## 4. Connectivity

1. Find the connectivity numbers $\kappa(G)$ and $\lambda(G)$ of the following graphs:
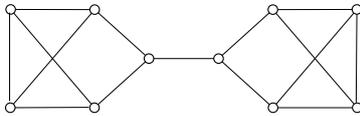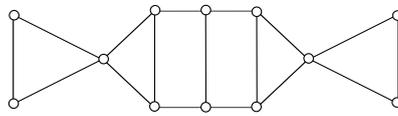
a)   b)   c)
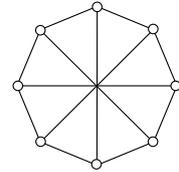
  d) Petersen graph;
  e) the complete bipartite graph $K_{m,n}$;

2. a) Prove without using Menger's theorem that every k-connected graph is also k-edge-connected.

  b) Construct a graph which is 2019-edge-connected, but not 2-connected.

3. In a k-connected graph G a vertex $s \in V(G)$ and a k-element set of vertices $T \subset V(G)$ is given, where T does not contain s. Prove that there exist k vertex-disjoint paths (apart from the initial vertex) from s to T.

4. Prove that in k-connected graph G (where $k \geq 2$), for any k vertices of G there exists a cycle containing them.