<u>Feladat:</u> Oldjuk meg:

$$(\text{k.é.})\begin{cases} a_0 = 4 \\ a_1 = -1 \\ a_n = a_{n-1} + 2a_{n-2}, \quad \text{ha } n \geq 2. \end{cases}$$

<u>Megoldás:</u> $a_n - a_{n-1} - 2a_{n-2} = 0, \quad \text{ha } n \geq 2$

$$\begin{aligned}
A(x) &= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots \\
-xA(x) &= - a_0 x - a_1 x^2 - a_2 x^3 - a_3 x^4 - \dots \\
\oplus\ -2x^2 A(x) &= - 2a_0 x^2 - 2a_1 x^3 - 2a_2 x^4 - \dots
\end{aligned}$$

$$(1 - x - 2x^2)A(x) = \underset{\substack{\uparrow \\ 4}}{a_0} + (\underset{\substack{\uparrow \\ -1}}{a_1} - \underset{\substack{\uparrow \\ 4}}{a_0})x \overset{(\text{k.é.})}{=} 4 - 5x.$$

$(\alpha, \beta \in \mathbb{R})$

$$\Rightarrow A(x) = \frac{4-5x}{1-x-2x^2} = \frac{4-5x}{(1-2x)(1+x)} = \frac{\alpha}{1-2x} + \frac{\beta}{1+x}$$

$$\left\lceil\ 1 - x - 2x^2 = \left(1 - \frac{x}{2}\right)^2 - \frac{x^2}{4} - 2x^2 = \left(1 - \frac{x}{2}\right)^2 - \frac{9}{4}x^2 = \right.$$

$$\left. = \left(1 - \frac{x}{2}\right)^2 - \left(\frac{3}{2}x\right)^2 = (1-2x)(1+x)\ \right\rfloor$$

$$\Leftrightarrow \quad \alpha(1+x) + \beta(1-2x) = 4 - 5x$$

$$(\alpha + \beta) + (\alpha - 2\beta)x = 4 - 5x$$

$$\Leftrightarrow \begin{cases} \alpha + \beta = 4 \\ \alpha - 2\beta = -5 \end{cases} \Leftrightarrow \quad \begin{matrix} \beta = 3 \\ \alpha = 1 \end{matrix}$$

Tehát $A(x) = \frac{1}{1-2x} + \frac{3}{1+x} = (*)$

$$\frac{1}{1-\boxed{2x}} = \sum_{n=0}^{\infty} (2x)^n = \sum_{n=0}^{\infty} 2^n x^n$$

$$\frac{1}{1+x} = \frac{1}{1-\boxed{(-x)}} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$(*) = \sum_{n=0}^{\infty} 2^n x^n + 3 \cdot \sum_{n=0}^{\infty} (-1)^n x^n = \sum_{n=0}^{\infty} \underbrace{\left(2^n + 3 \cdot (-1)^n\right)} x^n$$

$$\boxed{a_n = 2^n + 3 \cdot (-1)^n}$$