Name: .....

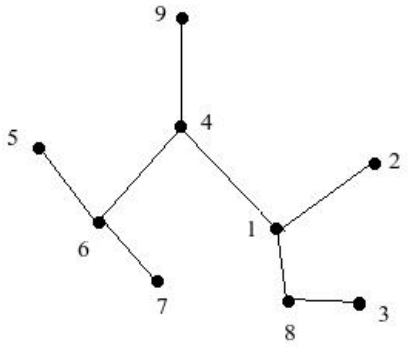
Sample exam #1

1. Counting trees

a) State Cayley's theorem about the number of labeled trees.

b) Draw all labeled trees on 3 vertices (with labels 1, 2, 3), and compare their number with Cayley's theorem.

c) Give the Prüfer code of the following labeled tree:

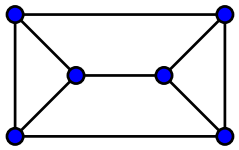


2. Edge coloring

a) What do we mean on proper edge coloring and edge chromatic number of a graph?

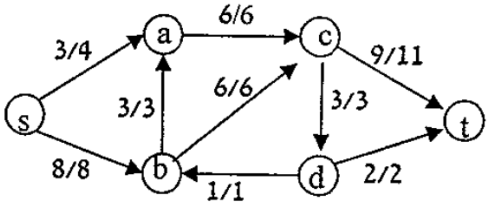
b) State Vizing's theorem.

c) Give a proper edge coloring of the following graph with 3 colors:



3. Network flows

a) Consider the network (and flow) given in the figure. The capacity of an edge is the second number on it.



Let $S = \{s, a, b\}$ and $T = \{t, c, d\}$. What is the capacity of this $\{S, T\}$-cut?

b) State the maximum flow – minimum cut theorem.

c) Let us consider an arbitrary network. Sketch (without rigorous proof) why the maximum flow value cannot exceed the capacity of an arbitrary cut. (This is the easier part of the MFMC theorem.)

4. Chinese postman problem

a) Present the chinese postman problem. (What is the setting and the optimization problem here?)

b) Consider the complete graph K_6 on 6 vertices, in which all edge lengths (weights) are defined to be 1. Solve the chinese postman problem for this edge-weighted graph. (You do not have to provide an optimal walk. You must compute its total length [cost] only.)