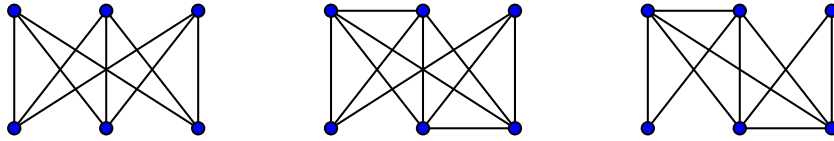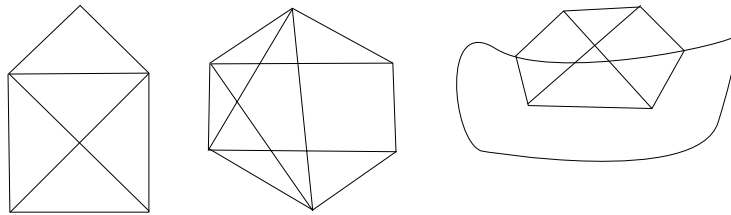## 5. Eulerian trail

1. Which of the following graphs contains an Eulerian trail or tour?

2. The graph G is defined as follows. The vertex set of G is $\{1, \ldots, 100\}$, and the numbers (vertices) i and j are adjacent if and only if $1 \leq |i - j| \leq 2$. Does G contains an Eulerian trail?

3. Which of the following figures can be drawn without lifting the pencil?

4. A connected graph G is given. Prove that there exists a closed walk in G which visits every edge of G exactly *twice*.

5. Prove that if exacty two vertices has odd degree in G, then there exists a path between these two vertices in G.

6.[+] A domino consists of two squares glued together. Every square contains the number 0, 1, 2, 3, 4, 5 or 6 (denoted by dots). Every possible unordered pair of these numbers appears on exactly one domino, so the total number of dominoes is $7 + \binom{7}{2} = 28$. Is it possible to put all the dominoes in a row so that the numbers on any two adjacent dominoes, written on their common side, coincide?