Polycirculant LCF codes for cubic graphs

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Every cubic Hamiltonian graph of order n can be represented by a sequence of length n with non-zero integer entries, known as the LCF code[3], which denotes the oriented spans along the Hamilton cycle. When the LCF code consists of a subsequence of length k, repeating m times it is termed a *polycirculant[4] LCF code of base k and exponent m*. We recall the following theorem:

Theorem 1. *The existence of a polycirculant LCF code with exponent $m, m > 1$ in a cubic graph is equivalent to the existence of a semi-regular automorphism of order m such that the quotient voltage graph has a Hamilton cycle with a net voltage relatively prime to m.*

We apply this result to an algorithm that generates all polycirculant LCF codes for a given graph. Characterizing cubic graphs that admit polycirculant LCF codes remains a challenging problem, even for highly symmetric graphs. For example, according to the online census of arc-transitive graphs[2], the graph F56B from the Foster census[1] is the smallest cubic arc-transitive Hamiltonian graph that does not possess a polycirculant LCF code. We present some observations based on the run of this algorithm on small generalized Petersen graphs.

References

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[3] R. Frucht. A canonical representation of trivalent Hamiltonian graphs. *J. Graph Theory*, 1:45–60, 1977.

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