```
restart:
with(linalg):

K := 4;

a0 := matrix(1, K):
a0[1, 1] := 2·(t − 1)·t·(t + 1)·(t + 2)·(t + 3):
a0[1, 2] := 3·(t − 3)·(t − 2)·(t − 1)·t·(t + 1):
a0[1, 3] := 4·(t − 3)·(t + 1)·(t + 2)·(t + 3):
a0[1, 4] := 3·(t − 2)·(t − 1)·t·(t + 1)·(t + 3):
print("Polynomials");
print(evalm(transpose(a0)));

a := matrix(1, K):
for k from 1 to K do:
a[1, k] := a0[1, k]:
od:

dd := vector(K):
for k from 1 to K do:
if expand(a[1, k]) = 0 then dd[k] := -1: fi:
if expand(a[1, k]) ≠ 0 then dd[k] := degree(a[1, k]): fi:
od:#k
print("degrees ", evalm(dd));

mc := vector(K):
for k from 1 to K do:
if dd[k] < 0 then mc[k] := 0: fi:
if dd[k] = 0 then mc[k] := a[1, k]: fi:
if dd[k] > 0 then
b := a[1, k]: for j from 1 to dd[k] do: b := diff(b, t)/j: od: mc[k] := b:
fi:
od:#k
print("maincoeff s ", evalm(mc));


X := matrix(K, K):
for i from 1 to K do: for j from 1 to K do: X[i, j] := 0: od: X[i, i] := 1: od:
print("X initial ", evalm(X));


S := 0:
for k from 1 to K do:
if expand(a[1, k]) = 0 then dd[k] := -1: fi:
if expand(a[1, k]) ≠ 0 then dd[k] := degree(a[1, k]): fi:
S := S + dd[k]:
od:


print("Sum of degrees ", S);
print("Degrees ", evalm(dd));
```

```
for s from 1 to S do:
print("_____"):
print("STEP ", s):
print("Normalization"):
for k from 1 to K do:
if expand(a[1, k]) = 0 then dd[k] := -1 : fi:
if expand(a[1, k]) ≠ 0 then dd[k] := degree(a[1, k]) : fi:
od:#k
for k from 1 to K do:
if dd[k] < 0 then mc[k] := 0 : fi:
if dd[k] = 0 then mc[k] := a[1, k] : fi:
if dd[k] ≥ 1 then
b := a[1, k] : for j from 1 to dd[k] do: b := diff(b, t)/j :od: mc[k] := b :
fi:
od:#k
print("polynomials", evalm(transpose(a))):
print("degrees ", evalm(dd)):
print("main coefficients ", evalm(mc)):

for k from 1 to K do:
if dd[k] ≥ 0 then a[1, k] := expand(mc[k]^-1 · a[1, k]) :fi:
od:#k
A := matrix(K, K):
for i from 1 to K do: for j from 1 to K do:
A[i, j] := 0 :
if i = j and dd[j] ≥ 0 then A[i, j] := mc[j]^-1 : fi:
if i = j and dd[j] < 0 then A[i, i] := 1 : fi:
od:od:#ij
X := evalm(X &* A):
print("New polynomials ", evalm(transpose(a))):
print("degrees ", evalm(dd)):
print("New X with X=XA ", evalm(X), "  A=", evalm(A)):

print("Reordering"):
P := matrix(K, K):
for i from 1 to K do:for j from 1 to K do: P[i, j] := 0 : od: P[i, i] := 1 :od:
for n from 1 to K do:
for i from 1 to K - 1 do:
j := i + 1 :
if (0 ≤ dd[i] and dd[i] < dd[j]) or dd[j] = -1 then
ddd := dd[j] : dd[j] := dd[i] : dd[i] := ddd :
aaa := a[1, j] : a[1, j] := a[1, i] : a[1, i] := aaa :
for k from 1 to K do:
ppp := P[k, j] : P[k, j] := P[k, i] : P[k, i] := ppp :
od:#k
fi:
#od:od: #ij
od:#i
```

```
od:#n
X := evalm(X&*P) :
print("New polynomials ") :
print(evalm(transpose(a))) :
print("degrees  ", evalm(dd)) :
print("New X=XP  ", evalm(X), "  with   P=  ", evalm(P)) :

print("Degree decreasing") :
L := 0 :
for k from 1 to K do:
if dd[k] =-1 then L := k  fi:
od:#k
L := L + 1 :
print("First nonzero pol with index   L=", L) :


#STOP condition
if L < K then

dddd := dd[L] - dd[L + 1] :
print("pol_L = ", a[1, L], "   pol_L+1 = ", a[1, L + 1], "    t^d =", t^dddd) :

a[1, L] := expand(a[1, L]-t^dddd·a[1, L + 1]) :
print("a_L-t^dddd a_L+1 = ", a[1, L]) :
B := matrix(K, K) :
for i from 1 to K do: for j from 1 to K do:
B[i, j] := 0 : od: B[i, i] := 1 :od:
B[L + 1, L] := -t^dddd :
X := evalm(X&*B) :
print("New polynomials") :
print(evalm(transpose(a))) :
print("New X=XB  ", evalm(X), "  with   B=  ", evalm(B)) :

fi: #STOPcond

#STOP
if L = K then s := S :fi:

print("CHEKING  a -a0 X=0") :
DDD := evalm(a - a0&*X) :
DD := evalm(DDD[1, 1]) :
for k from 1 to K do: DD := expand(DD) :od:
print(evalm(DD)) :

od:#s


print("_____________________________________________________________") :

print("_____________________________________________________________") :
```

```
GCD := evalm(a[1, K]) :
print(" GCD = ", GCD);

print("Cofactors in column K of X");

q := matrix(L, 1) :
for i from 1 to K do: q[i, 1] := X[i, L] : od:

print(evalm(q));

print("CHECKING GCD-a0_1 q_1-...-a0_K q_K =0");

DD := GCD :
for k from 1 to K do: DD := expand(DD − a0[1, k]·q[k, 1]) : od:
print(DD);
```

