MIDTERM #1 (SAMPLE)

1. [9 pts] Let A, B and C be the following matrices:

$$A = \begin{bmatrix} 2 & -2 \\ 1 & 3 \\ 4 & -1 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 0 & -2 \\ -3 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}.$$

Compute the following matrices:

$$C^T + 3C$$
, $A(B^T + C)$, $(B + A)C$.

2. [9 pts] Compute the following determinant:

$$\begin{vmatrix} 2 & 1 & -2 & 1 \\ 1 & 0 & 2 & 4 \\ 3 & 3 & -2 & 0 \\ 2 & 0 & -1 & 3 \end{vmatrix}.$$

3. [9 pts] Solve the following linear system by Cramer's rule:

$$\begin{cases} 7x_1 + 9x_2 = -1\\ 3x_1 + 4x_2 = 5. \end{cases}$$

4. [9 pts] Solve the following linear system using Gaussian elimination:

$$\begin{cases} x_1 - 2x_2 + x_3 = 0\\ -x_1 + 3x_2 + 2x_3 = -1\\ 2x_1 + x_2 + 17x_3 = -5. \end{cases}$$

5. [7 pts] Determine whether \mathbf{s} is a member of $\text{Span}(\mathbf{u}, \mathbf{v}, \mathbf{w})$, for

$$\mathbf{s} = [1, 2, 3]^T, \ \mathbf{u} = [-1, 2, 1]^T, \ \mathbf{v} = [1, -3, 2]^T, \ \mathbf{w} = [-1, 0, 7]^T.$$

6. [7 pts] Determine whether the following vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly independent in \mathbb{R}^4 :

$$\mathbf{u} = [1, 2, 1, 0]^T, \ \mathbf{v} = [2, 1, 0, 1]^T, \ \mathbf{w} = [1, 1, 1, 1]^T.$$

Always give detailed answer.