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Bicircular projections on some matrix and operator spaces. (English summary)
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The study of bicircular projections is motivated by complex analysis, more specifically by the study of Reinhardt domains. However, bicircular projections make sense in a purely Banach space theoretic setting. Let X be a complex Banach space in some norm $\|\cdot\|$, and let $P\colon X\to X$ be a bounded linear projection. We say that P is bicircular if the mappings of the form $e^{i\alpha}P+e^{i\beta}(1-P)$ are isometric for all pairs of real numbers α,β. A natural problem is to describe the bicircular projections of a given Banach space. Clearly, the answer depends on the norm of the space and can change if an equivalent norm is taken instead.

Motivated by the nice interplay between the geometry and the algebra of the spaces of bounded linear operators appearing in the holomorphic classification of bounded symmetric domains in complex Banach spaces, the authors consider some of these spaces of operators and give a complete description of the set of their bicircular projections. More precisely, they consider the classical spaces $B(H)$, $S(H)$ and $A(H)$ which consist, respectively, of all bounded linear, all symmetric bounded linear and all antisymmetric bounded linear operators $x\colon H\to H$, where H is a complex Hilbert space. The set of bicircular projections $P\colon B(H)\to B(H)$ consists of the transformations $P\colon x\mapsto px$ and $P\colon x\mapsto xp$ $(x\in B(H))$, where p is a selfadjoint projection $p\in B(H)$. The Jordan-Banach algebra $S(H)\subset B(H)$ admits only the trivial bicircular projections $P=0$ and $P=1$. If $P\colon A(H)\to A(H)$ is a bicircular projection on the Lie algebra $A(H)\subset B(H)$ of antisymmetric operators, then there exists a unit vector $\alpha\in H$ such that either $Px=px+xp^t$ $(x\in A(H))$ or $(1-P)x=px+xp^t$ $(x\in A(H))$, where $p=\alpha\otimes\alpha$ and p^t denotes the transpose of p. In the second case one also can write $Px=qxq^t$ where $q=1-p$.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.