On the graph and the set of determined directions of functions over $\mathrm{GF}(q)$

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Let q be a power of a prime p, and let f be a function from $\mathrm{GF}(q)$ to $\mathrm{GF}(q)$. The graph of f is the set of points in the affine plane $\mathrm{AG}(2,q)$ of the form $(x, f(x))$, where x ranges over $\mathrm{GF}(q)$. We will denote this point set by U_f. The directions determined by the graph of f are the points at infinity corresponding to the slopes of lines connecting pairs of points of the graph. We will denote this point set by D_f.

In this talk, we will show how properties of D_f yield information about f. To be more precise, we will use some new ideas and some old results (due to Carlitz, McConnel; Ball, Blokhuis, Brouwer, Storme, Szőnyi) to prove the following conjecture of Sziklai (which extends a result of McGuire and Göloğlu):

Theorem 1. *Let M denote a multiplicative subgroup of $\mathrm{GF}(q)$ and let f denote a function from $\mathrm{GF}(q)$ to $\mathrm{GF}(q)$. If $f(0) = 0$, $f(1) = 1$ and $D_f \subseteq M \cup \{0\}$, then f is an automorphism of $\mathrm{GF}(q)$.*

Then we will explain how U_f and D_f can be used to construct the smallest (known) t-fold blocking sets ($t = 1, 2, 3$) of $\mathrm{PG}(2, q)$, that is, point sets meeting every line in at least t points. During the 2025 Budapest Research Experience for Undergraduates (REU) Math Program, together with M.R. Kepes, E. Robin, B. Sógor, S. Wang, E. Williams, we proved the following:

Theorem 2. *In $\mathrm{PG}(2, q^h)$, $h > 1$, there exist t-fold blocking sets of size $t(q^h + q^{h-1} + 1)$ for $t = 2, 3$.*

References

[1] B. Csajbók: Extending a result of Carlitz and McConnel to polynomials which are not permutations, Finite Fields Appl. 108 (2025) 102683.