CLASSIFYING VERTEX-TRANSITIVE GRAPHS OF ORDER A PRODUCT OF TWO DISTINCT PRIMES

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(Joint work with Ted Dobson)

In the 1990's two distinct groups of researchers worked on classifying vertex-transitive graphs of order qp, where q and p are distinct primes. This work is mainly concerned with such vertex-transitive graphs whose automorphism group contains a transitive subgroup with a normal block system. Intuitively, this means they primarily consider those graphs with automorphism group an almost simple group.

We give a refined classification when the automorphism group contains a transitive subgroup with a normal block system. It was stated in the 1990's that all such graphs are isomorphic to metacirculant graphs, and we give a classification of metacirculant graphs of order qp into disjoint families. As the isomorphism problem has been solved for metacirculant graphs of order qp, this will lead to an enumeration of vertex-transitive graphs of order qp in future work, a longstanding open problem for which several partial results have been given. Our work also holds for digraphs.

References

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