The random generation of Latin rectangles based on the assignment problem

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(Joint work with G.P. Nagy)

Let A be a $k \times n$ Latin rectangle, viewed as an ordered tuple of pairwise orthogonal permutations. We study a randomized construction of such rectangles based on the classical assignment problem. Given a random cost matrix $w \in [0,1]^{n \times n}$, the Hungarian algorithm produces successive minimum-cost permutations $\alpha_1, \ldots, \alpha_k$, which form the rows of A. Each rectangle A thus corresponds to a convex polytope $P_\Gamma(A)$ in the cost space, and the probability of generating A equals $\mathrm{Vol}(P_\Gamma(A))$.

We establish structural properties of these polytopes. In particular, their volume is invariant under simultaneous column and symbol permutations (CS-equivalence), and we show that disjoint unions of bipartite graphs correspond to prism products of polytopes, implying multiplicativity of volumes. Exact volume computations for small parameters $(k,n) \in \{(4,4),(3,5),(3,6)\}$ confirm that the process is efficient but non-uniform, giving a negative answer to a problem posed in 2009 and listed in the online compilation of open problems in loop theory and quasigroup theory [2]. For instance, among the CS-classes of Latin squares of order four, the probabilities differ by a factor of more than three.

References

[1] F. Iftikhar, G. P. Nagy, *The random generation of Latin rectangles based on the assignment problem*, Discrete Appl. Math. 378 (2026), 329–336.

[2] Wikipedia contributors, *List of problems in loop theory and quasigroup theory — Wikipedia, The Free Encyclopedia*, `https://en.wikipedia.org/w/index.php?title=List_of_problems_in_loop_theory_and_quasigroup_theory&oldid=1277495759`, [Online; accessed 6-May-2025].