4

"Polynomials"

$$\begin{bmatrix} 2\,(t-1)\,t\,(t+1)\,(t+2)\,(t+3) \\ 3\,(t-3)\,(t-2)\,(t-1)\,t\,(t+1) \\ 4\,(t-3)\,(t+1)\,(t+2)\,(t+3) \\ 3\,(t-2)\,(t-1)\,t\,(t+1)\,(t+3) \end{bmatrix}$$

"degrees ", $\begin{bmatrix} 5 & 5 & 4 & 5 \end{bmatrix}$

"maincoeff s ", $\begin{bmatrix} 2 & 3 & 4 & 3 \end{bmatrix}$

"X initial ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Sum of degrees ", 19

"Degrees ", $\begin{bmatrix} 5 & 5 & 4 & 5 \end{bmatrix}$

"———————————————————————————————————————————————————————————————————————————————"

"STEP ", 1

"Normalization"

$$\text{"polynomials"}, \begin{bmatrix} 2\,(t-1)\,t\,(t+1)\,(t+2)\,(t+3) \\ 3\,(t-3)\,(t-2)\,(t-1)\,t\,(t+1) \\ 4\,(t-3)\,(t+1)\,(t+2)\,(t+3) \\ 3\,(t-2)\,(t-1)\,t\,(t+1)\,(t+3) \end{bmatrix}$$

$$\text{"degrees "}, \begin{bmatrix} 5 & 5 & 4 & 5 \end{bmatrix}$$

$$\text{"main coefficients "}, \begin{bmatrix} 2 & 3 & 4 & 3 \end{bmatrix}$$

$$\text{"New polynomials "}, \begin{bmatrix} t^5+5\,t^4+5\,t^3-5\,t^2-6\,t \\ t^5-5\,t^4+5\,t^3+5\,t^2-6\,t \\ t^4+3\,t^3-7\,t^2-27\,t-18 \\ t^5+t^4-7\,t^3-t^2+6\,t \end{bmatrix}$$

$$\text{"degrees "}, \begin{bmatrix} 5 & 5 & 4 & 5 \end{bmatrix}$$

$$\text{"New X with X=XA "}, \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{bmatrix}, \text{" A="}, \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{bmatrix}$$

"Reordering"

"New polynomials "

$$\begin{bmatrix} t^5+5\,t^4+5\,t^3-5\,t^2-6\,t \\ t^5-5\,t^4+5\,t^3+5\,t^2-6\,t \\ t^5+t^4-7\,t^3-t^2+6\,t \\ t^4+3\,t^3-7\,t^2-27\,t-18 \end{bmatrix}$$

$$\text{"degrees "}, \begin{bmatrix} 5 & 5 & 5 & 4 \end{bmatrix}$$

"New X=XP ", $\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & 0 \end{bmatrix}$, " with P= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 1

"pol_L = ", $t^5 + 5\,t^4 + 5\,t^3 - 5\,t^2 - 6\,t$, " pol_L+1 = ", $t^5 - 5\,t^4 + 5\,t^3 + 5\,t^2 - 6\,t$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", $10\,t^4 - 10\,t^2$

"New polynomials"

$$\begin{bmatrix} 10\,t^4 - 10\,t^2 \\ t^5 - 5\,t^4 + 5\,t^3 + 5\,t^2 - 6\,t \\ t^5 + t^4 - 7\,t^3 - t^2 + 6\,t \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & 0 \end{bmatrix}$, " with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"_____"

"STEP ", 2

"Normalization"

"polynomials", $\begin{bmatrix} 10\,t^4 - 10\,t^2 \\ t^5 - 5\,t^4 + 5\,t^3 + 5\,t^2 - 6\,t \\ t^5 + t^4 - 7\,t^3 - t^2 + 6\,t \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} 4 & 5 & 5 & 4 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 10 & 1 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} t^4 - t^2 \\ t^5 - 5\,t^4 + 5\,t^3 + 5\,t^2 - 6\,t \\ t^5 + t^4 - 7\,t^3 - t^2 + 6\,t \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} 4 & 5 & 5 & 4 \end{bmatrix}$

"New X with X=XA ", $\begin{bmatrix} \frac{1}{20} & 0 & 0 & 0 \\ -\frac{1}{30} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & 0 \end{bmatrix}$, " A=", $\begin{bmatrix} \frac{1}{10} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$\begin{bmatrix} t^5 - 5\,t^4 + 5\,t^3 + 5\,t^2 - 6\,t \\ t^5 + t^4 - 7\,t^3 - t^2 + 6\,t \\ t^4 - t^2 \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} 5 & 5 & 4 & 4 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ \frac{1}{3} & 0 & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{3} & 0 & 0 \end{bmatrix}$, " with P= ", $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 1

"pol_L = ", $t^5 - 5\,t^4 + 5\,t^3 + 5\,t^2 - 6\,t$, " pol_L+1 = ", $t^5 + t^4 - 7\,t^3 - t^2 + 6\,t$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", $-6t^4+12t^3+6t^2-12t$

"New polynomials"

$$\begin{bmatrix} -6t^4+12t^3+6t^2-12t \\ t^5+t^4-7t^3-t^2+6t \\ t^4-t^2 \\ t^4+3t^3-7t^2-27t-18 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ \frac{1}{3} & 0 & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \end{bmatrix}$, " with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"_____"

"STEP ", 3

"Normalization"

"polynomials", $\begin{bmatrix} -6t^4+12t^3+6t^2-12t \\ t^5+t^4-7t^3-t^2+6t \\ t^4-t^2 \\ t^4+3t^3-7t^2-27t-18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} 4 & 5 & 4 & 4 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} -6 & 1 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} t^4-2t^3-t^2+2t \\ t^5+t^4-7t^3-t^2+6t \\ t^4-t^2 \\ t^4+3t^3-7t^2-27t-18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} 4 & 5 & 4 & 4 \end{bmatrix}$

