A lower bound on the minimum weight of some geometric codes

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Let $\mathcal{D}(m,q)$ be the $2-(v,q+1,1)$ design of points and lines of the m-dimensional finite projective space $\mathrm{PG}(m,q)$, where $q=p^h$ and $v=\frac{q^{m+1}-1}{q-1}$. The p-ary code $\mathcal{C}=\mathcal{C}(m,q)$ associated with this design is the $\mathbb{F}_p$-subspace generated by the incidence vectors of the lines. The dual code $\mathcal{C}^{\perp}(m,q)$ is the $\mathbb{F}_p$-subspace of vectors in $\mathbb{F}_q^v$ that are orthogonal to all vectors of $\mathcal{C}(m,q)$ with respect to the standard inner product. These are particular examples of so-called *geometric codes*.

Determining the minimum weight of $\mathcal{C}^{\perp}(m,q)$ is a difficult and challenging problem. In [1], Bagchi and Inamdar proved that the minimum weight of $\mathcal{C}^{\perp}(m,q)$ is bounded from below by

$$2\left(\frac{q^m-1}{q-1}\left(1-\frac{1}{p}\right)+\frac{1}{p}\right).$$

Such problems in coding theory can be naturally translated into questions concerning the size of sets or multi-sets of points in projective or affine spaces, with special intersection properties with respect to the lines of $\mathrm{PG}(m,q)$, as shown for instance in [2].

In this talk, using this geometric approach and exploiting properties of certain kinds of polynomials, we will present a significant improvement of the bound given in 2002 by Bagchi and Inamdar, in the case $h>1$ and $m,p>2$.

References

[1] B. Bagchi, S. P. Inamdar. Projective geometric codes. *J. Combin. Theory Ser. A*, **99**(1) (2002), 128–142.

[2] S. Ball, A. Blokhuis, A. Gács, P. Sziklai, Zs. Weiner. On linear codes whose weights and length have a common divisor. *Adv. Math.*, **211** (2007), 94–104.