Terwilliger algebra of a graph

Abstract

In algebraic combinatorics, the following situation occurs often. Let Γ be a combinatorial object and let H be a certain algebraic object, associated with Γ. In this case, one of the main motivations in our research is the following question: what could we say about the combinatorial properties of Γ, if we know that H has certain algebraic properties? And vice-versa: what could we say about the algebraic properties of H, if we know that Γ has certain combinatorial properties?

Perhaps the most well-known example of this interplay between combinatorics and algebra is obtained if H is the automorphism group of a graph Γ. In this case there are many relations between combinatorial properties of Γ and algebraic properties of H. For example, if H acts transitively on the set of vertices of Γ, then Γ is regular (in a sense that every vertex of Γ has the same number of neighbours). If we further know that the stabilizer H_x of a vertex x has exactly three orbits, then Γ is strongly regular. There are other examples of this interplay available in the literature.

In this talk the algebraic object, associated with Γ, will not be its automorphism group, but rather a certain matrix algebra, called a *Terwilliger algebra of a graph* Γ. The main motivation, however, remains the same: what could we say about the combinatorial properties of Γ, if we know that its Terwilliger algebra has certain algebraic properties? And vice-versa: what could we say about the algebraic properties of the Terwilliger algebra of Γ, if we know that Γ has certain combinatorial properties?