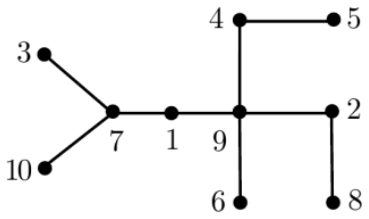
2. Enumeration of spanning trees

1.$^-$ Give the number of spanning trees of the graph
a) P_n, the path with n edges;
b) C_n, the cycle on n vertices.

2.$^-$ Among the spanning trees of K_n, how many stars and paths are?

3.$^-$ Count the number of spanning of K_n in which the fixed vertex u is a leaf. (Here $n \geq 2$.)

4. Determine the Prüfer code of the following labeled tree.



5. a) Find the labeled tree whose Prüfer code is $5, 3, 3, 3, 1, 4$.
b) Find the labeled tree whose Prüfer code is $1, 5, 1, 5, 9, 8, 2$.

6. Deduce Cayley's theorem from Kirchhoff's matrix tree theorem.

7. We leave an edge from the complete graph on $n \geq 3$ vertices. How many spanning trees does the obtained graph have?

8. Count the number of spanning of the complete biparite graph $K_{m,n}$.