 \tau^3 + \tau^2 - \frac{1}{6}$$

$$\text{"qw", 4, 1, "=" , } -\frac{345}{19456} \tau \sqrt{2} + \frac{637}{38912} \sqrt{2} + \frac{225}{4864} \sqrt{2} \tau^3 - \frac{645}{9728} \tau^2 \sqrt{2}$$

$$\text{"qw", 4, 2, "="}, \frac{833}{38912} \sqrt{2} - \frac{225}{19456} \tau \sqrt{2} - \frac{705}{9728} \tau^2 \sqrt{2} + \frac{225}{4864} \sqrt{2} \tau^3$$

$$\text{"qw", 4, 3, "="}, -\frac{225}{19456} + \frac{225}{9728} \tau$$

$$\text{"W["}, 5, \text{"}, \text{on points "}, 2, \text{"-->"}, 4, \text{""]"}$$

"Polynomials"

$$\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 - 62 \tau + 33) \tau^2$$

$$\frac{16}{3} \sqrt{2} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (30 \tau^2 + 2 \tau + 1) (\tau - 1)^2$$

$$- \frac{16}{3} (2 \tau - 1) (2 \tau^2 - 2 \tau - 1) (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1)$$

$$\text{" GCD over double edge "}, 5, \text{"="}, \tau^4 - 2 \tau^3 + \tau^2 - \frac{1}{6}$$

$$\text{"qw", 5, 1, "="}, \frac{345}{19456} \tau \sqrt{2} - \frac{637}{38912} \sqrt{2} - \frac{225}{4864} \sqrt{2} \tau^3 + \frac{645}{9728} \tau^2 \sqrt{2}$$

$$\text{"qw", 5, 2, "="}, \frac{833}{38912} \sqrt{2} - \frac{225}{19456} \tau \sqrt{2} - \frac{705}{9728} \tau^2 \sqrt{2} + \frac{225}{4864} \sqrt{2} \tau^3$$

$$\text{"qw", 5, 3, "="}, \frac{225}{19456} - \frac{225}{9728} \tau$$

$$\text{"W["}, 6, \text{"}, \text{on points "}, 3, \text{"-->"}, 4, \text{""]"}$$

"Polynomials"

0

$$- \frac{16}{3} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) \sqrt{2} (2 \tau^2 - 2 \tau - 1) (2 \tau - 1)$$

$$\frac{16}{3} (6 \tau^4 - 12 \tau^3 + 6 \tau^2 - 1) (60 \tau^4 - 120 \tau^3 + 60 \tau^2 + 1)$$

$$\text{" GCD over double edge "}, 6, \text{"="}, \tau^4 - 2 \tau^3 + \tau^2 - \frac{1}{6}$$

$$\text{"qw", 6, 1, "="}, 0$$

$$\text{"qw", 6, 2, "="}, \frac{15}{4864} \tau \sqrt{2} - \frac{255}{19456} \sqrt{2} - \frac{225}{4864} \sqrt{2} \tau^3 + \frac{675}{9728} \tau^2 \sqrt{2}$$

$$\text{"qw", 6, 3, "="}, \frac{49}{9728} + \frac{15}{2432} \tau - \frac{15}{2432} \tau^2$$

" _____ \"

_____ "

" _____ \"

_____ "

"Z polynomials along double edges"

"Double edge ", 1, " ", 1, "-->", 2

$$- \frac{64}{3} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) \sqrt{2}$$

"Zq["], 1, 1, " resp. ZqL["], 1, 1, "]"

