Curves with a large automorphism group admitting a cyclic subgroup of index 2

Arianna Dionigi

University of Florence (Italy)

(Joint work with Massimo Giulietti and Marco Timpanella)

The Hurwitz bound on the order of the $\mathbb{K}$-automorphism group $\mathrm{Aut}(\mathcal{X})$ of an algebraic curve $\mathcal{X}$ of genus $g(\mathcal{X}) \geqslant 2$ defined over a field $\mathbb{K}$ of zero characteristic states that $|\mathrm{Aut}(\mathcal{X})| \leqslant 84(g(\mathcal{X}) - 1)$. Improved bounds are available for the order of certain types of subgroups within automorphism groups. For instance, if a subgroup G of $\mathrm{Aut}(\mathcal{X})$ is dihedral, then in the complex case, $|G| \leqslant 4g(\mathcal{X}) + 4$. More recently it has been shown that a tighter bound holds for G a generalized quasi-dihedral group. In this paper we explore the more general setting of a curve defined of field of any characteristic, and G a group admitting a cyclic subgroup of index two and order coprime to the characteristic of the ground field. We first prove that many classical results about dihedral groups extend from the complex case to the case of any characteristic p. Then we provide some new results about non-dihedral groups admitting a cyclic subgroup of index 2.

References

[1] A. Dionigi, M. Giulietti and M. Timpanella *Algebraic curves with a large cyclic automorphism group*, submitted, arXiv:2410.13590.