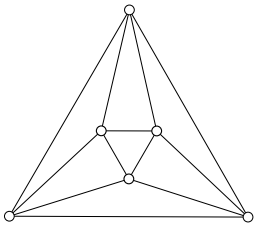
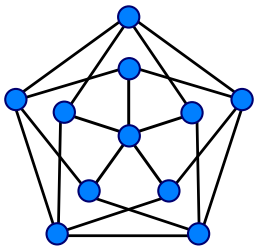
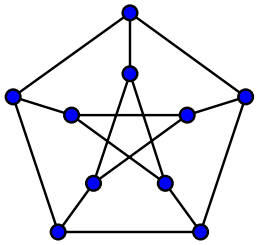
## 6. Edge coloring

**Vizing's theorem:** If $G$ is a *simple* graph, then $\chi_e(G) \leq \Delta(G) + 1$. (So for simple graphs, $\chi_e(G) = \Delta(G)$ or $\chi_e(G) = \Delta(G) + 1$.)

**1.** Find the edge chromatic number of the following graphs.

a) 

b) 

c) 

d) the graph obtained from the cycle $C_9$ by connecting every vertex to its two second neighbors on the cycle.

e) the complete graph $K_n$.

**2.** a) Show that every if a 3-regular graph have a Hamiltonian cycle, then the edge chromatic number of this graph is 3.

b) Does the Petersen graph have a Hamiltonian cycle?

**3.** $G$ is a 3-regular connected simple graph, which has an edge $e$ such that the graph $G - e$ is not connected. Prove that $\chi_e(G) = 4$.

**4.** a) Prove that the edge chromatic number of a $d$-regular *bipartite* graph is $d$.

b) Prove that if $G$ is a **bipartite graph**, then $\chi_e(G) = \Delta(G)$.

**5.**$^+$ Prove that the edge set of $K_{2n+1}$ can be partitioned into $n$ (edge sets of) Hamiltonian cycles of $K_{2n+1}$.