$$\begin{aligned}
& -\frac{5}{152} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) (30 \tau^2 - 30 \tau - 17) (2 \tau - 1) \\
& -\frac{5}{304} (4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63) (30 \lambda_2^2 - 30 \lambda_2 - 17) (-1 + 2 \lambda_2) \\
& + \frac{5}{304} (4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63) (30 \lambda_1^2 - 30 \lambda_1 - 17) (2 \lambda_1 - 1) \\
& \quad \text{"Zq["}, 1, 2, \text{" resp. ZqL["}, 1, 2, \text{"}]"} \\
& \quad 0 \\
& \quad 0 \\
& \quad \text{"Zq["}, 1, 3, \text{" resp. ZqL["}, 1, 3, \text{"}]"} \\
& -\frac{64}{3} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) \sqrt{2} \left(-\frac{49}{9728} - \frac{15}{2432} \tau + \frac{15}{2432} \tau^2 \right) \\
& -\frac{32}{3} (4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63) \sqrt{2} \left(-\frac{49}{9728} - \frac{15}{2432} \lambda_2 + \frac{15}{2432} \lambda_2^2 \right) \\
& -\frac{32}{3} (4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63) \sqrt{2} \left(-\frac{49}{9728} - \frac{15}{2432} \lambda_1 + \frac{15}{2432} \lambda_1^2 \right) \\
& \quad \text{"Double edge "}, 2, \text{" "}, 1, \text{"-->"}, 3 \\
& \quad \frac{64}{3} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) \sqrt{2} \\
& \quad \text{"Zq["}, 2, 1, \text{" resp. ZqL["}, 2, 1, \text{"}]"} \\
& \quad \frac{1}{912} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) (30 \tau - 13) (60 \tau^2 - 60 \tau - 49) \\
& -\frac{1}{1824} (4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63) (-17 + 30 \lambda_3) (60 \lambda_3^2 - 60 \lambda_3 - 49) \\
& + \frac{1}{1824} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63) (30 \lambda_1 - 13) (60 \lambda_1^2 - 60 \lambda_1 - 49) \\
& \quad \text{"Zq["}, 2, 2, \text{" resp. ZqL["}, 2, 2, \text{"}]"} \\
& -\frac{1}{912} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) (30 \tau - 17) (60 \tau^2 - 60 \tau - 49) \\
& -\frac{1}{1824} (4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63) (-13 + 30 \lambda_3) (60 \lambda_3^2 - 60 \lambda_3 - 49) \\
& -\frac{1}{1824} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63) (30 \lambda_1 - 17) (60 \lambda_1^2 - 60 \lambda_1 - 49) \\
& \quad \text{"Zq["}, 2, 3, \text{" resp. ZqL["}, 2, 3, \text{"}]"} \\
& \quad \frac{64}{3} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \tau \right) \\
& \frac{32}{3} (4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \lambda_3 \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{32}{3} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63 \right) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \lambda_1 \right) \\
& \quad \text{"Double edge ", 3, " ", 1, "-->", 4} \\
& - \frac{64}{3} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \sqrt{2} \\
& \quad \text{"Zq[", 3, 1, " resp. ZqL[", 3, 1, "]" } \\
& - \frac{1}{912} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) (30 \tau - 13) (60 \tau^2 - 60 \tau - 49) \\
& - \frac{1}{1824} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) (-17 + 30 \lambda_4) (60 \lambda_4^2 - 60 \lambda_4 - 49) \\
& + \frac{1}{1824} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) (30 \lambda_1 - 13) (60 \lambda_1^2 - 60 \lambda_1 - 49) \\
& \quad \text{"Zq[", 3, 2, " resp. ZqL[", 3, 2, "]" } \\
& - \frac{1}{912} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) (30 \tau - 17) (60 \tau^2 - 60 \tau - 49) \\
& - \frac{1}{1824} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) (-13 + 30 \lambda_4) (60 \lambda_4^2 - 60 \lambda_4 - 49) \\
& + \frac{1}{1824} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) (30 \lambda_1 - 17) (60 \lambda_1^2 - 60 \lambda_1 - 49) \\
& \quad \text{"Zq[", 3, 3, " resp. ZqL[", 3, 3, "]" } \\
& - \frac{64}{3} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \tau \right) \\
& - \frac{32}{3} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \lambda_4 \right) \\
& - \frac{32}{3} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \lambda_1 \right) \\
& \quad \text{"Double edge ", 4, " ", 2, "-->", 3} \\
& - \frac{64}{3} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \sqrt{2} \\
& \quad \text{"Zq[", 4, 1, " resp. ZqL[", 4, 1, "]" } \\
& - \frac{1}{912} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) (30 \tau - 13) (60 \tau^2 - 60 \tau - 49) \\
& - \frac{1}{1824} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) (-17 + 30 \lambda_3) (60 \lambda_3^2 - 60 \lambda_3 - 49) \\
& - \frac{1}{1824} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63 \right) (30 \lambda_2 - 13) (60 \lambda_2^2 - 60 \lambda_2 - 49) \\
& \quad \text{"Zq[", 4, 2, " resp. ZqL[", 4, 2, "]" } \\
& - \frac{1}{912} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) (30 \tau - 17) (60 \tau^2 - 60 \tau - 49)
\end{aligned}$$

$$\begin{aligned} & \frac{1}{1824} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \left(-13 + 30 \lambda_3 \right) \left(60 \lambda_3^2 - 60 \lambda_3 - 49 \right) \\ & - \frac{1}{1824} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63 \right) \left(30 \lambda_2 - 17 \right) \left(60 \lambda_2^2 - 60 \lambda_2 - 49 \right) \end{aligned}$$

"Zq[" , 4, 3, " resp. ZqL[" , 4, 3, "]"

$$\begin{aligned} & - \frac{64}{3} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \tau \right) \\ & - \frac{32}{3} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \lambda_3 \right) \\ & - \frac{32}{3} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63 \right) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \lambda_2 \right) \end{aligned}$$

