Name: ....

Graph theory exam #2

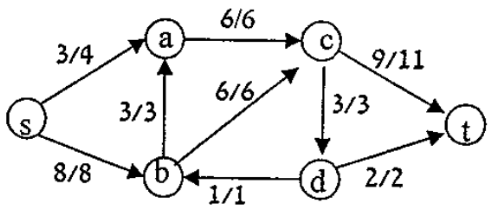
1. Degree sequences

a) When do we say that a sequence of numbers can be realized by simple graph?

b) Using Havel–Hakimi-algorithm, decide if the following sequence can be realized by simple graph or not: $6, 5, 5, 5, 4, 4, 2, 1$.

2. Network flows

a) Consider the network (and flow) given in the figure. The capacity of an edge is the second number on it.



Let $S = \{s, a, b\}$ and $T = \{t, c, d\}$. What is the capacity of this $\{S, T\}$-cut?

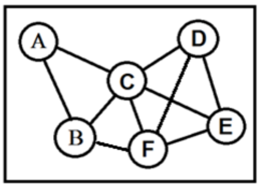
b) State the maximum flow – minimum cut theorem.

c) Let us consider an arbitrary network. Sketch (without rigorous proof) why the maximum flow value cannot exceed the capacity of an arbitrary cut. (This is the easier part of the MFMC theorem.)

3. Vertex coloring

a) What do we mean on proper vertex coloring? How is the chromatic number of a graph defined?

b) Color the vertices of the graph below by the greedy coloring algorithm, in the following order: A, B, C, D, E, F.

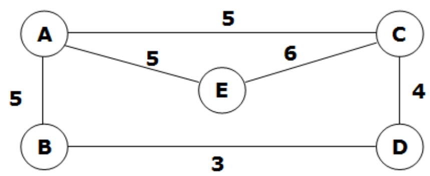


c) State the four color theorem.

4. Chinese postman problem

a) Present the chinese postman problem. (What is the setting and the optimization problem here?)

b) Consider the graph seen in the figure.



We want to take a closed walk in the graph so that we visit every edge (exactly) as many times as its label indicates (so we want to visit the edge AB exactly 5 times, we want to visit the edge BD exactly 3 times, and so on). Determine if such a closed walk exists or not. Justify your answer.