Find the eigenvalues of the following matrices:

a.) $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$,

b.) $B = \begin{bmatrix} 5 & -1 \\ 2 & 2 \end{bmatrix}$,

c.) $C = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$.

Sol.: a.) $\begin{vmatrix} 1-x & -2 \\ 1 & 1-x \end{vmatrix} = (1-x)(1-x) - (-2) =$

$= 1 - x - x + x^2 + 2 =$

$= x^2 - 2x + 3 = 0$

Quadratic formula: $x_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 3}}{2} = \frac{2 \pm \sqrt{-8}}{2}$

There are no solutions, so A has **NO** (real) eigenvalues.

b.) $\begin{vmatrix} 5-x & -1 \\ 2 & 2-x \end{vmatrix} = (5-x)(2-x) - (-2) =$

$= 10 - 5x - 2x + x^2 + 2 = x^2 - 7x + 12 = 0$

$x_{1,2} = \frac{7 \pm \sqrt{49 - 4 \cdot 12}}{2} = \frac{7 \pm \sqrt{1}}{2} \rightarrow \begin{matrix} x_1 = 4 \\ x_2 = 3 \end{matrix}$.

So the eigenvalues of B are **3 and 4**.

c.) $\begin{vmatrix} 1-x & 1 \\ -1 & 3-x \end{vmatrix} = (1-x)(3-x) - (-1) =$

$= 3 - x - 3x + x^2 + 1 = x^2 - 4x + 4 = 0$

$x_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot 4}}{2} = \frac{4 \pm \sqrt{0}}{2} = \frac{4 \pm 0}{2} \rightarrow \underline{x_1 = x_2 = 2}$.

So C has one eigenvalue: $\boxed{2}$. ☐