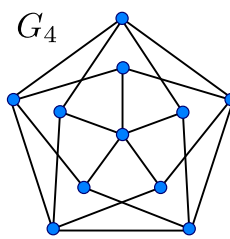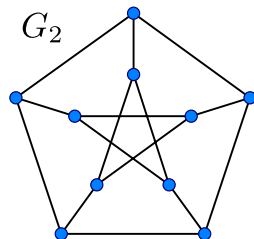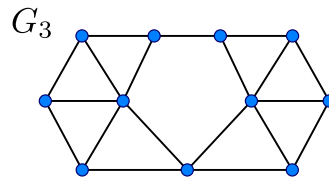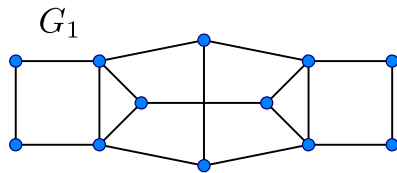## 6. Graph colorings

1. Which of the following graphs are bipartite?
  a) K_n, the complete graph on n vertices;
  b) C_n, the cycle graph on n vertices;
  c) S_n, the star graph with n edges;
  d) P_n, the path graph with n edges;
  e) The graphs in exercise 3.

2. Determine the chromatic number of the following graph.
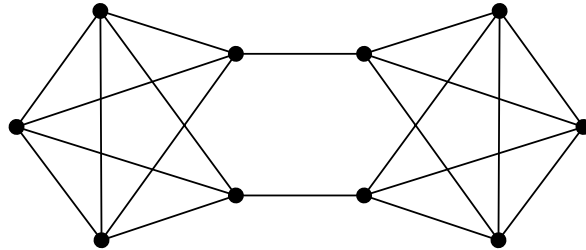
3. Determine the chromatic number of the following graphs.
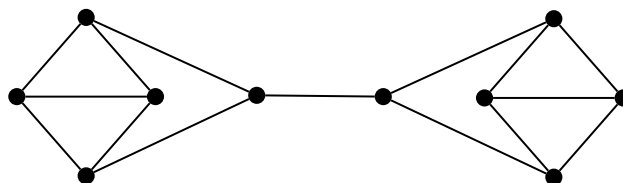

4. Let G be the following graph: The vertices of G are the squares of a chessboard, and two vertices (squares) are connected iff a king can move from one to the other with a valid move. What is the chromatic number of G?

5.$^+$ Let G be the graph on vertex set $\{1, \ldots, 100\}$ in which two different vertices (numbers) u and v are adjacent iff they are relative primes. Prove that $\chi(G) = \pi(100) + 1$, where $\pi(100)$ denotes the number of primes between 1 and 100.
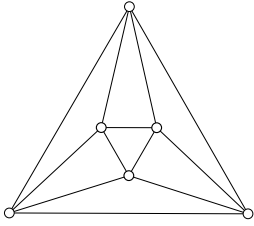
6.$^+$ Let G be the graph on vertex set $\{1, \ldots, 100\}$ in which two different vertices (numbers) are adjacent iff one is a divisor of the other. Determine $\chi(G)$

7. Determine the edge chromatic number of the following graph.

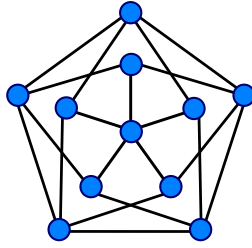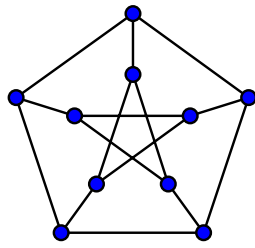**8.** Determine the edge chromatic number of the following graphs.

a)   b)   c)

d) the graph obtained the cycle C_9 by joining every pair of vertices of distance two in the cycle;

e) the complete graph K_n.