"Double edge " , 5, " " , 2, "-->" , 4

$$\frac{64}{3} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \sqrt{2}$$

"Zq[" , 5, 1, " resp. ZqL[" , 5, 1, "]"

$$\begin{aligned} & - \frac{1}{912} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \left(30 \tau - 13 \right) \left(60 \tau^2 - 60 \tau - 49 \right) \\ & \frac{1}{1824} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \left(-17 + 30 \lambda_4 \right) \left(60 \lambda_4^2 - 60 \lambda_4 - 49 \right) \\ & - \frac{1}{1824} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \left(30 \lambda_2 - 13 \right) \left(60 \lambda_2^2 - 60 \lambda_2 - 49 \right) \end{aligned}$$

"Zq[" , 5, 2, " resp. ZqL[" , 5, 2, "]"

$$\begin{aligned} & \frac{1}{912} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \left(30 \tau - 17 \right) \left(60 \tau^2 - 60 \tau - 49 \right) \\ & - \frac{1}{1824} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \left(-13 + 30 \lambda_4 \right) \left(60 \lambda_4^2 - 60 \lambda_4 - 49 \right) \\ & + \frac{1}{1824} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \left(30 \lambda_2 - 17 \right) \left(60 \lambda_2^2 - 60 \lambda_2 - 49 \right) \end{aligned}$$

"Zq[" , 5, 3, " resp. ZqL[" , 5, 3, "]"

$$\begin{aligned} & \frac{64}{3} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \tau \right) \\ & \frac{32}{3} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \lambda_4 \right) \\ & + \frac{32}{3} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \lambda_2 \right) \end{aligned}$$

"Double edge " , 6, " " , 3, "-->" , 4

$$- \frac{64}{3} \left(4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63 \right) \sqrt{2}$$

"Zq[" , 6, 1, " resp. ZqL[" , 6, 1, "]"

0
0

"Zq[" , 6, 2, " resp. ZqL[" , 6, 2, "]"

$$\begin{aligned} & -\frac{5}{152} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) (30 \tau^2 - 30 \tau - 17) (2 \tau - 1) \\ & -\frac{5}{304} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63) (30 \lambda_4^2 - 30 \lambda_4 - 17) (2 \lambda_4 - 1) \\ & + \frac{5}{304} (4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63) (30 \lambda_3^2 - 30 \lambda_3 - 17) (2 \lambda_3 - 1) \end{aligned}$$

"Zq[" , 6, 3, " resp. ZqL[" , 6, 3, "]"

$$\begin{aligned} & -\frac{64}{3} (4860 \tau^4 - 9720 \tau^3 + 4892 \tau^2 - 32 \tau - 63) \sqrt{2} \left(\frac{49}{9728} + \frac{15}{2432} \tau - \frac{15}{2432} \tau^2 \right) \\ & -\frac{32}{3} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63) \sqrt{2} \left(\frac{49}{9728} + \frac{15}{2432} \lambda_4 - \frac{15}{2432} \lambda_4^2 \right) \\ & -\frac{32}{3} (4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63) \sqrt{2} \left(\frac{49}{9728} + \frac{15}{2432} \lambda_3 - \frac{15}{2432} \lambda_3^2 \right) \end{aligned}$$

"Triangle " , 1, [1, 2, 3]

"Spline over triangle " , 1

$$\begin{aligned} & -\left(\frac{1}{1824} (4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63) (-17 + 30 \lambda_3) (60 \lambda_3^2 - 60 \lambda_3 - 49) \right. \\ & \quad - \frac{1}{1824} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63) (30 \lambda_2 - 13) (60 \lambda_2^2 - 60 \lambda_2 \\ & \quad - 49) \left. \right) \lambda_2^2 \lambda_3^2 \lambda_1 - \left(-\frac{1}{1824} (4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63) (-17 \right. \\ & \quad + 30 \lambda_3) (60 \lambda_3^2 - 60 \lambda_3 - 49) + \frac{1}{1824} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 \\ & \quad - 63) (30 \lambda_1 - 13) (60 \lambda_1^2 - 60 \lambda_1 - 49) \left. \right) \lambda_1^2 \lambda_3^2 \lambda_2 - \left(-\frac{5}{304} (4860 \lambda_1^4 - 9720 \lambda_1^3 \right. \\ & \quad + 4892 \lambda_1^2 - 32 \lambda_1 - 63) (30 \lambda_2^2 - 30 \lambda_2 - 17) (-1 + 2 \lambda_2) + \frac{5}{304} (4860 \lambda_2^4 - 9720 \lambda_2^3 \\ & \quad + 4892 \lambda_2^2 - 32 \lambda_2 - 63) (30 \lambda_1^2 - 30 \lambda_1 - 17) (2 \lambda_1 - 1) \left. \right) \lambda_1^2 \lambda_2^2 \lambda_3 + 92 \lambda_2^2 \lambda_3^2 \lambda_1 - 92 \\ & \quad \lambda_1^2 \lambda_3^2 \lambda_2 - 2 \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 + 10) + \frac{4}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_2 - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 + 2 \\ & \quad \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) - \frac{4}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_1 + \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 - 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 \\ & \quad + 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_2 \\ & -\left(\frac{1}{1824} (4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63) (-13 + 30 \lambda_3) (60 \lambda_3^2 - 60 \lambda_3 - 49) \right. \\ & \quad - \frac{1}{1824} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63) (30 \lambda_2 - 17) (60 \lambda_2^2 - 60 \lambda_2 \end{aligned}$$