"New X  with X=XA  ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ -\frac{1}{18} & 0 & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{18} & \frac{1}{3} & 0 & 0 \end{bmatrix}$, "  A=", $\begin{bmatrix} -\frac{1}{6} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$$\begin{bmatrix} t^5 + t^4 - 7\,t^3 - t^2 + 6\,t \\ t^4 - 2\,t^3 - t^2 + 2\,t \\ t^4 - t^2 \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$$

"degrees  ", $\begin{bmatrix} 5 & 4 & 4 & 4 \end{bmatrix}$

"New X=XP  ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ 0 & -\frac{1}{18} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{18} & 0 & 0 \end{bmatrix}$, "  with  P=  ", $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index  L=", 1

"pol_L = ", $t^5 + t^4 - 7\,t^3 - t^2 + 6\,t$, "  pol_L+1 = ", $t^4 - 2\,t^3 - t^2 + 2\,t$, "  t^d =", $t$

"a_L-t^dddd a_L+1 = ", $3\,t^4 - 6\,t^3 - 3\,t^2 + 6\,t$

"New polynomials"

$$\begin{bmatrix} 3\,t^4 - 6\,t^3 - 3\,t^2 + 6\,t \\ t^4 - 2\,t^3 - t^2 + 2\,t \\ t^4 - t^2 \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ \frac{1}{18}\,t & -\frac{1}{18} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{3}-\frac{1}{18}\,t & \frac{1}{18} & 0 & 0 \end{bmatrix}$, " with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -t & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"$\frac{}{}$"

"STEP ", 4

"Normalization"

"polynomials", $\begin{bmatrix} 3\,t^4-6\,t^3-3\,t^2+6\,t \\ t^4-2\,t^3-t^2+2\,t \\ t^4-t^2 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} 4 & 4 & 4 & 4 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 3 & 1 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} t^4-2\,t^3-t^2+2\,t \\ t^4-2\,t^3-t^2+2\,t \\ t^4-t^2 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} 4 & 4 & 4 & 4 \end{bmatrix}$

"New X with X=XA ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ \frac{1}{54}\,t & -\frac{1}{18} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{9}-\frac{1}{54}\,t & \frac{1}{18} & 0 & 0 \end{bmatrix}$, " A=", $\begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$$\begin{bmatrix} t^4-2\,t^3-t^2+2\,t \\ t^4-2\,t^3-t^2+2\,t \\ t^4-t^2 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \end{bmatrix}$$

"degrees ", $\begin{bmatrix} 4 & 4 & 4 & 4 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ \frac{1}{54}\,t & -\frac{1}{18} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{9}-\frac{1}{54}\,t & \frac{1}{18} & 0 & 0 \end{bmatrix}$, " with P= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 1

"pol_L = ", $t^4-2\,t^3-t^2+2\,t$, " pol_L+1 = ", $t^4-2\,t^3-t^2+2\,t$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", 0

"New polynomials"

$$\begin{bmatrix} 0 \\ t^4-2\,t^3-t^2+2\,t \\ t^4-t^2 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{18} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{18}-\frac{1}{54}\,t & \frac{1}{18} & 0 & 0 \end{bmatrix}$, " with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"_____"

"STEP ", 5

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ t^4 - 2\,t^3 - t^2 + 2\,t \\ t^4 - t^2 \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 4 & 4 & 4 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ t^4 - 2\,t^3 - t^2 + 2\,t \\ t^4 - t^2 \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 4 & 4 & 4 \end{bmatrix}$

"New X with X=XA ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ \frac{1}{54}\,t + \frac{1}{18} & -\frac{1}{18} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{18} - \frac{1}{54}\,t & \frac{1}{18} & 0 & 0 \end{bmatrix}$, " A=", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$\begin{bmatrix} 0 \\ t^4 - 2\,t^3 - t^2 + 2\,t \\ t^4 - t^2 \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 4 & 4 & 4 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & 0 & \frac{1}{20} & 0 \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{18} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{18}-\frac{1}{54}\,t & \frac{1}{18} & 0 & 0 \end{bmatrix}$, " with P= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t^4-2\,t^3-t^2+2\,t$, " pol_L+1 = ", $t^4-t^2$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", $-2\,t^3+2\,t$

"New polynomials"

$$\begin{bmatrix} 0 \\ -2\,t^3+2\,t \\ t^4-t^2 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & -\frac{1}{20} & \frac{1}{20} & 0 \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{45} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{18}-\frac{1}{54}\,t & \frac{1}{18} & 0 & 0 \end{bmatrix}$, " with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"_____"

"STEP ", 6

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ -2\,t^3+2\,t \\ t^4-t^2 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 3 & 4 & 4 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & -2 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ t^3 - t \\ t^4 - t^2 \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 3 & 4 & 4 \end{bmatrix}$

"New X with X=XA ", $\begin{bmatrix} 0 & \frac{1}{40} & \frac{1}{20} & 0 \\ \frac{1}{54}\,t + \frac{1}{18} & \frac{1}{90} & -\frac{1}{30} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{18} - \frac{1}{54}\,t & -\frac{1}{36} & 0 & 0 \end{bmatrix}$, " A=", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$\begin{bmatrix} 0 \\ t^4 - t^2 \\ t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18 \\ t^3 - t \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 4 & 4 & 3 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & \frac{1}{20} & 0 & \frac{1}{40} \\ \frac{1}{54}\,t + \frac{1}{18} & -\frac{1}{30} & 0 & \frac{1}{90} \\ 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{18} - \frac{1}{54}\,t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, " with P= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t^4 - t^2$, " pol_L+1 = ", $t^4 + 3\,t^3 - 7\,t^2 - 27\,t - 18$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", $-3\,t^3+6\,t^2+27\,t+18$

"New polynomials"

$$\begin{bmatrix} 0 \\ -3\,t^3+6\,t^2+27\,t+18 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \\ t^3-t \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & \frac{1}{20} & 0 & \frac{1}{40} \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{30} & 0 & \frac{1}{90} \\ 0 & -\frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{18}-\frac{1}{54}\,t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, " with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"$\dfrac{}{}$"

