

ON COMBINATORIAL STRUCTURE OF IMPRIMITIVE GRAPHS, PART I

Safet Penjić

University of Primorska (Slovenia)

Let (X, \mathcal{R}) denote a symmetric d -class association scheme on v vertices with primitive (ordinary) idempotents E_0, E_1, \dots, E_d . An association scheme is said to be *imprimitive* if some graph in the scheme is disconnected. It is well known that the following statements are equivalent:

- (i) (X, \mathcal{R}) is imprimitive;
- (ii) For some subset $\mathcal{J} = \{j_0 = 0, j_1, \dots, j_s\} \subseteq \{0, 1, \dots, d\}$ and some ordering of the vertices,

$$\sum_{h=0}^s E_{j_h} = \frac{1}{r} (I_w \otimes J_r),$$

for some integers w, r with $v = wr$ and $1 < w, r < v$.

Let Γ denote a connected k -regular graph on v vertices with adjacency algebra \mathcal{A} . In this talk, we present a combinatorial structure of Γ under the assumption that $I_w \otimes J_r \in \mathcal{A}$ for some integers w, r with $v = wr$ and $1 < w, r < v$. Some of the techniques that we use, as well as the necessary background material, can be found in [1, 2, 3].

This is work in progress and is joint work with Edwin van Dam and Giusy Monzillo.

References

- [1] M. A. Fiol and S. Penjić, On symmetric association schemes and associated quotient-polynomial graphs, *Algebr. Comb.* **4** (2021), 947–969, doi:10.5802/alco, <https://doi.org/10.5802/alco>.
- [2] G. Monzillo and S. Penjić, On commutative association schemes and associated (directed) graphs, *Electron. J. Combin.* **32** (2025), Paper No. 1.54, 38, doi:10.37236/12973, <https://doi.org/10.37236/12973>.
- [3] G. Monzillo and S. Penjić, On the combinatorial structure and algebraic characterizations of distance-regular digraphs, *Discrete Math.* **348** (2025), Paper No. 114512, 26, DOI:10.1016/j.disc.2025.114512, <https://doi.org/10.1016/j.disc.2025.114512>.