VI/6/g.) megoldása:

$$\begin{cases} a_0 = 1 \\ a_1 = 1 \\ a_n = 6a_{n-1} - 9a_{n-2}, & \text{ha } n \geq 2. \end{cases}$$

Mo.:

$$a_n - 6a_{n-1} + 9a_{n-2} = 0, \quad \text{ha } n \geq 2.$$

Legyen

$$\begin{aligned} A(x) &= a_0 + a_1 x + \cancel{a_2 x^2} + \cancel{a_3 x^3} + \cancel{a_4 x^4} + \dots \\ -6xA(x) &= -6a_0 x - \cancel{6a_1 x^2} - \cancel{6a_2 x^3} - \cancel{6a_3 x^4} - \dots \\ \oplus \quad +9x^2 A(x) &= \cancel{9a_0 x^2} + \cancel{9a_1 x^3} + \cancel{9a_2 x^4} + \dots \end{aligned}$$

$$(1-6x+9x^2)A(x) = a_0 + (a_1 - 6a_0)x = 1 - 5x.$$

$a_0 = 1$ $\quad$ $a_1 = 1$

parc. törtekre bontás (a nevező négyzet!)

$$\Rightarrow A(x) = \frac{1-5x}{1-6x+9x^2} = \frac{1-5x}{(1-3x)^2} = \frac{\alpha}{1-3x} + \frac{\beta}{(1-3x)^2}$$

$(\alpha, \beta \in \mathbb{R})$

$$\Updownarrow$$

$$1-5x = \alpha(1-3x) + \beta$$

$$1-5x = \alpha + \beta - 3\alpha x$$

$$\Updownarrow$$

$$\begin{cases} \alpha + \beta = 1 \\ -3\alpha = -5 \end{cases} \Rightarrow \alpha = \frac{5}{3}$$

$$\beta = -\frac{2}{3}.$$

Tehát $A(x) = \frac{5}{3} \cdot \frac{1}{1-3x} - \frac{2}{3} \cdot \frac{1}{(1-3x)^2}$

Mivel $\frac{1}{1-y} = \sum_{n=0}^{\infty} y^n \xRightarrow{y=3x} \frac{1}{1-3x} = \sum_{n=0}^{\infty} (3x)^n = \sum_{n=0}^{\infty} 3^n x^n.$

Mivel $\frac{1}{(1-y)^2} = \sum_{n=0}^{\infty} (n+1) y^n$ (ld. előadásjegyzet),

$$\xRightarrow{y=3x} \frac{1}{(1-3x)^2} = \sum_{n=0}^{\infty} (n+1)(3x)^n = \sum_{n=0}^{\infty} (n+1) 3^n x^n.$$

Tehát $A(x) = \frac{5}{3} \cdot \frac{1}{1-3x} - \frac{2}{3} \cdot \frac{1}{(1-3x)^2} =$

$$= \sum_{n=0}^{\infty} \underbrace{\left(\frac{5}{3} \cdot 3^n - \frac{2}{3} (n+1) 3^n \right)} x^n$$

$$\underline{\underline{a_n}} = [x^n] A = \frac{5}{3} 3^n - \frac{2}{3} (n+1) 3^n = \underline{\underline{\left(1 - \frac{2}{3} n\right) 3^n}}.$$

$\square$