"STEP ", 7

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ -3\,t^3+6\,t^2+27\,t+18 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \\ t^3-t \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 3 & 4 & 3 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & -3 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ t^3-2\,t^2-9\,t-6 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \\ t^3-t \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 3 & 4 & 3 \end{bmatrix}$

"New X  with X=XA  ", $\begin{bmatrix} 0 & -\frac{1}{60} & 0 & \frac{1}{40} \\ \frac{1}{54}\,t+\frac{1}{18} & \frac{1}{90} & 0 & \frac{1}{90} \\ 0 & \frac{1}{12} & \frac{1}{4} & 0 \\ \frac{1}{18}-\frac{1}{54}\,t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, "  A=", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$$\begin{bmatrix} 0 \\ t^4+3\,t^3-7\,t^2-27\,t-18 \\ t^3-2\,t^2-9\,t-6 \\ t^3-t \end{bmatrix}$$

"degrees  ", $\begin{bmatrix} -1 & 4 & 3 & 3 \end{bmatrix}$

"New X=XP  ", $\begin{bmatrix} 0 & 0 & -\frac{1}{60} & \frac{1}{40} \\ \frac{1}{54}\,t+\frac{1}{18} & 0 & \frac{1}{90} & \frac{1}{90} \\ 0 & \frac{1}{4} & \frac{1}{12} & 0 \\ \frac{1}{18}-\frac{1}{54}\,t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, "  with  P=  ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index  L=", 2

"pol_L = ", $t^4+3\,t^3-7\,t^2-27\,t-18$, "  pol_L+1 = ", $t^3-2\,t^2-9\,t-6$, "  t^d =", $t$

"a_L-t^dddd a_L+1 = ", $5\,t^3+2\,t^2-21\,t-18$

"New polynomials"

$$\begin{bmatrix} 0 \\ 5\,t^3+2\,t^2-21\,t-18 \\ t^3-2\,t^2-9\,t-6 \\ t^3-t \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & \frac{1}{60}\,t & -\frac{1}{60} & \frac{1}{40} \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{90}\,t & \frac{1}{90} & \frac{1}{90} \\ 0 & \frac{1}{4}-\frac{1}{12}\,t & \frac{1}{12} & 0 \\ \frac{1}{18}-\frac{1}{54}\,t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, " with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -t & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"$\frac{}{}$"

"STEP ", 8

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ 5\,t^3+2\,t^2-21\,t-18 \\ t^3-2\,t^2-9\,t-6 \\ t^3-t \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 3 & 3 & 3 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & 5 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ t^3+\frac{2}{5}\,t^2-\frac{21}{5}\,t-\frac{18}{5} \\ t^3-2\,t^2-9\,t-6 \\ t^3-t \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 3 & 3 & 3 \end{bmatrix}$

"New X with X=XA ", $\begin{bmatrix} 0 & \frac{1}{300}\,t & -\frac{1}{60} & \frac{1}{40} \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{450}\,t & \frac{1}{90} & \frac{1}{90} \\ 0 & \frac{1}{20}-\frac{1}{60}\,t & \frac{1}{12} & 0 \\ \frac{1}{18}-\frac{1}{54}\,t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, " A=", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$$\begin{bmatrix} 0 \\ t^3+\frac{2}{5}\,t^2-\frac{21}{5}\,t-\frac{18}{5} \\ t^3-2\,t^2-9\,t-6 \\ t^3-t \end{bmatrix}$$

"degrees ", $\begin{bmatrix} -1 & 3 & 3 & 3 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & \frac{1}{300}\,t & -\frac{1}{60} & \frac{1}{40} \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{450}\,t & \frac{1}{90} & \frac{1}{90} \\ 0 & \frac{1}{20}-\frac{1}{60}\,t & \frac{1}{12} & 0 \\ \frac{1}{18}-\frac{1}{54}\,t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, " with P= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t^3+\frac{2}{5}\,t^2-\frac{21}{5}\,t-\frac{18}{5}$, " pol_L+1 = ", $t^3-2\,t^2-9\,t-6$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", $\frac{12}{5}\,t^2+\frac{24}{5}\,t+\frac{12}{5}$

"New polynomials"

$$\begin{bmatrix} 0 \\ \frac{12}{5}\,t^2+\frac{24}{5}\,t+\frac{12}{5} \\ t^3-2\,t^2-9\,t-6 \\ t^3-t \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & \frac{1}{60}+\frac{1}{300}\,t & -\frac{1}{60} & \frac{1}{40} \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{90}-\frac{1}{450}\,t & \frac{1}{90} & \frac{1}{90} \\ 0 & -\frac{1}{30}-\frac{1}{60}\,t & \frac{1}{12} & 0 \\ \frac{1}{18}-\frac{1}{54}\,t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, " with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"_____"

"STEP ", 9

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ \frac{12}{5} t^2 + \frac{24}{5} t + \frac{12}{5} \\ t^3 - 2 t^2 - 9 t - 6 \\ t^3 - t \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 2 & 3 & 3 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & \frac{12}{5} & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ t^2 + 2 t + 1 \\ t^3 - 2 t^2 - 9 t - 6 \\ t^3 - t \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 2 & 3 & 3 \end{bmatrix}$

"New X with X=XA ", $\begin{bmatrix} 0 & \frac{1}{144} + \frac{1}{720} t & -\frac{1}{60} & \frac{1}{40} \\ \frac{1}{54} t + \frac{1}{18} & -\frac{1}{216} - \frac{1}{1080} t & \frac{1}{90} & \frac{1}{90} \\ 0 & -\frac{1}{72} - \frac{1}{144} t & \frac{1}{12} & 0 \\ \frac{1}{18} - \frac{1}{54} t & 0 & 0 & -\frac{1}{36} \end{bmatrix}$, " A=",

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$$\begin{bmatrix} 0 \\ t^3 - 2\,t^2 - 9\,t - 6 \\ t^3 - t \\ t^2 + 2\,t + 1 \end{bmatrix}$$

