

MRD Codes, Shortest Minimal Codes, and Intersecting Codes from Scattered Subspaces

Giuseppe Marino

University of Naples Federico II

Let $\Lambda = \text{PG}(V, \mathbb{F}_{q^n}) = \text{PG}(r-1, q^n)$, where V is a vector space of dimension r over \mathbb{F}_{q^n} , and let L be a subset of points of Λ . The set L is called an \mathbb{F}_q -linear set of rank k if it is defined by the nonzero vectors of an \mathbb{F}_q -subspace U of V of dimension k . Such a linear set is said to be *scattered* (and U is called *\mathbb{F}_q -scattered subspace*) if it has maximum possible size, namely $q^{k-1} + q^{k-2} + \dots + q + 1$. Scattered subspaces are of central interest in Galois Geometry due to their strong connections with rank-metric codes.

In this talk, I will present recent constructions of scattered subspaces that give rise to new families of maximum rank distance (MRD) codes [2] and to shortest minimal rank-metric codes [3]. Furthermore, taking inspiration from intersecting codes in the Hamming metric, I will introduce the notion of rank-metric intersecting codes [1]. These are codes in which any two nonzero codewords have nontrivially intersecting supports. We will explore their structure from both geometric and coding-theoretic viewpoints, establishing connections with MRD codes, minimal codes, and 2-spannable q -systems. Structural properties, parameter bounds, and explicit constructions will also be discussed. The talk will conclude with a selection of open problems and directions for future research.

References

- [1] D. BARTOLI, M. BORELLO, G. MARINO AND M. SCOTTI: Linear rank-metric intersecting codes, arXiv:2507.00569.
- [2] D. BARTOLI, G. MARINO AND A. NERI: New MRD codes from linear cutting blocking sets, *Annali di Matematica Pura ed Applicata* **202** (2023), 115–142.
- [3] S. LIA, G. LONGOBARDI, G. MARINO AND R. TROMBETTI: Short rank-metric codes and scattered subspaces, *SIAM Journal on Discrete Mathematics*, **38**(4) (2024), 2578–2598.

University of Naples Federico II, Dipartimento di Matematica e Applicazioni "Renato Caccioppoli", Via Cintia, Monte S. Angelo I-80126 Naples, Italy
e-mail: giuseppe.marino@unina.it