$$\begin{aligned}
& -49) \Big) \lambda_2^2 \lambda_3^2 \lambda_1 - \left(\frac{1}{1824} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \left(-13 \right. \right. \\
& \left. \left. + 30 \lambda_3 \right) \left(60 \lambda_3^2 - 60 \lambda_3 - 49 \right) - \frac{1}{1824} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 \right. \right. \\
& \left. \left. - 63 \right) \left(30 \lambda_1 - 17 \right) \left(60 \lambda_1^2 - 60 \lambda_1 - 49 \right) \Big) \lambda_1^2 \lambda_3^2 \lambda_2 - 92 \lambda_2^2 \lambda_3^2 \lambda_1 - 92 \lambda_1^2 \lambda_3^2 \lambda_2 - 2 \lambda_1^3 \left(4 \right. \\
& \left. - 3 \lambda_1 \right) \lambda_3 - 2 \lambda_2^3 \left(4 - 3 \lambda_2 \right) \lambda_3 - 2 \lambda_3^3 \left(6 \lambda_3^2 - 15 \lambda_3 + 10 \right) - \frac{2}{3} \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_1 - \frac{2}{3} \\
& \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_2 \\
& - \left(-\frac{32}{3} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \lambda_3 \right) \right. \\
& \left. - \frac{32}{3} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63 \right) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \lambda_2 \right) \right) \lambda_2^2 \\
& \lambda_3^2 \lambda_1 - \left(\frac{32}{3} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \lambda_3 \right) \right. \\
& \left. + \frac{32}{3} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63 \right) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \lambda_1 \right) \right) \lambda_1^2 \\
& \lambda_3^2 \lambda_2 - \left(-\frac{32}{3} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \sqrt{2} \left(-\frac{49}{9728} - \frac{15}{2432} \lambda_2 \right. \right. \\
& \left. \left. + \frac{15}{2432} \lambda_2^2 \right) - \frac{32}{3} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \sqrt{2} \left(-\frac{49}{9728} \right. \right. \\
& \left. \left. - \frac{15}{2432} \lambda_1 + \frac{15}{2432} \lambda_1^2 \right) \right) \lambda_1^2 \lambda_2^2 \lambda_3 + 92 \lambda_1^2 \lambda_2^2 \lambda_3 \sqrt{2} + \lambda_1^3 \left(6 \lambda_1^2 - 15 \lambda_1 + 10 \right) \sqrt{2} \\
& + \frac{4}{3} \lambda_1^3 \left(4 - 3 \lambda_1 \right) \lambda_2 \sqrt{2} - \frac{2}{3} \lambda_1^3 \left(4 - 3 \lambda_1 \right) \lambda_3 \sqrt{2} + \lambda_2^3 \left(6 \lambda_2^2 - 15 \lambda_2 + 10 \right) \sqrt{2} \\
& + \frac{4}{3} \lambda_2^3 \left(4 - 3 \lambda_2 \right) \lambda_1 \sqrt{2} - \frac{2}{3} \lambda_2^3 \left(4 - 3 \lambda_2 \right) \lambda_3 \sqrt{2} - \lambda_3^3 \left(6 \lambda_3^2 - 15 \lambda_3 + 10 \right) \sqrt{2} \\
& + \frac{2}{3} \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_1 \sqrt{2} + \frac{2}{3} \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_2 \sqrt{2}
\end{aligned}$$

"Triangle ", 2, [1, 2, 4]