"degrees ", $\begin{bmatrix} -1 & 3 & 3 & 2 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & -\frac{1}{60} & \frac{1}{40} & \frac{1}{144} + \frac{1}{720}\,t \\ \frac{1}{54}\,t + \frac{1}{18} & \frac{1}{90} & \frac{1}{90} & -\frac{1}{216} - \frac{1}{1080}\,t \\ 0 & \frac{1}{12} & 0 & -\frac{1}{72} - \frac{1}{144}\,t \\ \frac{1}{18} - \frac{1}{54}\,t & 0 & -\frac{1}{36} & 0 \end{bmatrix}$, " with P= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t^3 - 2\,t^2 - 9\,t - 6$, " pol_L+1 = ", $t^3 - t$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", $-2\,t^2 - 8\,t - 6$

"New polynomials"

$$\begin{bmatrix} 0 \\ -2\,t^2 - 8\,t - 6 \\ t^3 - t \\ t^2 + 2\,t + 1 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & -\frac{1}{24} & \frac{1}{40} & \frac{1}{144} + \frac{1}{720}\,t \\ \frac{1}{54}\,t + \frac{1}{18} & 0 & \frac{1}{90} & -\frac{1}{216} - \frac{1}{1080}\,t \\ 0 & \frac{1}{12} & 0 & -\frac{1}{72} - \frac{1}{144}\,t \\ \frac{1}{18} - \frac{1}{54}\,t & \frac{1}{36} & -\frac{1}{36} & 0 \end{bmatrix}$, " with B= ",

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"CHEKING  a -a0 X=0"

0

"_____"

"STEP  ", 10

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ -2\,t^2-8\,t-6 \\ t^3-t \\ t^2+2\,t+1 \end{bmatrix}$

"degrees  ", $\begin{bmatrix} -1 & 2 & 3 & 2 \end{bmatrix}$

"main coefficients  ", $\begin{bmatrix} 0 & -2 & 1 & 1 \end{bmatrix}$

"New polynomials  ", $\begin{bmatrix} 0 \\ t^2+4\,t+3 \\ t^3-t \\ t^2+2\,t+1 \end{bmatrix}$

"degrees  ", $\begin{bmatrix} -1 & 2 & 3 & 2 \end{bmatrix}$

"New X  with X=XA  ", $\begin{bmatrix} 0 & \frac{1}{48} & \frac{1}{40} & \frac{1}{144}+\frac{1}{720}\,t \\ \frac{1}{54}\,t+\frac{1}{18} & 0 & \frac{1}{90} & -\frac{1}{216}-\frac{1}{1080}\,t \\ 0 & -\frac{1}{24} & 0 & -\frac{1}{72}-\frac{1}{144}\,t \\ \frac{1}{18}-\frac{1}{54}\,t & -\frac{1}{72} & -\frac{1}{36} & 0 \end{bmatrix}$, "  A=",

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$$\begin{bmatrix} 0 \\ t^3 - t \\ t^2 + 4\,t + 3 \\ t^2 + 2\,t + 1 \end{bmatrix}$$

"degrees ", $\begin{bmatrix} -1 & 3 & 2 & 2 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & \frac{1}{40} & \frac{1}{48} & \frac{1}{144} + \frac{1}{720}\,t \\ \frac{1}{54}\,t + \frac{1}{18} & \frac{1}{90} & 0 & -\frac{1}{216} - \frac{1}{1080}\,t \\ 0 & 0 & -\frac{1}{24} & -\frac{1}{72} - \frac{1}{144}\,t \\ \frac{1}{18} - \frac{1}{54}\,t & -\frac{1}{36} & -\frac{1}{72} & 0 \end{bmatrix}$, " with P= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t^3 - t$, " pol_L+1 = ", $t^2 + 4\,t + 3$, " t^d =", $t$

"a_L-t^dddd a_L+1 = ", $-4\,t^2 - 4\,t$

"New polynomials"

$$\begin{bmatrix} 0 \\ -4\,t^2 - 4\,t \\ t^2 + 4\,t + 3 \\ t^2 + 2\,t + 1 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & \frac{1}{40} - \frac{1}{48}\,t & \frac{1}{48} & \frac{1}{144} + \frac{1}{720}\,t \\ \frac{1}{54}\,t + \frac{1}{18} & \frac{1}{90} & 0 & -\frac{1}{216} - \frac{1}{1080}\,t \\ 0 & \frac{1}{24}\,t & -\frac{1}{24} & -\frac{1}{72} - \frac{1}{144}\,t \\ \frac{1}{18} - \frac{1}{54}\,t & -\frac{1}{36} + \frac{1}{72}\,t & -\frac{1}{72} & 0 \end{bmatrix}$, " with B= ",

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -t & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"CHEKING  a -a0 X=0"

0

"_____"

"STEP  ", 11

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ -4\,t^2-4\,t \\ t^2+4\,t+3 \\ t^2+2\,t+1 \end{bmatrix}$

"degrees  ", $\begin{bmatrix} -1 & 2 & 2 & 2 \end{bmatrix}$

"main coefficients  ", $\begin{bmatrix} 0 & -4 & 1 & 1 \end{bmatrix}$

"New polynomials  ", $\begin{bmatrix} 0 \\ t^2+t \\ t^2+4\,t+3 \\ t^2+2\,t+1 \end{bmatrix}$

"degrees  ", $\begin{bmatrix} -1 & 2 & 2 & 2 \end{bmatrix}$

"New X  with X=XA  ", $\begin{bmatrix} 0 & -\frac{1}{160}+\frac{1}{192}\,t & \frac{1}{48} & \frac{1}{144}+\frac{1}{720}\,t \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{360} & 0 & -\frac{1}{216}-\frac{1}{1080}\,t \\ 0 & -\frac{1}{96}\,t & -\frac{1}{24} & -\frac{1}{72}-\frac{1}{144}\,t \\ \frac{1}{18}-\frac{1}{54}\,t & \frac{1}{144}-\frac{1}{288}\,t & -\frac{1}{72} & 0 \end{bmatrix}$, "  A=", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Reordering"

