

3.6. Feladat:

$$\begin{pmatrix} 2 & 1 & 4 & 8 & | & -1 \\ 1 & 3 & -6 & 2 & | & 3 \\ 3 & -2 & 2 & -2 & | & 8 \\ 2 & -1 & 2 & 0 & | & 4 \end{pmatrix} \xrightarrow{5} \begin{pmatrix} 1^* & 3 & -6 & 2 & | & 3 \\ 2 & 1 & 4 & 8 & | & -1 \\ 3 & -2 & 2 & -2 & | & 8 \\ 2 & -1 & 2 & 0 & | & 4 \end{pmatrix} \xrightarrow{\begin{matrix} -2 \\ -3 \\ -2 \end{matrix}} \begin{pmatrix} 1 & 3 & -6 & 2 & | & 3 \\ 0 & -5 & 16 & 4 & | & -7 \\ 0 & -11 & 20 & -8 & | & -1 \\ 0 & -7 & 14 & -4 & | & -2 \end{pmatrix} \xrightarrow{\begin{matrix} / \cdot 5 \\ / \cdot 5 \end{matrix}}$$

$$\sim \begin{pmatrix} 1 & 3 & -6 & 2 & | & 3 \\ 0 & -5^* & 16 & 4 & | & -7 \\ 0 & -55 & 100 & -40 & | & -5 \\ 0 & -35 & 70 & -20 & | & -10 \end{pmatrix} \xrightarrow{\begin{matrix} -11 \\ -7 \end{matrix}} \sim \begin{pmatrix} 1 & 3 & -6 & 2 & | & 3 \\ 0 & -5 & 16 & 4 & | & -7 \\ 0 & 0 & -76 & -84 & | & 72 \\ 0 & 0 & -42 & -48 & | & 39 \end{pmatrix} \xrightarrow{\begin{matrix} / : 4 \\ / : 3 \end{matrix}}$$

$$\sim \begin{pmatrix} 1 & 3 & -6 & 2 & | & 3 \\ 0 & -5 & 16 & 4 & | & -7 \\ 0 & 0 & -19 & -21 & | & 18 \\ 0 & 0 & -14 & -16 & | & 13 \end{pmatrix} / \cdot 19 \quad \sim \begin{pmatrix} 1 & 3 & -6 & 2 & | & 3 \\ 0 & -5 & 16 & 4 & | & -7 \\ 0 & 0 & -19^* & -21 & | & 18 \\ 0 & 0 & -266 & -304 & | & 247 \end{pmatrix} \xrightarrow{-14}$$

$$\sim \begin{pmatrix} 1 & 3 & -6 & 2 & | & 3 \\ 0 & -5 & 16 & 4 & | & -7 \\ 0 & 0 & -19 & -21 & | & 18 \\ 0 & 0 & 0 & -10 & | & -5 \end{pmatrix}$$

lépcsős alak

$$-10x_4 = -5 \quad / : (-10) \Rightarrow \underline{x_4 = \frac{1}{2}}$$

$$-19x_3 - 21x_4 = 18$$

$$-19x_3 = 18 + 21x_4 = 18 + \frac{21}{2} = \frac{36}{2} + \frac{21}{2} = \frac{57}{2}$$

$$/ : (-19) \Rightarrow$$

$$\underline{x_3 = -\frac{3}{2}}$$

$$-5x_2 + 16x_3 + 4x_4 = -7$$

$$-5x_2 = 7 - 16x_3 - 4x_4 = -7 + 16 \cdot \frac{3}{2} - \frac{4}{2} = 15$$

$$/ : (-5) \Rightarrow$$

$$\underline{x_2 = -3}$$

$$\underline{\underline{24}} \quad \underline{\underline{2}}$$

$$x_1 + 3x_2 - 6x_3 + 2x_4 = 3$$

$$\begin{aligned} \underline{\underline{x_1}} &= 3 - 3x_2 + 6x_3 - 2x_4 = 3 - 3 \cdot (-3) + 6 \cdot \left(-\frac{3}{2}\right) - 2 \cdot \frac{1}{2} = \\ &= 3 + 9 - 9 - 1 = \underline{\underline{2}} \end{aligned}$$

Tehát 1 db megoldás van:

$$x_1 = 2, \quad x_2 = -3, \quad x_3 = -\frac{3}{2}, \quad x_4 = \frac{1}{2}$$

Vektor-alakban: $(2, -3, -\frac{3}{2}, \frac{1}{2})$.