"Spline over triangle ", 2

$$\begin{aligned}
& - \left(\frac{1}{1824} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \left(-17 + 30 \lambda_4 \right) \left(60 \lambda_4^2 - 60 \lambda_4 - 49 \right) \right. \\
& \left. - \frac{1}{1824} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \left(30 \lambda_2 - 13 \right) \left(60 \lambda_2^2 - 60 \lambda_2 \right. \right. \\
& \left. \left. - 49 \right) \right) \lambda_2^2 \lambda_4^2 \lambda_1 - \left(-\frac{1}{1824} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \left(-17 \right. \right. \\
& \left. \left. + 30 \lambda_4 \right) \left(60 \lambda_4^2 - 60 \lambda_4 - 49 \right) + \frac{1}{1824} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 \right. \right. \\
& \left. \left. - 63 \right) \left(30 \lambda_1 - 13 \right) \left(60 \lambda_1^2 - 60 \lambda_1 - 49 \right) \right) \lambda_1^2 \lambda_4^2 \lambda_2 - \left(-\frac{5}{304} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 \right. \right. \\
& \left. \left. + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \left(30 \lambda_2^2 - 30 \lambda_2 - 17 \right) \left(-1 + 2 \lambda_2 \right) + \frac{5}{304} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \Big) \Big(30 \lambda_1^2 - 30 \lambda_1 - 17 \Big) \Big(2 \lambda_1 - 1 \Big) \Big) \lambda_1^2 \lambda_2^2 \lambda_4 + 92 \lambda_2^2 \lambda_4^2 \lambda_1 - 92 \\
& \lambda_1^2 \lambda_4^2 \lambda_2 - 2 \lambda_1^3 \Big(6 \lambda_1^2 - 15 \lambda_1 + 10 \Big) + \frac{4}{3} \lambda_1^3 \Big(4 - 3 \lambda_1 \Big) \lambda_2 - \frac{2}{3} \lambda_1^3 \Big(4 - 3 \lambda_1 \Big) \lambda_4 + 2 \\
& \lambda_2^3 \Big(6 \lambda_2^2 - 15 \lambda_2 + 10 \Big) - \frac{4}{3} \lambda_2^3 \Big(4 - 3 \lambda_2 \Big) \lambda_1 + \frac{2}{3} \lambda_2^3 \Big(4 - 3 \lambda_2 \Big) \lambda_4 - 2 \lambda_4^3 \Big(4 - 3 \lambda_4 \Big) \lambda_1 \\
& + 2 \lambda_4^3 \Big(4 - 3 \lambda_4 \Big) \lambda_2 \\
& - \Big(-\frac{1}{1824} \Big(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \Big) \Big(-13 + 30 \lambda_4 \Big) \Big(60 \lambda_4^2 - 60 \lambda_4 \\
& - 49 \Big) + \frac{1}{1824} \Big(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \Big) \Big(30 \lambda_2 - 17 \Big) \Big(60 \lambda_2^2 \\
& - 60 \lambda_2 - 49 \Big) \Big) \lambda_2^2 \lambda_4^2 \lambda_1 - \Big(-\frac{1}{1824} \Big(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \Big) \Big(-13 \\
