# Transformation groups: End–of–course examination

Dr. Gábor Nagy, Department of Geometry

April 7, 2009

In order to take part at the examination in this course you have to submit the solutions of the following exercises until Friday, May 15, 2009, 10 am, to me personally or in email attachment to `nagyg@math.u-szeged.hu`. The exam is successful if at least 50% of the questions is answered correctly. The submission may happen in pairs, no other collaboration is allowed.

**Exercise 1.** Let the group $G$ act on $X$ 2-transitively. Show that for any $x \in X$, $G_x$ is maximal in $G$. (3 points)

**Exercise 2.** Let $G$ act on $X$ and let $N$ be a regular normal subgroup of $G$. Show that for any $x \in X$, the map $n \mapsto n(x)$ defines a bijection between $N$ and $X$. Show that with respect to this bijection, the action of $G_x$ on $X$ and the action of $G_x$ on $N$ by conjugation are equivalent. (6 points)

**Exercise 3.** Let $F$ be a field, $A \in GL(n, F)$, $\boldsymbol{b} \in F^n$ and define of affine linear transformations $\boldsymbol{x} \mapsto A\boldsymbol{x} + \boldsymbol{b}$, where $\boldsymbol{x} \in F^n$. Denote by $AGL(n, F)$ the set of all the affine transformations. Show that $AGL(n, F)$ is a group which acts 2-transitively on $F^n$. (6 points)

**Exercise 4.** Let $V_1, V_2$ be subspaces over the field $F$ and $f : V_1 \to V_2$ a semilinear transformation. Show that for any subspace $W$ of $V_1$, $f(W)$ is a subspace of $V_2$. (3 points)

**Exercise 5.** Let $A$ be an $n \times n$ matrix which commutes with all elements of $GL(n, F)$. Show that $A = cI$ for some $c \in F$. (3 points)

**Exercise 6.** Show that the transvections $T_{\mu,a}$ and $T_{\nu,b}$ commute if and only if $\mu(b) = \nu(a) = 0$. (3 points)

**Exercise 7.** Let $B$ be a set of *seven* 3-element subsets of $\Omega = \{1, 2, 3, 4, 5, 6, 7\}$. Suppose that any pair of distinct elements of $B$ have at most one element in common. Show that each 2-element subset of $\Omega$ is contained in a unique element of $B$. Deduce that $\Omega$ and $B$ are the points of lines of $PG(2, 2)$. (6 points)