"New polynomials "

$$\begin{bmatrix} 0 \\ t^2+t \\ t^2+4\,t+3 \\ t^2+2\,t+1 \end{bmatrix}$$

"degrees ", $\begin{bmatrix} -1 & 2 & 2 & 2 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & -\frac{1}{160}+\frac{1}{192}\,t & \frac{1}{48} & \frac{1}{144}+\frac{1}{720}\,t \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{360} & 0 & -\frac{1}{216}-\frac{1}{1080}\,t \\ 0 & -\frac{1}{96}\,t & -\frac{1}{24} & -\frac{1}{72}-\frac{1}{144}\,t \\ \frac{1}{18}-\frac{1}{54}\,t & \frac{1}{144}-\frac{1}{288}\,t & -\frac{1}{72} & 0 \end{bmatrix}$, " with P= ",

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t^2+t$, " pol_L+1 = ", $t^2+4\,t+3$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", $-3\,t-3$

"New polynomials"

$$\begin{bmatrix} 0 \\ -3\,t-3 \\ t^2+4\,t+3 \\ t^2+2\,t+1 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & -\frac{13}{480}+\frac{1}{192}\,t & \frac{1}{48} & \frac{1}{144}+\frac{1}{720}\,t \\ \frac{1}{54}\,t+\frac{1}{18} & -\frac{1}{360} & 0 & -\frac{1}{216}-\frac{1}{1080}\,t \\ 0 & \frac{1}{24}-\frac{1}{96}\,t & -\frac{1}{24} & -\frac{1}{72}-\frac{1}{144}\,t \\ \frac{1}{18}-\frac{1}{54}\,t & \frac{1}{48}-\frac{1}{288}\,t & -\frac{1}{72} & 0 \end{bmatrix}$, " with B= ",

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"CHEKING a -a0 X=0"

0

"$\frac{}{}$"

"STEP ", 12

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ -3t-3 \\ t^2+4t+3 \\ t^2+2t+1 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 1 & 2 & 2 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & -3 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ t+1 \\ t^2+4t+3 \\ t^2+2t+1 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 1 & 2 & 2 \end{bmatrix}$

"New X with X=XA ", $\begin{bmatrix} 0 & \frac{13}{1440}-\frac{1}{576}t & \frac{1}{48} & \frac{1}{144}+\frac{1}{720}t \\ \frac{1}{54}t+\frac{1}{18} & \frac{1}{1080} & 0 & -\frac{1}{216}-\frac{1}{1080}t \\ 0 & -\frac{1}{72}+\frac{1}{288}t & -\frac{1}{24} & -\frac{1}{72}-\frac{1}{144}t \\ \frac{1}{18}-\frac{1}{54}t & -\frac{1}{144}+\frac{1}{864}t & -\frac{1}{72} & 0 \end{bmatrix}$, " A=",

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"Reordering"

"New polynomials "

$$\begin{bmatrix} 0 \\ t^2+4\,t+3 \\ t^2+2\,t+1 \\ t+1 \end{bmatrix}$$

"degrees ", $\begin{bmatrix} -1 & 2 & 2 & 1 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & \frac{1}{48} & \frac{1}{144}+\frac{1}{720}\,t & \frac{13}{1440}-\frac{1}{576}\,t \\ \frac{1}{54}\,t+\frac{1}{18} & 0 & -\frac{1}{216}-\frac{1}{1080}\,t & \frac{1}{1080} \\ 0 & -\frac{1}{24} & -\frac{1}{72}-\frac{1}{144}\,t & -\frac{1}{72}+\frac{1}{288}\,t \\ \frac{1}{18}-\frac{1}{54}\,t & -\frac{1}{72} & 0 & -\frac{1}{144}+\frac{1}{864}\,t \end{bmatrix}$, " with P= ",

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t^2+4\,t+3$, " pol_L+1 = ", $t^2+2\,t+1$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", $2\,t+2$

"New polynomials"

$$\begin{bmatrix} 0 \\ 2\,t+2 \\ t^2+2\,t+1 \\ t+1 \end{bmatrix}$$

"New X=XB ", $\begin{bmatrix} 0 & \frac{1}{72}-\frac{1}{720}t & \frac{1}{144}+\frac{1}{720}t & \frac{13}{1440}-\frac{1}{576}t \\ \frac{1}{54}t+\frac{1}{18} & \frac{1}{216}+\frac{1}{1080}t & -\frac{1}{216}-\frac{1}{1080}t & \frac{1}{1080} \\ 0 & -\frac{1}{36}+\frac{1}{144}t & -\frac{1}{72}-\frac{1}{144}t & -\frac{1}{72}+\frac{1}{288}t \\ \frac{1}{18}-\frac{1}{54}t & -\frac{1}{72} & 0 & -\frac{1}{144}+\frac{1}{864}t \end{bmatrix}$,

" with B= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"CHEKING a -a0 X=0"

0

"$\frac{}{}$"

"STEP ", 13

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ 2t+2 \\ t^2+2t+1 \\ t+1 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 1 & 2 & 1 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & 2 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ t+1 \\ t^2+2t+1 \\ t+1 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 1 & 2 & 1 \end{bmatrix}$

"New X with X=XA ",

$$\begin{bmatrix} 0 & \frac{1}{144}-\frac{1}{1440}t & \frac{1}{144}+\frac{1}{720}t & \frac{13}{1440}-\frac{1}{576}t \\ \frac{1}{54}t+\frac{1}{18} & \frac{1}{432}+\frac{1}{2160}t & -\frac{1}{216}-\frac{1}{1080}t & \frac{1}{1080} \\ 0 & -\frac{1}{72}+\frac{1}{288}t & -\frac{1}{72}-\frac{1}{144}t & -\frac{1}{72}+\frac{1}{288}t \\ \frac{1}{18}-\frac{1}{54}t & -\frac{1}{144} & 0 & -\frac{1}{144}+\frac{1}{864}t \end{bmatrix}, \text{" A="},$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"Reordering"