& + 30 \lambda_4 \Big) \Big(60 \lambda_4^2 - 60 \lambda_4 - 49 \Big) + \frac{1}{1824} \Big(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 \\
& - 63 \Big) \Big(30 \lambda_1 - 17 \Big) \Big(60 \lambda_1^2 - 60 \lambda_1 - 49 \Big) \Big) \lambda_1^2 \lambda_4^2 \lambda_2 + 92 \lambda_2^2 \lambda_4^2 \lambda_1 + 92 \lambda_1^2 \lambda_4^2 \lambda_2 + 2 \lambda_1^3 \Big(4 \\
& - 3 \lambda_1 \Big) \lambda_4 + 2 \lambda_2^3 \Big(4 - 3 \lambda_2 \Big) \lambda_4 + 2 \lambda_4^3 \Big(6 \lambda_4^2 - 15 \lambda_4 + 10 \Big) + \frac{2}{3} \lambda_4^3 \Big(4 - 3 \lambda_4 \Big) \lambda_1 + \frac{2}{3} \\
& \lambda_4^3 \Big(4 - 3 \lambda_4 \Big) \lambda_2 \\
& - \Big(\frac{32}{3} \Big(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \Big) \sqrt{2} \Big(-\frac{225}{19456} + \frac{225}{9728} \lambda_4 \Big) \\
& + \frac{32}{3} \Big(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \Big) \sqrt{2} \Big(\frac{225}{19456} - \frac{225}{9728} \lambda_2 \Big) \Big) \lambda_2^2 \\
& \lambda_4^2 \lambda_1 - \Big(-\frac{32}{3} \Big(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \Big) \sqrt{2} \Big(\frac{225}{19456} - \frac{225}{9728} \lambda_4 \Big) \\
& - \frac{32}{3} \Big(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \Big) \sqrt{2} \Big(-\frac{225}{19456} + \frac{225}{9728} \lambda_1 \Big) \Big) \lambda_1^2 \\
& \lambda_4^2 \lambda_2 - \Big(-\frac{32}{3} \Big(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \Big) \sqrt{2} \Big(-\frac{49}{9728} - \frac{15}{2432} \lambda_2 \\
& + \frac{15}{2432} \lambda_2^2 \Big) - \frac{32}{3} \Big(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \Big) \sqrt{2} \Big(-\frac{49}{9728} \\
& - \frac{15}{2432} \lambda_1 + \frac{15}{2432} \lambda_1^2 \Big) \Big) \lambda_1^2 \lambda_2^2 \lambda_4 + 92 \lambda_1^2 \lambda_2^2 \lambda_4 \sqrt{2} + \lambda_1^3 \Big(6 \lambda_1^2 - 15 \lambda_1 + 10 \Big) \sqrt{2} \\
& + \frac{4}{3} \lambda_1^3 \Big(4 - 3 \lambda_1 \Big) \lambda_2 \sqrt{2} - \frac{2}{3} \lambda_1^3 \Big(4 - 3 \lambda_1 \Big) \lambda_4 \sqrt{2} + \lambda_2^3 \Big(6 \lambda_2^2 - 15 \lambda_2 + 10 \Big) \sqrt{2} \\
& + \frac{4}{3} \lambda_2^3 \Big(4 - 3 \lambda_2 \Big) \lambda_1 \sqrt{2} - \frac{2}{3} \lambda_2^3 \Big(4 - 3 \lambda_2 \Big) \lambda_4 \sqrt{2} - \lambda_4^3 \Big(6 \lambda_4^2 - 15 \lambda_4 + 10 \Big) \sqrt{2} \\
& + \frac{2}{3} \lambda_4^3 \Big(4 - 3 \lambda_4 \Big) \lambda_1 \sqrt{2} + \frac{2}{3} \lambda_4^3 \Big(4 - 3 \lambda_4 \Big) \lambda_2 \sqrt{2}
\end{aligned}$$

