Generalizing two families of scattered quadrinomials in $\mathbb{F}_{q^{2t}}[X]$

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The $\mathbb{F}_{q^{2t}}$-linearized quadrinomial

$$\psi_{m,h,s} = m(x^{q^s} - h^{1-q^{s(t+1)}}x^{q^{s(t+1)}}) + x^{q^{s(t-1)}} + h^{1-q^{s(2t-1)}}x^{q^{s(2t-1)}} \tag{1}$$

has been the subject of intense study and has been generalized through several stages. This is shown to be scattered under the following hypotheses:

- (a) for $m = 1$ and $h \in \mathbb{F}_{q^{2t}}$ with $\mathrm{N}_{q^{2t}/q^t}(h) = -1$, see [1, 2, 3, 4, 6].
- (c) for $h \in \mathbb{F}_q$ and $m \in \mathbb{F}_{q^t}$ such that it is neither a $(q+1)$-th nor $(q-1)$-th power of an element beloging to $\ker \mathrm{Tr}_{q^{2t}/q^t}$, see [5].

In this talk, we will provide some sufficient conditions for the polynomial in (1) to be scattered. These will include and generalize those above obtained in previous works. Moreover, we highlight the relation with linear sets of the projective line and rank distance codes.

References

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