"New polynomials "

$$\begin{bmatrix} 0 \\ t^2+2t+1 \\ t+1 \\ t+1 \end{bmatrix}$$

"degrees ", $\begin{bmatrix} -1 & 2 & 1 & 1 \end{bmatrix}$

"New X=XP ", $\begin{bmatrix} 0 & \frac{1}{144}+\frac{1}{720}t & \frac{1}{144}-\frac{1}{1440}t & \frac{13}{1440}-\frac{1}{576}t \\ \frac{1}{54}t+\frac{1}{18} & -\frac{1}{216}-\frac{1}{1080}t & \frac{1}{432}+\frac{1}{2160}t & \frac{1}{1080} \\ 0 & -\frac{1}{72}-\frac{1}{144}t & -\frac{1}{72}+\frac{1}{288}t & -\frac{1}{72}+\frac{1}{288}t \\ \frac{1}{18}-\frac{1}{54}t & 0 & -\frac{1}{144} & -\frac{1}{144}+\frac{1}{864}t \end{bmatrix},$

" with P= ", $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t^2+2t+1$, " pol_L+1 = ", $t+1$, " t^d =", $t$

"a_L-t^dddd a_L+1 = ", $t+1$

"New polynomials"

$$\begin{bmatrix} 0 \\ t+1 \\ t+1 \\ t+1 \end{bmatrix}$$

"New X=XB ",

$$\left[\left[0,\ \frac{1}{144}+\frac{1}{720}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ \frac{1}{144}-\frac{1}{1440}\,t,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$
$$\left[\frac{1}{54}\,t+\frac{1}{18},\ -\frac{1}{216}-\frac{1}{1080}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{432}+\frac{1}{2160}\,t,\ \frac{1}{1080}\right],$$
$$\left[0,\ -\frac{1}{72}-\frac{1}{144}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ -\frac{1}{72}+\frac{1}{288}\,t,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$
$$\left.\left[\frac{1}{18}-\frac{1}{54}\,t,\ \frac{1}{144}\,t,\ -\frac{1}{144},\ -\frac{1}{144}+\frac{1}{864}\,t\right]\right],\ \text{" with B= ",}\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -t & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"CHEKING a -a0 X=0"

0

"_____"

"STEP ", 14

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ t+1 \\ t+1 \\ t+1 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 1 & 1 & 1 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ t+1 \\ t+1 \\ t+1 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & 1 & 1 & 1 \end{bmatrix}$

"New X with X=XA ",

$$\left[\left[0,\ \frac{1}{144}+\frac{1}{720}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ \frac{1}{144}-\frac{1}{1440}\,t,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$
$$\left[\frac{1}{54}\,t+\frac{1}{18},\ -\frac{1}{216}-\frac{1}{1080}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{432}+\frac{1}{2160}\,t,\ \frac{1}{1080}\right],$$
$$\left[0,\ -\frac{1}{72}-\frac{1}{144}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ -\frac{1}{72}+\frac{1}{288}\,t,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$
$$\left.\left[\frac{1}{18}-\frac{1}{54}\,t,\ \frac{1}{144}\,t,\ -\frac{1}{144},\ -\frac{1}{144}+\frac{1}{864}\,t\right]\right],\ \text{"}\quad A=\text{",}\ \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

"Reordering"

"New polynomials "

$$\begin{bmatrix}0\\t+1\\t+1\\t+1\end{bmatrix}$$

"degrees ", $\begin{bmatrix}-1&1&1&1\end{bmatrix}$

"New X=XP ",

$$\left[\left[0,\ \frac{1}{144}+\frac{1}{720}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ \frac{1}{144}-\frac{1}{1440}\,t,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$
$$\left[\frac{1}{54}\,t+\frac{1}{18},\ -\frac{1}{216}-\frac{1}{1080}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{432}+\frac{1}{2160}\,t,\ \frac{1}{1080}\right],$$
$$\left[0,\ -\frac{1}{72}-\frac{1}{144}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ -\frac{1}{72}+\frac{1}{288}\,t,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$
$$\left.\left[\frac{1}{18}-\frac{1}{54}\,t,\ \frac{1}{144}\,t,\ -\frac{1}{144},\ -\frac{1}{144}+\frac{1}{864}\,t\right]\right],\ \text{"}\quad \text{with}\quad P=\ \text{",}\ \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

"Degree decreasing"

"First nonzero pol with index L=", 2

"pol_L = ", $t+1$, "  pol_L+1 = ", $t+1$, "  t^d =", 1

"a_L-t^dddd a_L+1 = ", 0

"New polynomials"

$$\begin{bmatrix} 0 \\ 0 \\ t+1 \\ t+1 \end{bmatrix}$$

"New X=XB ",

$$\left[\left[0,\ \frac{1}{480}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ \frac{1}{144}-\frac{1}{1440}\,t,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$
$$\left[\frac{1}{54}\,t+\frac{1}{18},\ -\frac{1}{144}-\frac{1}{720}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{432}+\frac{1}{2160}\,t,\ \frac{1}{1080}\right],$$
$$\left[0,\ -\frac{1}{96}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ -\frac{1}{72}+\frac{1}{288}\,t,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$
$$\left.\left[\frac{1}{18}-\frac{1}{54}\,t,\ \frac{1}{144}+\frac{1}{144}\,t,\ -\frac{1}{144},\ -\frac{1}{144}+\frac{1}{864}\,t\right]\right],\ \text{" with B= ",}\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"CHEKING a -a0 X=0"

0

"_____"

"STEP ", 15

"Normalization"

"polynomials", $\begin{bmatrix} 0 \\ 0 \\ t+1 \\ t+1 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & -1 & 1 & 1 \end{bmatrix}$

"main coefficients ", $\begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}$