"Triangle ", 3, [1, 3, 4]

"Spline over triangle ", 3

$$\begin{aligned}
& - \left(-\frac{1}{1824} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \left(-17 + 30 \lambda_4 \right) \left(60 \lambda_4^2 - 60 \lambda_4 \right. \right. \\
& \quad \left. \left. - 49 \right) + \frac{1}{1824} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \left(30 \lambda_1 - 13 \right) \left(60 \lambda_1^2 \right. \right. \\
& \quad \left. \left. - 60 \lambda_1 - 49 \right) \right) \lambda_1^2 \lambda_4^2 \lambda_3 - \left(-\frac{1}{1824} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \left(-17 \right. \right. \\
& \quad \left. \left. + 30 \lambda_3 \right) \left(60 \lambda_3^2 - 60 \lambda_3 - 49 \right) + \frac{1}{1824} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 \right. \right. \\
& \quad \left. \left. - 63 \right) \left(30 \lambda_1 - 13 \right) \left(60 \lambda_1^2 - 60 \lambda_1 - 49 \right) \right) \lambda_1^2 \lambda_3^2 \lambda_4 - 92 \lambda_1^2 \lambda_4^2 \lambda_3 - 92 \lambda_1^2 \lambda_3^2 \lambda_4 - 2 \\
& \quad \lambda_1^3 \left(6 \lambda_1^2 - 15 \lambda_1 + 10 \right) - \frac{2}{3} \lambda_1^3 \left(4 - 3 \lambda_1 \right) \lambda_3 - \frac{2}{3} \lambda_1^3 \left(4 - 3 \lambda_1 \right) \lambda_4 - 2 \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_1 \\
& \quad - 2 \lambda_4^3 \left(4 - 3 \lambda_4 \right) \lambda_1 \\
& - \left(-\frac{5}{304} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63 \right) \left(30 \lambda_4^2 - 30 \lambda_4 - 17 \right) \left(2 \lambda_4 - 1 \right) \right. \\
& \quad \left. + \frac{5}{304} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \left(30 \lambda_3^2 - 30 \lambda_3 - 17 \right) \left(2 \lambda_3 - 1 \right) \right) \\
& \quad \lambda_3^2 \lambda_4^2 \lambda_1 - \left(-\frac{1}{1824} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \left(-13 + 30 \lambda_4 \right) \left(60 \lambda_4^2 \right. \right. \\
& \quad \left. \left. - 60 \lambda_4 - 49 \right) + \frac{1}{1824} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \left(30 \lambda_1 - 17 \right) \left(60 \right. \right. \\
& \quad \left. \left. \lambda_1^2 - 60 \lambda_1 - 49 \right) \right) \lambda_1^2 \lambda_4^2 \lambda_3 - \left(\frac{1}{1824} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \left(-13 \right. \right. \\
& \quad \left. \left. + 30 \lambda_3 \right) \left(60 \lambda_3^2 - 60 \lambda_3 - 49 \right) - \frac{1}{1824} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 \right. \right. \\
& \quad \left. \left. - 63 \right) \left(30 \lambda_1 - 17 \right) \left(60 \lambda_1^2 - 60 \lambda_1 - 49 \right) \right) \lambda_1^2 \lambda_3^2 \lambda_4 + 92 \lambda_1^2 \lambda_4^2 \lambda_3 - 92 \lambda_1^2 \lambda_3^2 \lambda_4 - 2 \lambda_1^3 \left(4 \right. \\
& \quad \left. - 3 \lambda_1 \right) \lambda_3 + 2 \lambda_1^3 \left(4 - 3 \lambda_1 \right) \lambda_4 - 2 \lambda_3^3 \left(6 \lambda_3^2 - 15 \lambda_3 + 10 \right) - \frac{2}{3} \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_1 + \frac{4}{3} \\
& \quad \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_4 + 2 \lambda_4^3 \left(6 \lambda_4^2 - 15 \lambda_4 + 10 \right) + \frac{2}{3} \lambda_4^3 \left(4 - 3 \lambda_4 \right) \lambda_1 - \frac{4}{3} \lambda_4^3 \left(4 - 3 \lambda_4 \right) \lambda_3 \\
& - \left(-\frac{32}{3} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63 \right) \sqrt{2} \left(\frac{49}{9728} + \frac{15}{2432} \lambda_4 - \frac{15}{2432} \lambda_4^2 \right) \right. \\
& \quad \left. - \frac{32}{3} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \sqrt{2} \left(\frac{49}{9728} + \frac{15}{2432} \lambda_3 - \frac{15}{2432} \right. \right. \\
& \quad \left. \left. \lambda_3^2 \right) \right) \lambda_3^2 \lambda_4^2 \lambda_1 - \left(-\frac{32}{3} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \sqrt{2} \left(\frac{225}{19456} \right. \right. \\
& \quad \left. \left. - \frac{225}{9728} \lambda_4 \right) - \frac{32}{3} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \sqrt{2} \left(-\frac{225}{19456} \right. \right. \\
& \quad \left. \left. + \frac{225}{9728} \lambda_1 \right) \right) \lambda_1^2 \lambda_4^2 \lambda_3 - \left(\frac{32}{3} \left(4860 \lambda_1^4 - 9720 \lambda_1^3 + 4892 \lambda_1^2 - 32 \lambda_1 - 63 \right) \sqrt{2} \left(\right. \right. \\
& \quad \left. \left. - \frac{225}{19456} + \frac{225}{9728} \lambda_3 \right) + \frac{32}{3} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -63) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \lambda_1 \right) \lambda_1^2 \lambda_3^2 \lambda_4 - 92 \lambda_3^2 \lambda_4^2 \lambda_1 \sqrt{2} + \lambda_1^3 (6 \lambda_1^2 - 15 \lambda_1 \\
& + 10) \sqrt{2} - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_3 \sqrt{2} - \frac{2}{3} \lambda_1^3 (4 - 3 \lambda_1) \lambda_4 \sqrt{2} - \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 \\
& + 10) \sqrt{2} + \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_1 \sqrt{2} - \frac{4}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_4 \sqrt{2} - \lambda_4^3 (6 \lambda_4^2 - 15 \lambda_4 \\
& + 10) \sqrt{2} + \frac{2}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_1 \sqrt{2} - \frac{4}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_3 \sqrt{2}
\end{aligned}$$

