

Name: .....

1. Find  $\chi'(G)$ , the **edge** chromatic number of the graph  $G$  in Figure 1.

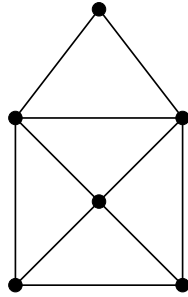


Figure 1

2. Consider the graph  $G$  in Figure 2 and the red matching  $M$  in  $G$ . Using the Hungarian method, find an augmenting path in  $G$  with respect to  $M$ , and augment  $M$  along it.

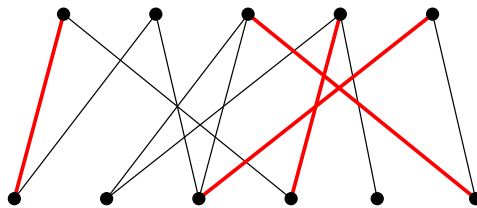


Figure 2

3. Let  $G$  be the following graph: The vertices of  $G$  are the numbers 1, 2, 3, 4, 5, 6, 7, 8 and two different vertices (numbers)  $i$  and  $j$  are adjacent in  $G$  if and only if one is divisible by the other (i.e. if  $i|j$  or  $j|i$ ). Determine the chromatic number of  $G$ .
4. Prove that every  $d$ -regular bipartite multigraph contains a perfect matching, if  $d \geq 1$ .

*Justify your answers!*