The many faces of the Möbius–Kantor configuration

Gábor Gévay

University of Szeged (Hungary)

(Joint work with Tomaž Pisanski)

The Möbius–Kantor configuration is a unique (8_3) combinatorial configuration that was described by Möbius in 1828; he proved that it cannot be realized geometrically with points and lines in the real Euclidean plane; Kantor described it in 1881 as a geometric point-line configuration in the complex plane [4].

Based on a simple elementary geometry theorem, we have proven that it has a geometric realization in the real Euclidean plane such that its blocks are equilateral triangles. Moreover, the realization with triangles expresses a configuration theorem which states that if any seven triangles are equilateral, then the last, eighth triangle is also equilateral; thus, this theorem follows the logical pattern of the classical configuration theorems (like e.g. the Pappus or Desargues thereom).

We found that several consequences make this realization particularly interesting. To mention some of them, it can be lifted to the real Euclidean 3-space, which leads to a novel, highly self-intersecting polyhedron which is topologically equivalent to the double torus. Moreover, this polyhedron is homeomorphic to a semiregular map which can be obtained by a specific truncation from the dual of the regular map R2.1 (using Conder's notation [1]). This is consistent with the fact that the Möbius–Kantor graph (the incidence graph of the configuration) can be embedded in the double torus [2, 3].

Our new realization of the configuration MK(8_3) also leads to an interesting geometric representation of the Möbius–Kantor graph. This representation may serve as a starting example for defining a novel family of geometric graphs.

References

[1] M. D. E. Conder, All regular orientable maps on surfaces of genus 2 to 101, `http://www.math.auckland.ac.nz/~conder`

[2] H. S. M. Coxeter and W. O. J. Moser, Generators and Relations for Discrete Groups, Springer-Verlag, Berlin - Heidelberg, 1980.

[3] D. Marušič and T. Pisanski, The remarkable generalized Petersen graph, *Math. Slovaca*, **50** (2000), 117–121.

[4] T. Pisanski and B. Servatius, *Configurations from a Graphical Viewpoint*, Birkhäuser Advanced Texts, Birkhäuser, New York, 2013.