"Triangle ", 4, [2, 3, 4]

"Spline over triangle ", 4

$$\begin{aligned}
& - \left(\frac{1}{1824} (4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63) (-17 + 30 \lambda_4) (60 \lambda_4^2 - 60 \lambda_4 - 49) \right. \\
& \quad - \frac{1}{1824} (4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63) (30 \lambda_2 - 13) (60 \lambda_2^2 - 60 \lambda_2 \\
& \quad - 49) \left. \right) \lambda_2^2 \lambda_4^2 \lambda_3 - \left(\frac{1}{1824} (4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63) (-17 \right. \\
& \quad + 30 \lambda_3) (60 \lambda_3^2 - 60 \lambda_3 - 49) - \frac{1}{1824} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 \\
& \quad - 63) (30 \lambda_2 - 13) (60 \lambda_2^2 - 60 \lambda_2 - 49) \left. \right) \lambda_2^2 \lambda_3^2 \lambda_4 + 92 \lambda_2^2 \lambda_4^2 \lambda_3 + 92 \lambda_2^2 \lambda_3^2 \lambda_4 + 2 \\
& \quad \lambda_2^3 (6 \lambda_2^2 - 15 \lambda_2 + 10) + \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_3 + \frac{2}{3} \lambda_2^3 (4 - 3 \lambda_2) \lambda_4 + 2 \lambda_3^3 (4 - 3 \lambda_3) \lambda_2 \\
& \quad + 2 \lambda_4^3 (4 - 3 \lambda_4) \lambda_2 \\
& - \left(-\frac{5}{304} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63) (30 \lambda_4^2 - 30 \lambda_4 - 17) (2 \lambda_4 - 1) \right. \\
& \quad + \frac{5}{304} (4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63) (30 \lambda_3^2 - 30 \lambda_3 - 17) (2 \lambda_3 - 1) \left. \right) \\
& \quad \lambda_3^2 \lambda_4^2 \lambda_2 - \left(-\frac{1}{1824} (4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63) (-13 + 30 \lambda_4) (60 \lambda_4^2 \right. \\
& \quad - 60 \lambda_4 - 49) + \frac{1}{1824} (4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63) (30 \lambda_2 - 17) (60 \\
& \quad \lambda_2^2 - 60 \lambda_2 - 49) \left. \right) \lambda_2^2 \lambda_4^2 \lambda_3 - \left(\frac{1}{1824} (4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63) (-13 \right. \\
& \quad + 30 \lambda_3) (60 \lambda_3^2 - 60 \lambda_3 - 49) - \frac{1}{1824} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 \\
& \quad - 63) (30 \lambda_2 - 17) (60 \lambda_2^2 - 60 \lambda_2 - 49) \left. \right) \lambda_2^2 \lambda_3^2 \lambda_4 + 92 \lambda_2^2 \lambda_4^2 \lambda_3 - 92 \lambda_2^2 \lambda_3^2 \lambda_4 - 2 \lambda_2^3 (4 \\
& \quad - 3 \lambda_2) \lambda_3 + 2 \lambda_2^3 (4 - 3 \lambda_2) \lambda_4 - 2 \lambda_3^3 (6 \lambda_3^2 - 15 \lambda_3 + 10) - \frac{2}{3} \lambda_3^3 (4 - 3 \lambda_3) \lambda_2 + \frac{4}{3} \\
& \quad \lambda_3^3 (4 - 3 \lambda_3) \lambda_4 + 2 \lambda_4^3 (6 \lambda_4^2 - 15 \lambda_4 + 10) + \frac{2}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_2 - \frac{4}{3} \lambda_4^3 (4 - 3 \lambda_4) \lambda_3 \\
& - \left(-\frac{32}{3} (4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 - 63) \sqrt{2} \left(\frac{49}{9728} + \frac{15}{2432} \lambda_4 - \frac{15}{2432} \lambda_4^2 \right) \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{32}{3} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \sqrt{2} \left(\frac{49}{9728} + \frac{15}{2432} \lambda_3 - \frac{15}{2432} \right. \\
& \left. \lambda_3^2 \right) \lambda_3^2 \lambda_4^2 \lambda_2 - \left(\frac{32}{3} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 - 63 \right) \sqrt{2} \left(-\frac{225}{19456} \right. \right. \\
& \left. \left. + \frac{225}{9728} \lambda_4 \right) + \frac{32}{3} \left(4860 \lambda_4^4 - 9720 \lambda_4^3 + 4892 \lambda_4^2 - 32 \lambda_4 - 63 \right) \sqrt{2} \left(\frac{225}{19456} \right. \right. \\
& \left. \left. - \frac{225}{9728} \lambda_2 \right) \right) \lambda_2^2 \lambda_4^2 \lambda_3 - \left(-\frac{32}{3} \left(4860 \lambda_2^4 - 9720 \lambda_2^3 + 4892 \lambda_2^2 - 32 \lambda_2 \right. \right. \\
& \left. \left. - 63 \right) \sqrt{2} \left(\frac{225}{19456} - \frac{225}{9728} \lambda_3 \right) - \frac{32}{3} \left(4860 \lambda_3^4 - 9720 \lambda_3^3 + 4892 \lambda_3^2 - 32 \lambda_3 \right. \right. \\
& \left. \left. - 63 \right) \sqrt{2} \left(-\frac{225}{19456} + \frac{225}{9728} \lambda_2 \right) \right) \lambda_2^2 \lambda_3^2 \lambda_4 - 92 \lambda_3^2 \lambda_4^2 \lambda_2 \sqrt{2} + \lambda_2^3 \left(6 \lambda_2^2 - 15 \lambda_2 \right. \\
& \left. + 10 \right) \sqrt{2} - \frac{2}{3} \lambda_2^3 \left(4 - 3 \lambda_2 \right) \lambda_3 \sqrt{2} - \frac{2}{3} \lambda_2^3 \left(4 - 3 \lambda_2 \right) \lambda_4 \sqrt{2} - \lambda_3^3 \left(6 \lambda_3^2 - 15 \lambda_3 \right. \\
& \left. + 10 \right) \sqrt{2} + \frac{2}{3} \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_2 \sqrt{2} - \frac{4}{3} \lambda_3^3 \left(4 - 3 \lambda_3 \right) \lambda_4 \sqrt{2} - \lambda_4^3 \left(6 \lambda_4^2 - 15 \lambda_4 \right. \\
& \left. + 10 \right) \sqrt{2} + \frac{2}{3} \lambda_4^3 \left(4 - 3 \lambda_4 \right) \lambda_2 \sqrt{2} - \frac{4}{3} \lambda_4^3 \left(4 - 3 \lambda_4 \right) \lambda_3 \sqrt{2}
\end{aligned}$$

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"Final result displayed"

