

# MRD Codes, Shortest Minimal Codes, and Intersecting Codes from Scattered Subspaces

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Let  $\Lambda = \text{PG}(V, \mathbb{F}_{q^n}) = \text{PG}(r - 1, q^n)$ , where  $V$  is a vector space of dimension  $r$  over  $\mathbb{F}_{q^n}$ , and let  $L$  be a subset of points of  $\Lambda$ . The set  $L$  is called an  $\mathbb{F}_q$ -linear set of rank  $k$  if it is defined by the nonzero vectors of an  $\mathbb{F}_q$ -subspace  $U$  of  $V$  of dimension  $k$ . Such a linear set is said to be *scattered* (and  $U$  is called  *$\mathbb{F}_q$ -scattered subspace*) if it has maximum possible size, namely  $q^{k-1} + q^{k-2} + \dots + q + 1$ . Scattered subspaces are of central interest in Galois Geometry due to their strong connections with rank-metric codes.

In this talk, I will present recent constructions of scattered subspaces that give rise to new families of maximum rank distance (MRD) codes [2] and to shortest minimal rank-metric codes [3]. Furthermore, taking inspiration from intersecting codes in the Hamming metric, I will introduce the notion of rank-metric intersecting codes [1]. These are codes in which any two nonzero codewords have nontrivially intersecting supports. We will explore their structure from both geometric and coding-theoretic viewpoints, establishing connections with MRD codes, minimal codes, and 2-spannable  $q$ -systems. Structural properties, parameter bounds, and explicit constructions will also be discussed. The talk will conclude with a selection of open problems and directions for future research.

## References

- [1] D. BARTOLI, M. BORELLO, G. MARINO AND M. SCOTTI: Linear rank-metric intersecting codes, arXiv:2507.00569.
- [2] D. BARTOLI, G. MARINO AND A. NERI: New MRD codes from linear cutting blocking sets, *Annali di Matematica Pura ed Applicata* **202** (2023), 115–142.
- [3] S. LIA, G. LONGOBARDI, G. MARINO AND R. TROMBETTI: Short rank-metric codes and scattered subspaces, *SIAM Journal on Discrete Mathematics*, **38(4)** (2024), 2578–2598.

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