"New polynomials ", $\begin{bmatrix} 0 \\ 0 \\ t+1 \\ t+1 \end{bmatrix}$

"degrees ", $\begin{bmatrix} -1 & -1 & 1 & 1 \end{bmatrix}$

"New X with X=XA ",

$$\left[\left[0,\ \frac{1}{480}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ \frac{1}{144}-\frac{1}{1440}\,t,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$

$$\left[\frac{1}{54}\,t+\frac{1}{18},\ -\frac{1}{144}-\frac{1}{720}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{432}+\frac{1}{2160}\,t,\ \frac{1}{1080}\right],$$
$$\left[0,\ -\frac{1}{96}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ -\frac{1}{72}+\frac{1}{288}\,t,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$
$$\left[\frac{1}{18}-\frac{1}{54}\,t,\ \frac{1}{144}+\frac{1}{144}\,t,\ -\frac{1}{144},\ -\frac{1}{144}+\frac{1}{864}\,t\right]\Big],\ \text{"}\quad A=\text{",}\ \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

"Reordering"

"New polynomials "

$$\begin{bmatrix}0\\0\\t+1\\t+1\end{bmatrix}$$

"degrees ", $\begin{bmatrix}-1&-1&1&1\end{bmatrix}$

"New X=XP ",

$$\left[\left[0,\ \frac{1}{480}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ \frac{1}{144}-\frac{1}{1440}\,t,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$
$$\left[\frac{1}{54}\,t+\frac{1}{18},\ -\frac{1}{144}-\frac{1}{720}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{432}+\frac{1}{2160}\,t,\ \frac{1}{1080}\right],$$
$$\left[0,\ -\frac{1}{96}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ -\frac{1}{72}+\frac{1}{288}\,t,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$
$$\left.\left[\frac{1}{18}-\frac{1}{54}\,t,\ \frac{1}{144}+\frac{1}{144}\,t,\ -\frac{1}{144},\ -\frac{1}{144}+\frac{1}{864}\,t\right]\right],\ \text{" with P= ",}\ \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

"Degree decreasing"

"First nonzero pol with index L=", 3

"pol_L = ", $t+1$, " pol_L+1 = ", $t+1$, " t^d =", 1

"a_L-t^dddd a_L+1 = ", 0

"New polynomials"

$$\begin{bmatrix}0\\0\\0\\t+1\end{bmatrix}$$

"New X=XB ",

$$\left[\left[0,\ \frac{1}{480}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ -\frac{1}{480}+\frac{1}{960}\,t,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$
$$\left[\frac{1}{54}\,t+\frac{1}{18},\ -\frac{1}{144}-\frac{1}{720}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{720}+\frac{1}{2160}\,t,\ \frac{1}{1080}\right],$$
$$\left[0,\ -\frac{1}{96}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ 0,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$
$$\left.\left[\frac{1}{18}-\frac{1}{54}\,t,\ \frac{1}{144}+\frac{1}{144}\,t,\ -\frac{1}{864}\,t,\ -\frac{1}{144}+\frac{1}{864}\,t\right]\right],\ \text{" with B= ",}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&-1&1\end{bmatrix}$$

"CHEKING a -a0 X=0"

0

"_____"

"STEP ", 16

"Normalization"

$$\text{"polynomials"},\begin{bmatrix}0\\0\\0\\t+1\end{bmatrix}$$

"degrees ", $\begin{bmatrix}-1&-1&-1&1\end{bmatrix}$

"main coefficients ", $\begin{bmatrix}0&0&0&1\end{bmatrix}$

$$\text{"New polynomials "},\begin{bmatrix}0\\0\\0\\t+1\end{bmatrix}$$

"degrees ", $\begin{bmatrix}-1&-1&-1&1\end{bmatrix}$

"New X with X=XA ",

$$\left[\left[0,\ \frac{1}{480}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ -\frac{1}{480}+\frac{1}{960}\,t,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$
$$\left[\frac{1}{54}\,t+\frac{1}{18},\ -\frac{1}{144}-\frac{1}{720}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{720}+\frac{1}{2160}\,t,\ \frac{1}{1080}\right],$$
$$\left[0,\ -\frac{1}{96}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ 0,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$

$$\left[\frac{1}{18}-\frac{1}{54}\,t,\ \frac{1}{144}+\frac{1}{144}\,t,\ -\frac{1}{864}\,t,\ -\frac{1}{144}+\frac{1}{864}\,t\right]\Big],\ \text{"}\quad A=\text{",}\ \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

"Reordering"

"New polynomials "

$$\begin{bmatrix}0\\0\\0\\t+1\end{bmatrix}$$

"degrees ", $\begin{bmatrix}-1&-1&-1&1\end{bmatrix}$

"New X=XP ",

$$\left[\left[\frac{1}{480}\,t-\left(\frac{1}{144}-\frac{1}{1440}\,t\right)t,\ -\frac{1}{480}+\frac{1}{960}\,t,\ 0,\ \frac{13}{1440}-\frac{1}{576}\,t\right],\right.$$
$$\left[-\frac{1}{144}-\frac{1}{720}\,t-\left(\frac{1}{432}+\frac{1}{2160}\,t\right)t,\ \frac{1}{720}+\frac{1}{2160}\,t,\ \frac{1}{54}\,t+\frac{1}{18},\ \frac{1}{1080}\right],$$
$$\left[-\frac{1}{96}\,t-\left(-\frac{1}{72}+\frac{1}{288}\,t\right)t,\ 0,\ 0,\ -\frac{1}{72}+\frac{1}{288}\,t\right],$$

$$\left[\frac{1}{144}+\frac{1}{144}\,t,\ -\frac{1}{864}\,t,\ \frac{1}{18}-\frac{1}{54}\,t,\ -\frac{1}{144}+\frac{1}{864}\,t\right]\Big],\ \text{"}\quad \text{with}\quad P=\text{",}\ \begin{bmatrix}0&0&1&0\\1&0&0&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$$

"Degree decreasing"

"First nonzero pol with index L=", 4

"CHEKING a -a0 X=0"

0

"_____"

"_____"

" GCD = ", $t+1$

"Cofactors in column K of X"

$$\begin{bmatrix} \frac{13}{1440} - \frac{1}{576}\,t \\ \frac{1}{1080} \\ -\frac{1}{72} + \frac{1}{288}\,t \\ -\frac{1}{144} + \frac{1}{864}\,t \end{bmatrix}$$

"CHECKING GCD-a0_1 q_1-...-a0_K q_K =0"

0 **(1)**

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