Strongly regular graphs with 2-transitive two-graphs

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A two-graph is a pair (V, T), where T is a set of unordered triples of a vertex set V, such that every (unordered) quadruple from V contains an even number of triples from T. The two-graph is called *regular* if each pair of vertices is in a constant number of triples.

Given a graph $\Gamma = (V, E)$, the set of triples T of the vertex set V, whose induced subgraph has an odd number of edges, forms a two-graph on the set V. The two-graph $\Omega(\Gamma) = (V, T)$ is the *associated two-graph* of Γ.

We study the following problem. Let $\Gamma = (V, E)$ be a nontrivial strongly regular graph with associated two-graph $\mathcal{T} = (V, T)$. Write $H = \mathrm{Aut}(\Gamma)$ and $G = \mathrm{Aut}(\mathcal{T})$. Assume that G acts 2-transitively on on V, H is transitive and maximal subgroup of G not containing the unique largest perfect subgroup. We characterize G, H and Γ. This construction would lead to a new construction for strongly regular graphs.

References

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