Nets, Webs and Squabs of Conics over Finite Fields

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The classification of "inequivalent" objects satisfying a given set of axioms, typically up to a "natural" action of some group on the set of such objects, has long fascinated mathematicians. Throughout history such problems have led to elegant results, sometimes requiring centuries to resolve.

In combinatorics these questions lie at the heart of enumeration problems. In finite geometry they appear in the study of ovals, hyperovals, unitals, arcs, ovoids, etc.; topics that have intrigued researchers for more than half a century, with origins in Dickson's work in the early 1900s. Beniamino Segre's seminal contributions and Jacques Tits' profound structural insights into incidence geometries provided a unifying framework that continues to shape the field today.

Besides incidence geometries, algebraic varieties over finite fields, such as algebraic curves, quadrics, cubic surfaces, and Veronese varieties, play a central role in understanding and classifying geometric objects, with important applications in coding theory and cryptography.

Some of these classification problems can be rephrased in terms of multilinear algebra, where tensors capture the essence of geometric configurations and equivalence is governed by group actions. This viewpoint sheds new light on classical problems and leads to surprising connections between geometry, algebra, and combinatorics.

In this talk, I will focus on recent advances on challenges involving nets, webs, and squabs of conics, based on joint work with John Sheekey, Tomasz Popiel, and Nour Alnajjarine.