On automorphism groups and p-rank of algebraic curves in positive characteristic

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In the study of algebraic curves, a fundamental problem is to determine the number of symmetries (or automorphisms) that a given curve can have. This question goes back to the nineteenth century, when significant results were obtained for curves over the complex numbers, particularly through the work of Hurwitz. Much of this theory extends to curves defined over arbitrary fields of characteristic zero, and over the past 125 years the structure and behavior of automorphism groups in this setting have been thoroughly investigated.

In positive characteristic, however, new and remarkable phenomena arise. Automorphism groups may be unexpectedly large compared to the genus of the curve, and the presence of points with stabilizers containing nontrivial p-subgroups (the so-called non-tame case) makes their analysis considerably more involved.

While the relationship between the genus of a curve and its automorphism group is relatively well understood, much less is known about the interaction between automorphism groups and another important birational invariant: the p-rank. In this talk we will delve into this topic and present some recent results in this direction.

References

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